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An Integrals Involving the Multivariable *H*-Functions

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Abstract

The aim of this paper is to evaluate an infinite integral involving the product of multivariable *H*-functions along with Srivastava polynomial and M-series by means of finite difference operations E. As the generalized hypergeometric function and multivariable *H*-functions are of a very general nature, the integral, on specializing the parameters, leads to a generalization of many results some of which are known and other are believed to be new.

Keywords: -Operator, Srivastava polynomial, M-series.

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Introduction

The multivariable *H*-functions[1] is defined and represented in the following manner:

$$H[z_1,\ldots\ldots,z_r]=H^{0,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\middle|\begin{matrix}\left(a_j,\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{1,p}:\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\\ \left(b_j,\beta_j^{(1)},\ldots,\beta_j^{(r)}\right)_{1,q}:\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\end{matrix}\right]$$

$$=\frac{1}{(2\pi i)^r}\int_{L_1}\cdots\int_{L_r}\phi_i(\xi_1)\ldots\phi_r(\xi_r)\,\psi(\xi_1,\ldots,\xi_r)z_1^{\xi_1},\ldots,z_r^{\xi_r}d\xi_1\ldots d\xi_r, \quad (1.1)$$

where $i=\sqrt{-1}$,

$$\psi(\xi_1,\ldots,\xi_r)=\frac{\prod_{j=1}^{n}\Gamma\left(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)}{\prod_{j=n+1}^{p}\Gamma\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)\prod_{j=1}^{q}\Gamma\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}\xi_i\right)}, \quad (1.2)$$

$$\beta_j^{(1)}=\beta_j^{(\prime)},\ \beta_j^{(2)}=\beta_j^{(\prime\prime)}$$

$$\phi_i(\xi_i)=\frac{\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}\xi_i\right)\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}+\gamma_j^{(i)}\xi_i\right)}{\prod_{j=m_i+1}^{q_i}\Gamma\left(1-d_j^{(i)}+\delta_j^{(i)}\xi_i\right)\prod_{j=n_i+1}^{p_i}\Gamma\left(c_j^{(i)}-\gamma_j^{(i)}\xi_i\right)},\ \text{for all } i\in(1,\ldots,r). \quad (1.3)$$

A general class of polynomials of real and complex variables is defined and studied by Batesman[2]Srivastava[3] in the following form:

$$S_n^m[x]=\sum_{K=0}^{[n/m]}\frac{(-n)_{mK}}{K!}A_{n,K}x^K, \qquad n=0,1,2,3,\ldots\ldots \quad (1.4)$$

The M-series is defined by Sharma,[4,5]

$${}_pM_q^{\alpha}[x^m]=\sum_{l=0}^{\infty}\frac{(a_1)_l\ldots\ldots(a_p)_l}{(b_1)_l\ldots\ldots(b_q)_l}\frac{x^{ml}}{\Gamma(\alpha l+1)} \quad (1.5)$$

here, $\alpha\in\mathbb{C},\Re(\alpha)>0$ and $(a_j)_{k'}(b_j)_k$ are pochammer symbols.

Formulae Used

Following formulae will be used in the work are present in this section:

From Erdélyi [6, p. 337, eq. (8)], we have

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$$\int_0^\infty x^{s-1}e^{-\alpha x}W_{\mu,\nu}(\beta x)dx=\beta^{\nu+1/2}\frac{\Gamma(\nu+s+1/2)\Gamma(-\nu+s+1/2)}{\Gamma(s-\mu+1)\left(\alpha+\beta/2\right)^{\nu+s+1/2}}$$

$${}_2F_1\left[(\nu+s+1/2),(\nu-\mu+1/2);(s-\mu+1);\frac{2\alpha-\beta}{2\alpha+\beta}\right] \quad (2.1)$$

where $Re(s)>[Re(\mu)]-1/2$.

