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Image Compression Reimagined through Frequency domain to Wavelet Trees

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Abstract

As the demand for efficient image storage and transmission grows across domains such as medical imaging, web development, and satellite communication, image compression has become a critical focus in digital media processing. This paper explores fundamental image compression principles and evaluates two prominent approaches: Fast Fourier Transform (FFT) and wavelet-based techniques, including Haar and Set Partitioning in Hierarchical Trees (SPIHT). While FFT leverages global frequency analysis for high-speed compression with moderate detail preservation, wavelet transforms provide superior localization in both time and frequency domains, allowing for better edge preservation and scalability. Experimental comparisons using grayscale images reveal that SPIHT achieves higher Peak Signal-to-Noise Ratio (PSNR) and compression ratio, whereas Haar excels in speed and simplicity. The findings emphasize the complementary nature of these techniques and highlight emerging hybrid approaches that combine FFT's efficiency with the adaptive precision of wavelets to meet diverse compression needs.

Keywords: Image compression, Wavelet, Haar, SPIHIT

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Introduction

As digital communication and multimedia technologies continue to grow, the demand for efficient image compression has become increasingly important. From medical imaging and web applications to satellite data and mobile photography, the ability to store and transmit images quickly and efficiently is essential. Most natural images contain a high degree of redundancy due to strong correlations between neighboring pixels [1]. Image compression reduces this redundancy by transforming the image into a more compact representation that retains essential information while discarding irrelevant or repetitive data. It results a significant reduction in file size with minimal impact on visual quality.

Compression methods are generally divided into two categories[2]: lossless method preserves the original data exactly, whereas lossy approaches allow some degradation in image quality in exchange for significantly higher compression ratios where perceptual quality over perfect reconstruction is preferred[3]. Lossless compression is critical in fields like medical imaging and legal documentation, where every pixel must be preserved exactly. In contrast, lossy compression is better suited to multimedia and web applications, where minor differences from the original are acceptable in exchange for smaller file sizes.

A standard image compression process typically involves three key stages [4]: transformation to convert image data into a more compressible form, quantization to reduce precision and eliminate less significant information, and encoding to represent the remaining data efficiently.

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Image Compression Techniques

In lossy techniques, the term "noise" refers to small reconstruction errors introduced during stages like quantization and thresholding [5]. These discrepancies are typically imperceptible but contribute to higher compression efficiency. As digital data continues to grow in size and complexity, striking the right balance between compression ratio, computational cost, and visual quality remains a key research challenge. A range of mathematical tools and algorithms have been developed to address this [4, 6-7], one of the most widely used being the Fast Fourier Transform (FFT).

FFT in Image Compression

Data The Fast Fourier Transform (FFT) is a powerful technique for image compression, particularly effective when maintaining the image's overall frequency structure is more important than preserving local detail. FFT is an efficient implementation of the Discrete Fourier Transform (DFT)[8,9], which converts spatial image data into the frequency domain using sinusoidal basis functions. In the frequency domain, most of an image's essential information is concentrated in low-frequency components. These components correspond to broader, perceptually significant features of the image. High-frequency components, which often represent fine textures or noise, can be reduced or removed without noticeably affecting quality. This makes FFT especially suitable for lossy compression, where the aim is to minimize file size while retaining acceptable visual fidelity [10]. By discarding less important frequency data and encoding the rest efficiently, FFT-based compression reduces storage and bandwidth requirements with relatively low computational overhead. A typical workflow of FFT based image compression process has depicted in Figure. 1.

One of FFT's key strengths is its ability to extract meaningful frequency features [11]. After transformation, low-frequency components are positioned centrally, while high-frequency components appear near the edges of the spectrum. This structure supports intuitive filtering and feature selection during compression. However, FFT comes with important limitations. Its use of global sinusoidal basis functions reduces spatial localization, resulting less effective for images with sharp edges or localized features. Additionally, FFT assumes signal periodicity, which can lead to artifacts at image boundaries unless special techniques are used to mitigate discontinuities.

For 2D images, FFT is applied in two stages: a 1D FFT across rows followed by a 1D FFT across columns. The 2D Discrete Fourier Transform of an image *f*(x,y) with size MxN is mathematically represented as:

$$F(u,v)=\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{j2\pi(\frac{u}{M}+\frac{y}{N})} \qquad (1)$$

Where *F*(u,v) is the complex frequency coefficients, and (u, v) denote the frequency indices. The FFT reduces the computational complexity of this operation from $O(N^2)$ to $O(N\log N)$, making it well-suited for real-time image compression applications. This separable structure is computationally efficient but assumes orthogonality in the transform, which may restrict its adaptability in some compression contexts.

Despite these drawbacks, FFT has seen wide adoption in early image compression standards like JPEG [12, 13] and remains valuable in applications where global frequency characteristics are more important than fine spatial detail. Its ability to rapidly analyze an image's frequency content also makes it useful in hybrid systems, where FFT is combined with more localized methods like, wavelet transforms, to improve overall performance.

