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Fractional Thermal Analysis in a Generalized two Dimensional Infinite half Space with Heat Source

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ABSTRACT

A generalized time fractional thermoelastic study under one relaxation time consideration is presented here for the two dimensional model having infinite half space. To analyze the thermal response of the problem, the infinite space is subjected to periodical varying heat source with respect to time. Futher, time derivative involved in heat transfer equation is of Caputo type having order α . Certains thermomechanical boundaries are applied at lower surface. Here the solution is obtained directly by employing Laplace and Hankel transformation but inversion for Laplace is evaluated by numerical method as given by Gaver-Stehfast algorithm. Properties of Copper metal is selected for the numerical analysis and results of temperature and stress distributions are presented graphically by using MATLAB software.

Keywords: Fractional Calculus, half-space, Generalized thermoelasticity,Heat source, Laplce and Hankel Transform.

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INTRODUCTION

In theory of thermoelasticity the problems are mainly classified into two categories, namely static and dynamic thermoelastic problems. The dynamic problem of thermal stresses are widely important and applicable in many engineering processes which fundamentally deals with high temperatures for example aerodynamic structures and nuclear reactors. It is noted that for classical uncoupled thermoelastic modelling (a) No elastic terms contained in heat transfer equation. (b) Heat transfer equation is of parabolic type which predicting the in-finite speeds of propagation of heat wave. Because of above two phenomenon the theory is found not compatible with physical observations. The first overcome is shortcome by the introduction of coupled thermoelastic theory by Biot [1] but parabolic in nature still exist in both coupled and uncoupled thermoelasticity.

Further, two new generalizations were introduced in the coupled thermoelasticity theory. The first generalization is done by Lord and Shulman [2], in which wave type heat equation obtained by replacing classical Fourier law with a new modified law. And for both the coupled and uncoupled cases second shortcome is automatically eliminated since associated heat eaqaton is a hyperbolic one.

The thermoelastic theory which deals with concept of two relaxation times or having temperature dependency is known as second generalization. The thermoelastic theory with two relaxation times was developed by Green and Lindsay [3].From last few years, a lot of research work has developed in theory of fractional [4-19].

In this present study, we modify the work done by [20] by considering heat transfer equation with the time derivative of Caputo type having order α .

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MATHEMATICAL MODELING OF THE PROBLEM

Following [20], let's assume a generalized problem of thermoelasticity of two dimensional homogeneous isotropic solid, whose z -axis and bounding plane are kept perpendicular. The above problem is formulated for one relaxation time and fractional time derivative having order $0 < \alpha \le 2$. To observe the thermal behavior of solid a known function of temperature flow is applied and free mechanical loads at the surface medium is maintained. For formulation of problem cylindrical coordinate system is used and initial temperature flow at T_0.

For generalized thermoelasticity the governing equation with variables $r,\ z,\ t$ represented as in [20] following:

The equations of motion:

$$\mu\nabla^2 u - \frac{\mu}{r^2}u + (\lambda+\mu)\frac{\partial e}{\partial r} - \gamma\frac{\partial \theta}{\partial r} = \rho\frac{\partial^2 u}{\partial t^2} \quad (1)$$

$$\mu\nabla^2 w + (\lambda+\mu)\frac{\partial e}{\partial z} - \gamma\frac{\partial \theta}{\partial z} = \rho\frac{\partial^2 w}{\partial t^2} \quad (2)$$

The heat transfer equation with fractional derivative of order $0 < \alpha \le 2$ [7]

$$k\, I^{\alpha-1}\nabla^2 T = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)(\rho C_E T + \gamma T_0 e)$$

$$-\rho\left(1+\tau_0\frac{\partial}{\partial t}\right)Q \quad (3)$$

In which the Riemann Liouville integral is I^{α} expressed in convolution type form as:

$$I^{\alpha}g(t) = \int_0^t \frac{(t-s)^{\alpha-1}}{\Gamma(\alpha)}g(s)\,ds; \qquad \alpha > 0 \quad (4)$$

Here, Lebesgue integrable function is $g(t)$ and the Gamma function is $\Gamma(\alpha)$.

for absolutely continuity of , we have

$$\lim_{\alpha\to 1}\frac{d^{\alpha}}{dt^{\alpha}}g(t) = g'(t) \quad (5)$$

where, thermal conductivity is denoted by k, τ_0 refers for relaxation time, strain tensor components is e_{ij}, Specific heat at constant strain is denoted by C_E, absolute value of temperature is T, in natural state medium temperature is T_0, with assumption as $(|T-T_0)/T| << 1$.

