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# Modeling of Transport Problem in Linear Programming with Python (PULP)

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## Abstract

Integer programming is a framework for rapid conversion and global optimum. The basic issue in transportation problem is to minimize the cost or time (distance) of delivery. There are two things in our consideration, one is cost of transportation and other demand or how much can be supply. This paper will discuss the transportation problem and apply the zero-one integer assignment to reduce the complexity of the transportation solving concept. The paper will develop the concept to solve the transportation problem and apply the zero-one integer to find the optimum cost of transportation. This paper convert transportation problem in integer programming than solve with machine learning. The paper will present a model of transportation problem solution with the PULP software. We will introduce the programming in Python to make a model to minimize the cost (time or distance) in any transportation problem and Mathematical algorithm of zero-one Transportation programming to solve these types of problems.

**Keywords:** Zero-One programming, Transportation, Integer programming, Minimization, Python, PULP, Algorithm.

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## Introduction

A rough integer linear programming (FRILP) algorithm is presented by Ammar.[1] They found optimal solution variables in the rough intervals. They used the slice-sum and branch and bound method for construct upper approximation interval and lower approximation interval. The rough intervals are useful new tools to tackle the uncertainty, vague and imprecise data in decision-making problems. Integer programming has affected many innovation and operations research models, according to Fred.[2] Some of these innovations may be linked with Artificial intelligence. Many approaches of the heuristic procedure will increase way to more possibilities for future developments.[3] Discussed the three equation Roseboro's parabolic valley, Powell's quartic function, fletch and Powell hellica. Integer programming mathematical modeling of traveling sales men problems was provided by.[4] They have developed several such efficient models in terms of generality, number of variables and constraints. A salesman must visit n cities and leave the base city mark 0 and visit n cities then return to 0. According to them

Minimize the linear form $\sum_{0\le i\ne j\le n}^{n} d_{ij}x_{ij}$. over the set determinedbytherelation[5] $\sum_{\substack{i=0\\ i\ne j}}^{n} x_{ij}=1$ $u_i - u_j + px_{ij} \le p-1\ (1\le i\ne j\le n)$ discussed some integer programming models with only two constraints and found solution with simplex method. They correlated the analysis of models with operations research and convert problems in linear integer programming.[6] Presented clipping method that was modified method for unpromising variants to solve the integer linear programming with Boolean variables. It can improve and reduce the complexity of calculation in integer linear programming. This proposed work can be used in different integer linear programming tasks interpreted as problems. The proposed method can reduce time for the solution and reduce the algorithmic complexity.[7] Discussed the material dispatch problem for Albert David Company to optimize transportation cost in all over India. They solved the transportation problem with the help of dual simplex and two-phase method. They found the solution using Tora software and compared the obtained optimal solution with the Vogel approximation method.[8] Suggested to use inter programming for solve the issues of economy management, communication and engineering. They used integer programming model and branch and bound method to study the project investment problem. Integer programming can also be used for a variety of assignment problem in real-life applications. In real-life

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problems X task to be complete by Y individuals, each person also has the different characteristics and efficiency to complete various tasks about to use integer programming for personnel assigned. In combinatorial problems in real life, the method of zero-one integer programming is used to find the optimal solution in set of feasible solutions.[9] Presented mixed integer linear programming is a framework for rapid conversion and global optimum. They presented a model and method which form the fundamentals of process integration, restriction and problem formulation. Their study included capabilities and results for simple, fictional frameworks and applicable across scale, time and plant complexity. The method deal with material and energies of different types and Incorporates pinch analysis fundamentals to obey the second law of thermodynamics.[10] Showed the planning sequence of actions in each network to make a feasible route. They presented a number of integer programming formulations that have different degree of flexibility. The problem has many ordering constraints so implemented branch and cut algorithm in which constraints are dynamically generated. Their results improved previous planning of integer programming approaches and cost optimal planning. They presented a series of IP formulations that represent the planning problem as a set of loosely coupled network flow problems and network flow problem corresponds to one of the state variables in the planning domain.[11] Worked about solve problems with thousands of integer variables on a personal computer and obtain approximate solution with millions of binary variables. Their worked solve the linear inequality and equality constraints where some or all of the variables are required to be integral. The model