Wavelet Transform in Image Compression

Wavelet transforms have become a cornerstone of modern image compression due to their ability to localize image features in both spatial and frequency domains [14]. Unlike global transforms such as the FFT, wavelets use scale-adaptive, localized basis functions, making them highly effective for compressing images with fine details. This localized analysis enables sparse signal representation,

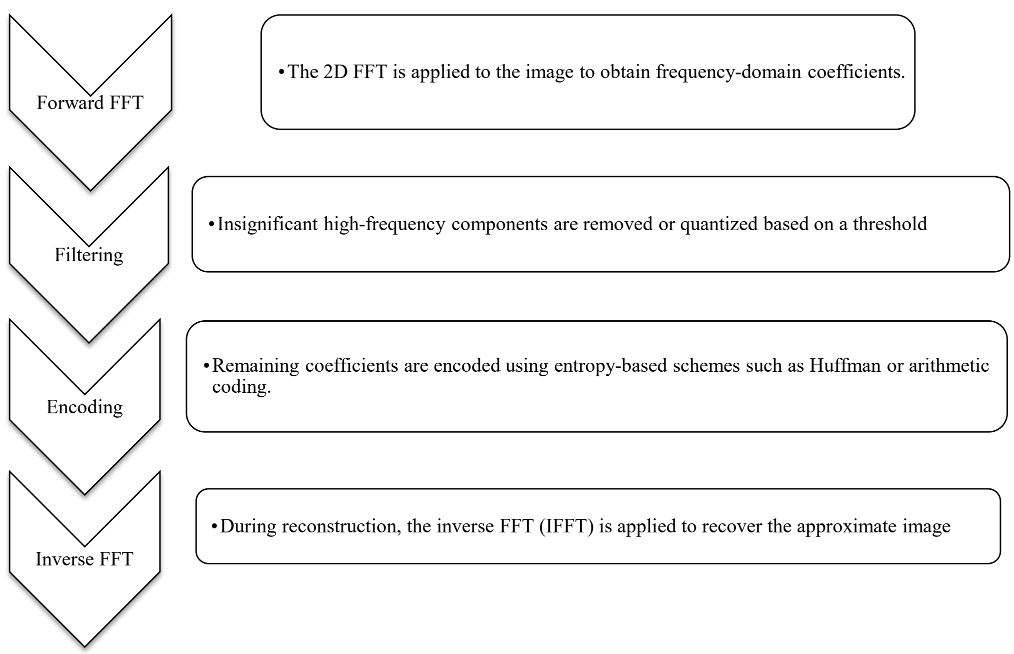

Figure 1: Standard workflow of FFT-based image compression

improving compression performance while also benefiting tasks like denoising and edge detection. Wavelet transform is initiated and developed on the basis of Fourier transform [15]. A typical wavelet based image compression process can be described as:

Transformation

The compression process begins by transforming the image from the spatial domain into a frequency representation by the Discrete Wavelet Transform (DWT). It decomposes the image into approximation (low-frequency) and detail (high-frequency) sub-bands [6, 16]. Most of the image's perceptual energy is concentrated in the approximation sub-band, while less significant high-frequency components can be selectively discarded or quantized. Discrete Cosine Transform (DCT) and FFT are widely used in standards like JPEG whereas; DWT offers improved multiresolution analysis and better localization, making it more suitable for modern compression needs [17].

Quantization and Thresholding

Following transformation, quantization reduces the precision of wavelet coefficients, while thresholding removes values below a set magnitude. This step introduces the primary loss in lossy compression but allows for substantial data reduction with minimal impact on perceived quality.

Encoding

The final stage applies entropy coding, such as Huffman or arithmetic coding, to efficiently represent the quantized data by removing statistical redundancy. This reduces file size further without introducing additional distortion.

A block diagram of the process and its reverse are shown in Figure 2.

Advanced Techniques: Haar and SPIHT

The Haar Wavelet, one of the simplest DWTs, recursively splits the image into average and difference components [18]. It is computationally efficient and effective in real-time applications, providing good Peak Signal-to-Noise Ratio (PSNR) and minimal edge distortion.

The quality of compression can be quantitatively assessed using PSNR, which is derived from Mean Squared Error (MSE):

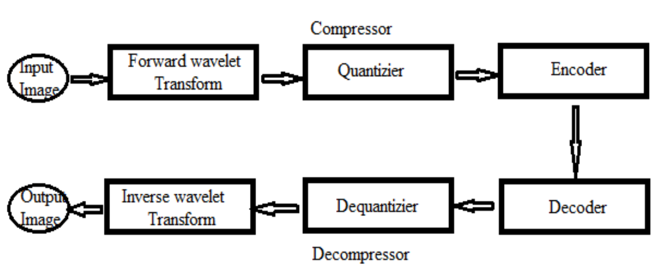

Figure 2: Block diagram of a typical image compression system

$$PSNR = 10 \log_{10} \frac{f_{max}^2}{MSE} \tag{2}$$

where f_{max} is the maximum pixel intensity, and MSE is the Mean Square Error between the original and reconstructed image. MSE can be determined by the relation

$$MSE = \frac{\sqrt{\sum \{f(x,y,z) - f_c(x,y,z)\}^2}}{N} \tag{3}$$

where N is total number of pixels, f(x,y,z) is the original image, and f_c(x,y,z) is the decompressed image.