The finite difference operator is [(7), p. 33, with $w = 1$]:

$$E_af(a)=f(a+1) \quad (2.2)$$

and

$$(\alpha)_n=\frac{\Gamma(\alpha+n)}{\Gamma(\alpha)} \quad (2.3)$$

Improper Integrals

Theorem

The integrals which is used to be established are given in this section

$$\int_0^\infty x^{s-1}e^{-1/2\beta x}W_{\mu,\nu}(\beta x)H^{0,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix}z_1x^{k_1}\\ \vdots\\ z_rx^{k_r}\end{matrix}\middle|\begin{matrix},\left(a_j,\alpha_j^{(1)},\ldots,\alpha_j^{(r)}\right)_{1,p}:\left(c_j^{(1)},\gamma_j^{(1)}\right)_{1,p_1};\ldots;\left(c_j^{(r)},\gamma_j^{(r)}\right)_{1,p_r}\\ \left(b_j,\beta_j^{(1)},\ldots,\beta_j^{(r)}\right)_{1,q},:\left(d_j^{(1)},\delta_j^{(1)}\right)_{1,q_1};\ldots;\left(d_j^{(r)},\delta_j^{(r)}\right)_{1,q_r}\end{matrix}\right]$$

$$S_N^M[x^\lambda]\,{}_PM_Q^{\alpha}[x^\gamma]dx$$

$$H^{0,n+2:m_1,n_1;\ldots;m_r,n_r}_{p+2,q+1:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1\beta^{-k_1}\\ \vdots \\ z_r\beta^{-k_r}\end{matrix}\middle|\begin{matrix}(1/2 \pm v - s - \lambda K - \gamma l, k_1,\ldots,k_r),(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q},(\mu - s - \lambda K - \gamma l, k_1,\ldots,k_r):\ (d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right] \quad (3.1)$$

where

$$Re(\beta) > 0, Re\left(s + {}^{b_j}/_{\beta_j}\right) > |Re(v)| - \tfrac{1}{2} : (j = 1,2,\ldots,k).$$

$$\int_0^\infty x^{s-1} e^{-1/2\beta x} W_{\mu,v}(\beta x)\, {}_uF_v(A_u; B_v; Cx^d)$$

$$H^{0,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1 x^{k_1}\\ \vdots \\ z_r x^{k_r}\end{matrix}\middle|\begin{matrix},(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q},:(d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right] S_N^M[x^\lambda]\, {}_PM_Q^\alpha[x^\gamma]dx$$

$$= \beta^{-s}\sum_{h=0}^{\infty}\frac{(A_v)_h C^h}{(B_v)_h}\sum_{K=0}^{[N/M]}\frac{(-N)_{MK}}{K!}A_{N,K}\beta^{-\lambda K}\sum_{l=0}^{\infty}\frac{(a_1)_l \ldots (a_p)_l}{(b_1)_l \ldots (b_Q)_l}\frac{\beta^{-\gamma l}}{\Gamma(\alpha l+1)}$$

$$H^{0,n+2:m_1,n_1;\ldots;m_r,n_r}_{p+2,q+1:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1\beta^{-k_1}\\ \vdots \\ z_r\beta^{-k_r}\end{matrix}\middle|\begin{matrix}(1/2 \pm v - dh - s - \lambda K - \gamma l, k_1,\ldots,k_r),(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q},(\mu - s - dh - \lambda K - \gamma l, k_1,\ldots,k_r):\ (d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right] \quad (3.2)$$

where

$$Re\left(s + {}^{b_j}/_{\beta_j}\right) > |Re(v)| - \tfrac{1}{2} : (j = 1,2,\ldots,k).$$

Proof

The integral (3.1) can be obtain by replacing themultivariable *H*-functions, $S_N^M[x]$ and ${}_PM_Q^\alpha[x]$ on the left-hand side by its equivalent counter integral (1.1), (1.2), (1.4) and (1.5) respectably. Then changing the order of integral as per required then evaluating the inner integral with the help of [6, p.337, eq. (8), with $\alpha = {}^{\beta}/_2$].