$$Max\ z = cx$$

$$Ax \leq b$$

$$l \leq x \leq u$$

$$x_j\ integral, j = 1 \dots \dots \dots . p$$

Present a mixed integer programming (MIP). They assumed $1 \leq p \leq n$ the problem is a linear program (LP) and if $p=n$, it convert in pure integer program (PIP). A PIP containing all variables that have to be 0 or 1 is called a binary Integer program (BIP). A MIP in which all integer variables are 0 or 1 is called a mixed binary integer program (MBIP). These will discussed discrete optimization, including constraint programming, Langrangian duality, basis reduction simulated TABU search and genetic algorithms.[12] Presented a general framework for mining constraints and consider structured output as an integer linear programming (ILP) problem. They verified the proposed constraint mining algorithm in synthetic and real world problems. Their technique can solve 9*9 Sudoku puzzles and minimal spanning tree problems. Their algorithm can also integrate with a neural network model to learn the hierarchical label structure of a multi-label classification task. The framework is general and is able to identify underlying constraints in structured prediction problems. They formulated ILP constraints mining and focused on foundation for this potential area. [13] Presented a complete review of deterministic mix integer linear program (MILP) non-linear program solution methods. A mixed integer programming problem is used for decision where some constrained decision variables are non-integer values. MILP can use to find the minimum and maximum of the non-linear objective function with linear constraints. Minimization and maximization of a non-linear objective function subject to non-linear constraints with continuous and discrete variables are performed by MINLP solution methods. They described MILP and MINLP solution methods with methodologies, algorithms, software and solver. Different types of MILP and MINLP methods were introduced with software to solve the real time problems in engineering and science.[14] Discussed the problems in computer programming and integer programming. They found

- Feasible values are not found in usual linear economic function.
- The values depended on integer parameters.

For resolve these issues they suggested a classical simplex algorithm. In this method not difficulty in lexicographic order. This algorithm gave the solution of continuous parametric problems.

They implemented an algorithm and found the semantics analysis of computer programs. The problem table is in unit matrix and then reduced complexity by apply the factor $\frac{m}{(n+m)}$ as the method was called revised form of simplex algorithm. The termination of the algorithm depended on integers and non-integers to avoid rounding errors.[15] Focused on the two ways to care of the TSP issue of a book shop. They examined after effect of Hungarian strategy hand approach with AMPL. They presented a model of books retailer to visit the five urban areas to satisfy the interest.[16] Developed a fixed point iterative method that satisfied mapping properties in Integer programming. They implied that the uniqueness of Tarski's fixed point is an NP-hard problem, and compute all integer or mixed-integer points in a poly tope and extended to convex non-linear integer programming. The comparative analysis of weak and strong integer programming formulations for the traveling salesman problem using commercial IP software and a short MATLAB code have done by.[17] In this article, we used the zero- one integer algorithm for the first time to solve the transportation problem. It reduces the complexity of assign the quantity according to the demand. We introduce the programming with PULP to find a way to solve the transportation problem.

# Materials and Methods

IF the variables of LPP can be restricted with the values zero and one then LPP can be say Zero-One Programming Problem. This type of programming can be applied in various method of L.P.P. It can also apply for fixed charge problems, traveling sales men problems, etc. Here we will generate a model to solve the transportation problem with zero-

**Table 1:** Zero-One transportation matrix

| *Factory* | $W_1$ | $W_2$ | …….. | $W_j$ | ……… | $W_n$ | *Capacities* |
|---|---|---|---|---|---|---|---|
| $F_1$ | $c_{11}x_{11}$ (1) | $c_{12}x_{12}$(1) | | $c_{1j}x_{1j}$ (0) | | $c_{1n}x_{1n}$ (0) | $a_1$ |
| $F_2$ | $c_{21}x_{21}$ (0) | $c_{22}x_{22}$(1) | | $c_{2j}x_{2j}$ (1) | | $c_{2n}x_{2n}$ (0) | $a_2$ |
| ….. | ….. | ….. | …. | …… | …. | ….. | |
| $F_i$ | $c_{i1}x_{i1}$(0) | $c_{i2}x_{i2}$(0) | ….. | $c_{ii}x_{ii}$ (1) | …. | $c_{in}x_{in}$ (0) | $a_i$ |
| $F_m$ | $c_{m1}x_{m1}$ (0) | $c_{m2}x_{m2}$(0) | ….. | $c_{mi}x_{mi}$(0) | …. | $c_{mn}x_{mn}$(1) | $a_m$ |
| Requirement | $b_1$ | $b_2$ | ….. | $b_j$ | ….. | $b_n$ | $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ |

**Table 2:** Allotment matrix

| *Factory* | $W_1$ | $W_2$ | …….. | $W_j$ | ……… | $W_n$ | *Capacities* |
|---|---|---|---|---|---|---|---|
| $F_1$ | $c_{11}x_{11}$ (1) | $c_{12}x_{12}$(1) | | | | | $a_1$ |
| $F_2$ | | $c_{22}x_{22}$ (1) | | $c_{2i}x_{2i}$(1) | | | $a_2$ |
| ….. | | | ….. | | | | |
| $F_i$ | | | | $c_{ii}x_{ii}$(1) | | | $a_i$ |
| $F_m$ | | | | | | $c_{mn}x_{mn}$(1) | $a_m$ |
| Requirement | $b_1$ | $b_2$ | | $b_j$ | | $b_n$ | $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ |

**Table 3:**

| *Factory* | *Product to be supply* | *Cost of transport* |
|---|---|---|
| $F_1$ | $x_{11}$ & $x_{12}$ | $c_{11}x_{11} + c_{12}x_{12}$ |
| $F_2$ | $x_{12}$ & $x_{13}$ | $c_{12}x_{12} + c_{13}x_{13}$ |
| ….. | …… | …………… |
| $F_i$ | $x_{ii}$ | $c_{ii}x_{ii}$ |
| $F_m$ | $x_{mn}$ | $c_{mn}x_{mn}$ |

one Programming. Let us assume that n source supply the material on m destination. There are different process and $a_i$ units of certain product and $j^{th}$ required $b_j$ units. There is $c_{ij}$ being the cost of transport from *i* origin to *j* destination. Then by the feasible solution is ecit if it satisfies the necessary and sufficient condition of transportation $\sum a_i = \sum b_j$ = where $1 \le i \le m, 1 \le j \le n$.

In case the $\sum a_i \ne \sum b_j$ there required some manipulation for convert the situation in $\sum a_i = \sum b_j$. Let $C_{ij}$ be the cost of transportation per unit product from *i* to *j* and $x_{ij}$ be the number of product. The problem is to determine the $x_{ij} \ge 0$ and also satisfying the

$$\sum_{j=1}^{n} x_{ij} = a_i \ \forall\ i = 1,2,3 \ldots .m$$

And as well as the requirement constraints:

$$\sum_{i=1}^{m} x_j = b_j \ \forall\ j = 1,2,3 \ldots .n$$

The problem Minimize Z = CX, $X \in R^{mn}$ S.t constraint AX = b, $X \ge 0, b \in R^{m+n}$

Where $X = \{x_{11}, x_{12}, \ldots\ldots\}$ and the cost vector $b = \{a_1, a_2 \ldots\ldots a_m, b_1, b_2 \ldots\ldots bn\}$

It can be present as

$$A = \begin{matrix} e_{mn}^{(1)} & e_{mn}^{(2)} \cdots\cdots & e_{mn}^{m} \\ I_n & I_n \cdots\cdots & I_n \end{matrix}$$

Let $a_{ij} = e_i + e_{m+j}$ where $e_i, e_{m+j} \in R^n$ are unit vectors. By the Theorem, The number of basic variables in a transportation problem is at the most m+n-1.

## Discussion and Result

Now the Zero-One programming can apply in Travelling Salesmen problem to minimize the cost or distance or time. It can be formulated as:

$$MinZ = \sum \sum_{j} \sum_{k} d_{ij} x_{ijk}$$

Where $d_{ij}$ denotes the distance from station *i* to *j* station

$$x_{ijk} = \begin{cases} 1, if\ the\ kth\ directed\ arc\ is\ from\ station\ i\ to\ j \\ o, if\ otherwise\ any\ condition\ exist \end{cases}$$

Zero-One programming can be applied to solve the fixed charge problem. In this type of problem it is required to produce at least M units of certain products on n different machines.

Let $xj$ be the number of units produced on machine $j$, $j = 1,2,3\ldots\ldots\ldots n$. There is machine installation cost is ficed K. So the total cost function for $j^{th}$ machine is given by

$$c_j(x_j) = \begin{cases} k_j + c_j x_j, x_j > 0 \\ 0, x_j = 0 \end{cases}$$

Where $kj$ is the setup cost of $j$ machine

$$Min\ Z = \sum_{j=1}^{n} c_j x_j$$

The installation of machine required huge amount let be M of unit. Then the $y_i$ unit installation cost $\sum My_i$. Now the problem is

$$MinZ = \sum My_{i.} + \sum_{j=1}^{n} c_j x_j$$

Suppose there are $m$ sources and $n$ destination where the requirement of product. Here we consider a transportation problem with zero-one programming. Suppose there are m sources or factory and n-destination or users stations. The purpose of the problem is to minimize the transportation cost (time or distance). There are two things in our consideration, one is cost of transportation and other demand or how much can be supplied. Now we are introducing the table two tabular representations of cost ($c_{ij}$) and product variables ($x_{ij}$) The combined table is showing the cost of transport of $x_{ij}$ product with $c_{ij}$. The major problem is to allot the number of goods supply from different factories to different stations to minimize the cost (time or distance). The distribution of quantities is according to demand and capacity of factory. Now

We use the Zero-One programming model for supply the product. In this method we apply the steps.