The Set Partitioning in Hierarchical Trees (SPIHT) algorithm builds on DWT's strengths by encoding wavelet coefficients using a hierarchical tree structure that tracks relationships across scales [19]. It produces an embedded bitstream that supports both progressive transmission and scalable quality. SPIHT is suitable for both lossy and lossless compression, achieving high PSNR and compression ratios. Here compression ratio is defined by the ration between original and compressed image size and plays a crucial role in computational effort. Wavelet-based methods, particularly Haar and SPIHT, are the foundation of modern standards like JPEG2000. These methods outperform traditional techniques, especially for images with high spatial complexity [18].

Result And Discussion

A comparative analysis between Haar and SPIHIT techniques has been performed to determine their applicability here. When applied to a test grayscale image [20], both Haar and SPIHT demonstrated strong compression performance as shown in Figure 3. The compressed image and its quality, measured by the PSNR are detailed in Table 1.

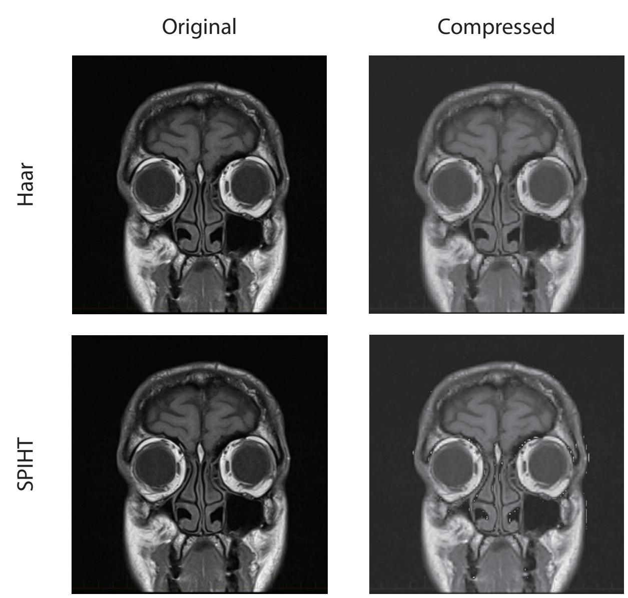

Figure 3: Different wavelet applications to compress an image

Table 1: A comparison chart between Haar and SPIHT wavelet based image compression parameters

Technique	*PSNR (dB)*	*Compression ratio*
Haar	32.3	6.1:1
SPIHT	34.7	7.4:1

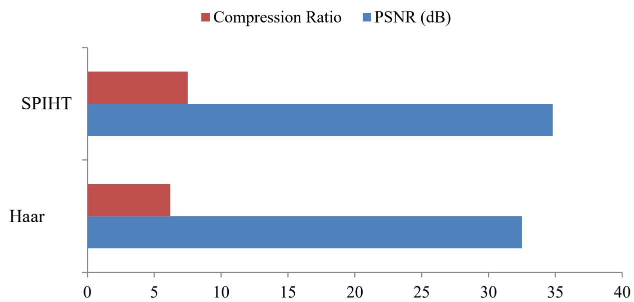

Figure 4: PSNR and Compression Ratio comparison

In both cases the number of decomposition levels, which determines the level of detail in the compressed image are kept at 4 and the number of bits used to represent the coefficients after compression, which affects the quality of the compressed image are at 11. While SPIHT achieved higher PSNR and compression ratio, Haar excelled in speed, making it well-suited for real-time applications. Visually, both methods preserved acceptable image quality, with SPIHT offering finer detail reproduction. The result is displayed graphically in Figure 4.

While wavelets have become more popular for their adaptability and localized analysis, FFT continues to play a foundational role in image compression research. The rise of hybrid techniques that integrate FFT with wavelet-based methods reflects a broader trend toward leveraging the strengths of both global and local transformations for improved compression efficiency.

Conclusion

Image compression plays a vital role in how we store and share visuals efficiently, especially in a world overloaded with digital content. This study looked at two popular ways to shrink image sizes namely, the Fast Fourier Transform (FFT) and wavelet-based techniques like Haar and SPIHT. Each with their own strengths and weakness are discussed here. FFT stands out for being fast and straightforward, making it a good fit when processing speed is a priority and fine details aren't as critical. However, it tends to struggle with preserving sharp edges or local patterns in images. On the other hand, wavelet-based methods offer more flexibility. They do a better job at handling details and textures, especially in images with lots of complexity. SPIHT, in particular, produced the best balance between quality and file size in our tests, while Haar proved useful when faster performance was needed.

Overall, there's no one-size-fits-all solution. The choice between these methods depends on the specific needs. A balanced technique should be chosen in terms of preserving quality, saving space, or keeping things fast. In many cases, combining techniques may provide the best of both worlds, and that's where the future of image compression seems to be heading.

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