While the integral (3.2) can be obtain on multiplying both side of (3.1) by

$$\frac{\prod_{j=1}^{u}\Gamma(A_j+\delta)C^\delta}{\prod_{j=1}^{v}\Gamma(B_j+\delta)}$$

and applying the operator , we get

$$exp(E_\delta^d E_\delta)\int_0^\infty x^{s-1}e^{-1/2\beta x}W_{\mu,v}(\beta x)\, H^{0,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1 x^{k_1}\\ \vdots \\ z_r x^{k_r}\end{matrix}\middle|\begin{matrix},(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q},:(d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

$$S_N^M[x^\lambda]\, {}_PM_Q^\alpha[x^\gamma]dx\,\frac{\prod_{j=1}^{u}\Gamma(A_j+\delta)C^\delta}{\prod_{j=1}^{v}\Gamma(B_j+\delta)}$$

$$= exp(E_\delta^d E_\delta)\beta^{-s}\sum_{K=0}^{[N/M]}\frac{(-N)_{MK}}{K!}A_{N,K}\beta^{-\lambda K}\sum_{l=0}^{\infty}\frac{(a_1)_l \ldots (a_P)_l}{(b_1)_l \ldots (b_Q)_l}\frac{\beta^{-\gamma l}}{\Gamma(\alpha l+1)}$$

$$H^{0,n+2:m_1,n_1;\ldots;m_r,n_r}_{p+2,q+1:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1\beta^{-k_1}\\ \vdots \\ z_r\beta^{-k_r}\end{matrix}\middle|\begin{matrix}(1/2 \pm v - s - \lambda K - \gamma l, k_1,\ldots,k_r),(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q},(\mu - s - \lambda K - \gamma l, k_1,\ldots,k_r):\ (d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

$$\frac{\prod_{j=1}^{u}\Gamma(A_j+\delta)C^\delta}{\prod_{j=1}^{v}\Gamma(B_j+\delta)}. \quad (3.3)$$

Expanding both side of (3.3) and using (2.2), we get

$$\sum_{h=0}^{\infty}\int_0^\infty x^{s-1}e^{-1/2\beta x}W_{\mu,v}(\beta x)\, H^{0,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1 x^{k_1}\\ \vdots \\ z_r x^{k_r}\end{matrix}\middle|\begin{matrix}(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q}:(d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

$$S_N^M[x^\lambda]\, {}_PM_Q^\alpha[x^\gamma]\frac{\prod_{j=1}^{u}\Gamma(A_j+\delta+h)C^{\delta+h}}{\prod_{j=1}^{v}\Gamma(B_j+\delta+h)h!}dx$$

$$= \beta^{-s}\frac{\prod_{j=1}^{u}\Gamma(A_j+\delta+h)C^{\delta+h}}{\prod_{j=1}^{v}\Gamma(B_j+\delta+h)h!}\sum_{K=0}^{[N/M]}\frac{(-N)_{MK}}{K!}A_{N,K}\beta^{-\lambda K}\sum_{l=0}^{\infty}\frac{(a_1)_l \ldots (a_P)_l}{(b_1)_l \ldots (b_Q)_l}\frac{\beta^{-\gamma l}}{\Gamma(\alpha l+1)}$$

$$H^{0,n+2:m_1,n_1;\ldots;m_r,n_r}_{p+2,q+1:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1\beta^{-k_1}\\ \vdots \\ z_r\beta^{-k_r}\end{matrix}\middle|\begin{matrix}(1/2 \pm v - dh - s - \lambda K - \gamma l, k_1,\ldots,k_r),(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q},(\mu - s - dh - \lambda K - \gamma l, k_1,\ldots,k_r):\ (d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right] \quad (3.4)$$

Now using (2.3) for changing the order of integration and summation on the left-hand side, which is justified due to [(8), p. 178 75(III)], and replacing *Aj*+ *δ* by *Aj* and *Bj*+ *δ* by *Bj*, we get the required result (3.2).

Special Case

- On removing $S_N^M[x^\lambda]$ and ${}_PM_Q^\alpha[x^\gamma]$ from both (3.1) and (3.2), we get the result of [9, pp. 27-33].
- On taking *u* = 2, *v* = 3, *d* = 1 in (3.2) and using the result [10, pp. 105, 106]:

$${}_2F_1\left(\frac{1}{2}a + \frac{1}{2}b, \frac{1}{2}a + \frac{1}{2}b - \frac{1}{2}; a, b, a+b-1; 4x\right) = {}_0F_1(-; a; x)\,{}_0F_1(-; b; x)$$

and ${}_2F_3\left(a, b-a; b, \frac{1}{2}b, \frac{1}{2}b + \frac{1}{2}; \frac{1}{4}x^2\right) = {}_1F_1(a; b; x)\,{}_1F_1(a; b; -x)$ we obtain