The Caputo derivative for $g(t)$ having order α is

$$D_{0,t}^{\alpha}g(t) = D_{0,t}^{-(m-\alpha)}\frac{d^m}{dt^m}f(t) = \int_0^t \frac{(t-\tau)^{m-\alpha-1}}{\Gamma(m-\alpha)}g^m(\tau)d\tau,$$

$$m-1 < \alpha < m \quad (6)$$

Where $D_t^{\alpha}[.]$ denotes differential operator of fractional order α with respect to time t.

Further,

$$e = u/r + \partial u/\partial r + \partial w/\partial z \quad (7)$$

For $\alpha \to 1$ the equation of heat conduction (3), becomes

$$k\nabla^2 T = (\partial/\partial t + \tau_0\partial^2/\partial t^2)(\rho C_E.T + \gamma.T_0 e) - \rho(1+\tau_0\partial/\partial t)Q \quad (8)$$

Equation (8) is nothing but the equation in generalized theory with one relaxation time subjected to heat source.

For $\alpha \to 1$ with $\tau_0 \to 0$ the equation (3), transform

$$k\nabla^2 T = \left(\rho C_E\frac{\partial T}{\partial t} + \gamma T_0\frac{\partial e}{\partial t}\right) - \rho Q \quad (9)$$

Which is the heat equation obtained is same as in coupled theory of thermoelasticity.

$$\nabla^2 = \partial^2/\partial r^2 + 1/r.\partial/\partial r + \partial w/\partial z \quad (10)$$

$$\sigma_{rr} = 2\mu\,\partial u/\partial r + \lambda e - \gamma(T-T_0) \quad (11)$$

$$\sigma_{zz} = 2\mu\frac{\partial w}{\partial z} + \lambda e - \gamma(T-T_0) \quad (12)$$

$$\sigma_{rz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \quad (13)$$

The following dimensionless variables are introduced:

$$r^* = c_1\eta\, r,\ z^* = c_1\eta\, z,\ u^* = c_1\eta\, u,\ w^* = c_1\eta\, w,$$

$$t^* = c_1^2\eta\, t,\ \tau_0^* = c_1^2\eta\,\tau_0,\ \sigma_{ij}^* = \frac{\sigma_{ij}}{\mu},$$

$$\theta = \frac{\gamma(T-T_0)}{(\lambda+2\mu)},\ \eta = \frac{\rho\, c_E}{k},\ Q^* = \frac{\rho\gamma Q}{k\, c_1^2\eta^2(\lambda+2\mu)},$$

$$c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

Here, c_1 is the speed of isothermal elastic waves propagation.

Using (13), the governing equations as well as constitutive relations take the form (for convenience we drop the primes)

$$\nabla^2 u - \frac{u}{r^2} + \left(\frac{\lambda+\mu}{\mu}\right)e - \frac{\lambda+2\mu}{\mu}\frac{\partial\theta}{\partial r} = \frac{\lambda+2\mu}{\mu}\frac{\partial^2 u}{\partial t^2} \quad (14)$$

$$\nabla^2 w + \left(\frac{\lambda+\mu}{\mu}\right)\frac{\partial e}{\partial z} - \frac{\lambda+2\mu}{\mu}\frac{\partial\theta}{\partial z} = \frac{\lambda+2\mu}{\mu}.\frac{\partial^2 w}{\partial t^2} \quad (15)$$

$$I^{\alpha-1}.\nabla^2\theta = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)(\theta + \varepsilon e) - \left(1 + \tau_0\frac{\partial}{\partial t}\right)Q \quad (16)$$

$$\sigma_{rr} = 2\frac{\partial u}{\partial r} + \left(\frac{\lambda}{\mu}\right)e - \frac{\lambda+2\mu}{\mu}\theta \quad (17)$$

$$\sigma_{zz} = 2\frac{\partial w}{\partial z} + \left(\frac{\lambda}{\mu}\right)e - \frac{\lambda+2\mu}{\mu}\theta \quad (18)$$

$$\sigma_{rz} = \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \quad (19)$$