- First make the combined table of cost and product quantities.
- Second start the allotment according to minimum cost.
- The place Zero at the place where is no allotment of quantity.
- Place one where the allotment the quantity as per minimum cost.
- In Next table we take only data where quantities allotted.
- It can be done in single table and without confusion.
- Calculate the total cost.

Mathematical presentation of Zero-One Transportation programming is

$$P\{zero\ \&\ one\} = \begin{cases} 0, when\ c_{ij}x_{ij}\ is\ not\ in\ range \\ 1\ when\ c_{ij}x_{ij}\ is\ in\ range \end{cases}$$

The allotment 0 and 1 is basically defined according to supplier requirement. The cost of transportation range is decided by the supplier. If the shipping charge from one station to another destination is in the range, then it allocates it one mark. If transportation cost is not in the range or not convenient for supplier to ship, then it can be zero mark. The tabular presentation of Zero–One transportation programming can be done as:

## Zero- One Transportation Matrix

This is the advantage of this zero-one programming that now we take only element which assigned one. In next step we consider only destinations where allotment is given. There is no confusion and easy to understand the places where transport possible. The next table

## Allotment Matrix

This table clearly shows the allotment of product and how much requirement of each destination. The Matrix shows that the F1 supply the quantities $x_{11}$ and $x_{12}$ to W1 and W2. Similarly, we can insert the transport cost from each factory.

## Transportation Cost from Each Factory

Now the Total cost of transportation from factory wise can be easily calculated.

The transportation cost

$$\sum c_{ij}x_{ij} = c_{11}x_{11} + c_{12}x_{12} + c_{12}x_{12} + c_{13}x_{13} + \cdots \ldots \ldots + c_{ii}x_{ii} + c_{mn}x_{mn}$$

Now setup a linear system of equations which can be solved (read: optimized) *via* PuLP. By defining the cost function:

```
costi,j = distancei,j * wagei,j
we can minimize the cost subject to:
Supply = Demand
for key, Value in supply. Items ():
 lab[key] = laborPrice[key] * Value
 # create revenue matrix
 rev = {}
 for key, Value in demand. Items():
 rev[key] = Value * sell Price[key]
 # create a cost matrix and convert to a list
 costs = (cost Mile * dist)/capacity
 costs = costs.tolist()
 costs = make Dict ([suppliers, destinations], costs, 0)
 # setup the cost minimization problem
 prob = Lp Problem ('Canning Distribution Problem', Lp Minimize)
 # create a list of all possible routes
 routes = [(S,D) for S in Suppliers for D in Destinations]
 # create a dictionary to contain all referenced variables
 route_var = LpVariable.dicts ("Route", (Suppliers, Destinations), 0, None, LpInteger)
 # add the objective function to the problem
 prob += lpSum ([route_var[S][D]*costs[S][D] for (S,D) in routes])
 # constraints functions
 for S in Suppliers:
 prob += lpSum([route_var[S][D] for D in Destinations]) <= supply[S]
 "sum_of_products_out_of_suppliers_%s" %S
 for D in Destinations:
 prob += lpSum([route_var[S][D] for S in Suppliers]) >= demand[D]
 "sum_of_products_into_destinations_%s" %D
 prob.write LP('Transport_problem.lp')
 prob.solve ().
```

## Conclusion

The paper presents the tabular form of cost and product variables by assigning the zero-one integer. This method will reduce the complexity of assigning the quantity of delivery material. The Paper will convert the Transport problem in linear programming than make a solving model in PULP software. The model finds the way to reduce or minimize the cost of transport the material.

Industries make a range of transportation problem and tried to fit the cost in given range.

The mathematical presentation of Zero-One Transportation programming is

$$P\{zero\ \&\ one\} = \begin{cases} 0, when\ c_{ij}x_{ij}\ is\ not\ in\ range \\ 1\ when\ c_{ij}x_{ij}\ is\ in\ range \end{cases}$$

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## Conflict of Interests

The Author(s) declare(s) that there is no conflict of interest.

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