$$\int_0^\infty x^{s-1}e^{-1/2\beta x}W_{\mu,v}(\beta x)\,{}_0F_1\left(-; B_1; \frac{1}{4}Cx\right){}_0F_1\left(-; B_2; \frac{1}{4}Cx\right)$$

$$H^{0,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1 x^{k_1}\\ \vdots \\ z_r x^{k_r}\end{matrix}\middle|\begin{matrix},(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q},:(d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right] S_N^M[x^\lambda]\, {}_PM_Q^\alpha[x^\gamma]dx$$

$$= \beta^{-s}\sum_{K=0}^{[N/M]}\frac{(-N)_{MK}}{K!}A_{N,K}\beta^{-\lambda K}\sum_{l=0}^{\infty}\frac{(a_1)_l \ldots (a_P)_l}{(b_1)_l \ldots (b_Q)_l}\frac{\beta^{-\gamma l}}{\Gamma(\alpha l+1)}\sum_{h=0}^{\infty}\frac{\left(\frac{B_1}{2}+\frac{B_2}{2}\right)_h\left(\frac{B_1}{2}+\frac{B_2}{2}-\frac{1}{2}\right)_h C^h}{(B_1)_h(B_2)_h(B_1+B_2-1)_h h!\,\beta^h}$$

$$H^{0,n+2:m_1,n_1;\ldots;m_r,n_r}_{p+2,q+1:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1\beta^{-k_1}\\ \vdots \\ z_r\beta^{-k_r}\end{matrix}\middle|\begin{matrix}(1/2 \pm v - s - \lambda K - \gamma l, k_1,\ldots,k_r),(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q},(\mu - s - \lambda K - \gamma l, k_1,\ldots,k_r):\ (d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right] \quad (4.1)$$

and

$$\int_0^\infty x^{s-1}e^{-1/2\beta x}W_{\mu,v}(\beta x)\,{}_1F_1\left(A_1; B_1; 2(\sqrt{Cx})^2\right){}_1F_1\left(A_1; B_1; -2(\sqrt{Cx})^2\right)$$

$$H^{0,n:m_1,n_1;\ldots;m_r,n_r}_{p,q:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1 x^{k_1}\\ \vdots \\ z_r x^{k_r}\end{matrix}\middle|\begin{matrix},(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q},:(d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right] S_N^M[x^\lambda]\, {}_PM_Q^\alpha[x^\gamma]dx$$

$$= \beta^{-s}\sum_{K=0}^{[N/M]}\frac{(-N)_{MK}}{K!}A_{N,K}\beta^{-\lambda K}\sum_{l=0}^{\infty}\frac{(a_1)_l \ldots (a_P)_l}{(b_1)_l \ldots (b_Q)_l}\frac{\beta^{-\gamma l}}{\Gamma(\alpha l+1)}$$

$$H^{0,n+2:m_1,n_1;\ldots;m_r,n_r}_{p+2,q+1:p_1,q_1;\ldots;p_r,q_r}\left[\begin{matrix} z_1\beta^{-k_1}\\ \vdots \\ z_r\beta^{-k_r}\end{matrix}\middle|\begin{matrix}(1/2 \pm v - dh - s - \lambda K - \gamma l, k_1,\ldots,k_r),(a_j;\alpha_j^{(1)},\ldots,\alpha_j^{(r)})_{1,p}:(c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta_j^{(1)},\ldots,\beta_j^{(r)})_{1,q},(\mu - s - dh - \lambda K - \gamma l, k_1,\ldots,k_r):\ (d_j^{(1)},\delta_j^{(1)})_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right] \quad (4.2)$$

respectively. The conditions of validity for (4.1) and (4.2) are same as given in (3.1).

Conclusion

In the present paper is to evaluate an infinite integral involving the product of multivariable *H*-functions along with Srivastava polynomial and *M*-series by means of finite difference operations *E*. As the generalized hypergeometric function and multivariable *H*-functions are of a very general nature, the integral, on specializing the parameters, leads to a generalization of many results some of which are known and other are believed to be new. We can also obtainthe number of special functions as the special cases of our main results, which are related with Multivariable *H*-functions.

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