On combining (14) and (15) and using equation (10), we obtains

$$\nabla^2 e - \nabla^2\theta = \frac{\partial^2 e}{\partial t^2} \quad (20)$$

The corresponding initial conditions are given as

$$\theta\big|_{t=0} = \frac{\partial\theta}{\partial t}\bigg|_{t=0} = 0$$

The mechanical and thermal boundaries for the region $0 < r < \infty$ defined as

$$\left.\begin{aligned} \theta(r,0,t) &= f(r,t) \\ \sigma_{zz}(r,0,t) &= 0 \\ \sigma_{rz}(r,0,t) &= 0 \end{aligned}\right\} \quad (21)$$

ANALYTICAL SOLUTION OF THE MODEL

The Laplace transform (L.T.) for $g(r,z,t)$ is defined as

$$L.T.of\,[g(r,z,t)] = \bar{g}(r,z,s) = \int_0^\infty e^{-st} g(r,z,t)\,dt \quad (22)$$

On following Povstenko [9] for Riemann–Liouville fractional integral the Laplace transform has the from

$$L[I^n g(t)] = \frac{1}{s^n} L[g(t)] \qquad n > 0 \quad (23)$$

Now, applying Laplace transform to (14) to (20) consecutively and using (23), we obtain

$$\nabla^2\bar{u} - \frac{\bar{u}}{r^2} + \left(\frac{\lambda+\mu}{\mu}\right)\bar{e} - \frac{\lambda+2\mu}{\mu}\partial\bar{\theta}/\partial r = \frac{\lambda+2\mu}{\mu}s^2\bar{u} \quad (24)$$

$$\nabla^2\bar{w} + \left(\frac{\lambda+\mu}{\mu}\right)\partial\bar{e}/\partial z - \frac{\lambda+2\mu}{\mu}\partial\bar{\theta}/\partial z = \frac{\lambda+2\mu}{\mu}s^2\bar{w} \quad (25)$$

$$\left(\nabla^2 - s^{\alpha-1}(s + \tau_0 s^2)\right)\bar{\theta} = s^{\alpha-1}(1 + \tau_0 s)(s\varepsilon\,\bar{e} - \bar{Q}) \quad (26)$$

$$(\nabla^2 - s^2)\bar{e} = \nabla^2\bar{\theta} \quad (27)$$

$$\bar{\sigma}_{rr} = 2\frac{\partial\bar{u}}{\partial r} + \left(\frac{\lambda}{\mu}\right)\bar{e} - \frac{\lambda+2\mu}{\mu}\bar{\theta} \quad (28)$$

$$\bar{\sigma}_{zz} = 2\frac{\partial\bar{w}}{\partial z} + \left(\frac{\lambda}{\mu}\right)\bar{e} - \frac{\lambda+2\mu}{\mu}\bar{\theta} \quad (29)$$

$$\bar{\sigma}_{rz} = \left(\frac{\partial\bar{u}}{\partial z} + \frac{\partial\bar{w}}{\partial r}\right) \quad (30)$$

The condition as in (21) are written as

$$\bar{\theta} = \bar{f}.(r,s) \quad (31)$$

$$\bar{\sigma}_{zz} = \bar{\sigma}_{rz} = 0 \quad (32)$$

After removing $\bar{e}$ from the (26) and (27), we obtains

$$\left\{\nabla^4 - \nabla^2\left[s^2 + s^{\alpha-1}.(s + \tau_0 s^2)(1+\varepsilon)\right] + s^{\alpha+1}(s + \tau_0 s^2)\right\}\bar{\theta} = -s^{\alpha-1}(1 + \tau_0 s)(\nabla^2 - s^2)\bar{Q} \quad (33)$$

$$\left\{(\nabla^2 - k_1^2)(\nabla^2 - k_2^2)\right\}\bar{\theta} = -s^{\alpha-1}(1 + \tau_0 s)(\nabla^2 - s^2)\bar{Q} \quad (34)$$

where k_1^2 and k_2^2 denotes the roots of the characteristic equation

$$\left\{k^4 - \left(s^2 + s^{\alpha-1}(s+\tau_0 s^2)(1+\varepsilon)\right)k^2 + s^{\alpha+1}(s+\tau_0 s^2)\right\} = 0 \tag{35}$$

The solution of (34) can be presented as,

$$\overline{\theta} = \overline{\theta}_1 + \overline{\theta}_2 + \overline{\theta}_p \tag{36}$$

Where $\overline{\theta}_i$ is a solution of the

$$(\nabla^2 - k_1^2)\overline{\theta}_i = 0 \quad for \quad i = 1, 2 \tag{37}$$

Further, $\overline{\theta}_p$ denotes the particular integral of (34).

Next, Hankel transformation (H.T.) of $\overline{g}(\xi, z, s)$ w. r. t r of zero order defined as

$$\overline{g}^*(\xi, z, s) = H.T.of\left[\overline{g}(r, z, s)\right] = \int_0^\infty \overline{g}(r, z, s)\, r\, J_0(\xi r)\, dr \tag{38}$$

Inversion of above transformation is given by the relation as

$$\overline{g} = H^{-1}\left[\overline{g}^*(\xi)\right] = \int_0^\infty \overline{g}^*(\xi)\, \xi\, J_0(\xi r)\, d\xi \tag{39}$$

In which, J_0 denotes the first kind Bessel's function of order zero.

Employing the H.T. to (37) , we obtain

$$\left\{D^2 - (k_1^2 + \xi^2)\right\}\overline{\theta}_i^* = 0 \quad for\, i = 1, 2 \quad and\, D = \partial/\partial z \tag{40}$$

The solution of the (40) can be expressed in the form,

$$\overline{\theta}_i^* = A_i(\xi, s)\left(k_i^2 - s^2\right)e^{-q_i z} \tag{41}$$

Where $q_i = \sqrt{\alpha^2 + k_i^2}$

Next, employing the H.T. on (34), we obtains

$$\left\{(D^2 - q_1^2)\left(D^2 - q_2^2\right)\right\}\overline{\theta}_p^* = -s^{\alpha-1}\left(1+\tau_0 s\right)\left(D^2 - q^2\right)\overline{Q} \tag{42}$$

Where $q = \sqrt{\xi^2 + s^2}$

For cylindrical co-ordinates, the periodically varying heat source has taken the form,

$$Q = \begin{cases} \dfrac{Q_0 \delta(.r)}{2\pi .r}\dfrac{\sin \pi t}{\tau}; & 0 \le t \le \tau \\ 0; & t > \tau \end{cases} \tag{43}$$

Where, Q_0 - heat source strength

On employing Laplace & Hankel transformation (43) becomes

$$\overline{Q}^*(\xi, z, s) = \frac{Q_0\, \pi\, \tau\left(1 + e^{-s\tau}\right)}{\left(s^2 \tau^2 + \pi^2\right)} \tag{44}$$

On simplifying equation (42) becomes,

$$\overline{\theta}_p^* = \frac{s^{\alpha-1}(1+\tau_0 s)q^2}{q_1^2 q_2^2}\frac{Q_0\, \pi\, \tau\left(1 + e^{-s\tau}\right)}{\left(s^2 \tau^2 + \pi^2\right)} \tag{45}$$

The required solution of the problem in the transformed domain is

$$\overline{\theta}^*(\xi, z, s) = \sum_{i=1}^{2} A_i(\xi, s)\left(k_i^2 - s^2\right)e^{-q_i z} + \frac{s^{\alpha-1}(1+\tau_0 s)q^2}{q_1^2 q_2^2}\frac{Q_0\, \pi\, \tau\left(1 + e^{-s\tau}\right)}{\left(s^2 \tau^2 + \pi^2\right)} \tag{46}$$

Further, on taking the inversion of Hankel transformation of both sides, we obtain

$$\overline{\theta}(r, z, s) = \int_0^\infty \left\{\sum_{i=1}^{2} A_i(\xi, s)\left(k_i^2 - s^2\right)e^{-q_i z} + \frac{s^{\alpha-1}(1+\tau_0 s)q^2}{q_1^2 q_2^2}\frac{Q_0\, \pi\, \tau\left(1 + e^{-s\tau}\right)}{\left(s^2 \tau^2 + \pi^2\right)}\right\}\xi\, J_0(\xi\, r)d\xi \tag{47}$$

Similarly eliminating between (26) and (27), we get,

$$\left\{(\nabla^2 - k_1^2)\left(\nabla^2 - k_2^2\right)\right\}\overline{e} = -s^{\alpha-1}\left(1+\tau_0 s\right)\nabla^2\, \overline{Q} \tag{48}$$

On employing Hankel transform of (48), we get,

$$\left\{(D^2 - q_1^2)\left(D^2 - q_2^2\right)\right\}\overline{e}^* = -s^{\alpha-1}\left(1+\tau_0 s\right)\left(D^2 - \xi^2\right)\overline{Q}^* \tag{49}$$

The Complete solution of (49) is,

$$\overline{e}^*(\xi, z, s) = \sum_{i=1}^{2} A_i(\xi, s)\, k_i^2\, e^{-q_i z} + \frac{s^{\alpha-1}(1+\tau_0 s)\xi^2}{q_1^2 q_2^2}\frac{Q_0\, \pi\, \tau\left(1 + e^{-s\tau}\right)}{\left(s^2 \tau^2 + \pi^2\right)} \tag{50}$$

Further applying the inversion of Hankel Transformation, we obtain

$$\bar{e}(\xi, z, s) = \int_0^\infty \left\{ \sum_{i=1}^{2} A_i(\xi, s)\, k_i^2\, e^{-q_i z} + \frac{s^{\alpha-1}(1+\tau_0 s)\xi^2}{q_1^2 q_2^2} \frac{Q_0 \pi \tau (1+e^{-s\tau})}{(s^2\tau^2+\pi^2)} \right\} \xi J_0(\xi r) d\xi \quad (51)$$

The component of axial displacement is obtained by using equations (47) and (51) in Hankel transformation of equation (25) as

$$\bar{w}^*(\xi, z, s) = B(\xi, s) e^{-q_3 z} - \sum_{i=1}^{2} A_i(\xi, s)\, q_i\, e^{-q_i z} \quad (52)$$

$$q_3 = \sqrt{\xi^2 + \frac{s^2(\lambda + 2\mu)}{\mu}}$$

On employing inversion of H. T. of (52), we get

$$\bar{w}(r, z, s) = \int_0^\infty \left\{ B(\xi, s) e^{-q_3 z} - \sum_{i=1}^{2} A_i(\xi, s)\, q_i\, e^{-q_i z} \right\} \xi J_0(\xi r) d\xi \quad (53)$$

On using equations (50) and (52) in (7) after employing Hankel and Laplace transformation, we obtain

$$H\left(\frac{1}{r}\frac{\partial}{\partial r}(r\bar{u})\right) = B(\xi, s) q_3 e^{-q_3 z} - \xi^2 \left[\sum_{i=1}^{2} A_i(\xi, s) e^{-q_i z} - \frac{s^{\alpha-1}(1+\tau_0 s)}{q_1^2 q_2^2} \frac{Q_0 \pi \tau (1+e^{-s\tau})}{(s^2\tau^2+\pi^2)} \right] \quad (54)$$

After taking H.T. inversion on both sides of equation (54), we get,

$$\bar{u} = \int_0^\infty \left\{ B(\xi, s) q_3 e^{-q_3 z} - \xi^2 \left[\sum_{i=1}^{2} A_i(\xi, s) e^{-q_i z} - \frac{s^{\alpha-1}(1+\tau_0 s)}{q_1^2 q_2^2} \frac{Q_0 \pi \tau (1+e^{-s\tau})}{(s^2\tau^2+\pi^2)} \right] \right\} J_1(\xi r) d\xi \quad (55)$$

The stress components tensor are expressed as

$$\bar{\sigma}_{zz} = -\int_0^\infty 2B(\xi, s) q_3 e^{-q_3 z} + (\xi^2 + q_3^2) \left[\sum_{i=1}^{2} A_i(\xi, s) e^{-q_i z} - \frac{s^{\alpha-1}(1+\tau_0 s)}{q_1^2 q_2^2} \frac{Q_0 \pi \tau (1+e^{-s\tau})}{(s^2\tau^2+\pi^2)} \right] \} \times \xi J_0(\xi r) d\xi \quad (56)$$

$$\bar{\sigma}_{rz} = \int_0^\infty \left\{ \begin{array}{l} -(1+q_3^2) B(\xi, s) e^{-q_3 z} \\ + \left[\sum_{i=1}^{2} A_i(\xi, s)\, q_i\, (1+\xi^2) e^{-q_i z} \right] \end{array} \right\} J_1(\xi r) d\xi \quad (57)$$

After using the Hankel transform, the boundaries (31) and (32) become,

$$\bar{\theta}^*(\xi, 0, s) = \bar{f}^*(\xi, s) \quad (58)$$

$$\bar{\sigma}_{zz}^*(\xi, 0, s) = \bar{\sigma}_{rz}^*(\xi, 0, s) = 0 \quad (59)$$

Next, implying the boundaries (58) and (59) to determine the unknown parameters, we get,

$$\sum_{i=1}^{2} A_i(\xi, s)(k_i^2 - s^2) + \frac{s^{\alpha-1}(1+\tau_0 s) q^2}{q_1^2 q_2^2} \frac{Q_0 \pi \tau (1+e^{-s\tau})}{(s^2\tau^2+\pi^2)} = \bar{f}^*(\xi, s) \quad (60)$$

$$-2B(\xi, s) q_3 + (\xi^2 + q_3^2) \sum_{i=1}^{2} A_i(\xi, s) - \frac{s^{\alpha-1}(1+\tau_0 s)}{q_1^2 q_2^2} \frac{Q_0 \pi \tau (1+e^{-s\tau})}{(s^2\tau^2+\pi^2)} = 0 \quad (61)$$

$$-(1+q_3^2) B(\xi, s) + \sum_{i=1}^{2} A_i(\xi, s)\, q_i\, (\xi^2 + 1) = 0 \quad (62)$$

Numerical method is utilized to obtain the complete solution of the problem on using equations (60) to (62).

INVERSION OF DOUBLE TRANSFORMS

Numerical inverting method is adopted to remove the complexity in Laplace transform inversion by using the algorithm by Gaver-Stehfast [17-19, 20]. Gaver and Stehfast approximation modified formula is as

$$f(t) = \frac{\ln 2}{t} \sum_{p=1}^{\psi} D(p, \psi) = (-1)^{p+M} \sum_{n=m}^{\min(p,M)} \frac{n^M (2n)!}{(M-n)!(n-1)!(p-n)!(2n-p)!} F\left(p \frac{\ln 2}{t}\right) \quad (63)$$

Here, ψ - even integer, $M = \psi/2$, and m is the integer part of the $(p+1)/2$. Here with the use of Stehfast algorithm optimal value of ψ was chosen for fast convergence and high accuracy of results. In order to evaluate the involved integral the technique by Romberg numerical integration [19] with variable step size was used. Numerical analysis is done by MATLAB.

Numerical Results And Discussion

Following [20] we consider copper as the thermoelastic material with following physical constants

$k = 386\, J/.K.m.s \qquad \alpha_t = 1.78\times10^{-5}/K$

$C_E = 383.1 J\ K/g.K$

$\mu = 3.86\times10^{10}\, N./m \ \lambda = 7.76\times10^{10}\, N./m$

$\rho = 8954\, k.g/.m \ \tau_0 = 0.02s \ \ T_0 = 293K$

$\varepsilon = 0.0168 N.m./J$, $c_1 = 4.158\times10^{3} m./s$

$\eta = 8886.73 s./m$, $\beta^2 = 4$, $a = 1$, $\theta_0 = 1$, $b = 1$

The above said numerical investigation for different α when t = 0.15. In Figures 1–2, the distribution of temperature and axial stresses are plotted along radii with different α .

1. Finite speeds of propagation it is noted for heat flow and elastic effects.
2. The distribution of temperature and thermal stress found directly proportional to parameter α .
3. Distributions of temperature and stress show that there is strongly dependency of variation on α .

Hence, we can say that fractional parameter of order α significantly affects the temperature and stress distribution for infinite half space.

Conclusion

In this article, we studied a two dimensional time fractional problem of infinite half space problem with one relaxation time with varying periodically heat source. It is noticed that due to variation of heat source along the radial direction non-dimensional temperature and axial stress component changes non-uniformly with fractional parameter. Heat wave flow here assumes finite speed of propagation because of presence of relaxation parameter in the basic equations.

Also the graphical analysis shows a significant relationship between the thermal field propagation with fractional parameters. The distributions of temperature and axial stress increase with increasing of α . Consideration fractional order parameter gives new classification of materials which establish relation between material ability to conduct with heat flow.

Hence, from this study we can say that the effect of fractional theory on the generalized thermoelasticity more realistically describes the behavior of the particles as compare to integer order. Finally, it is concluded that the solutions discussed in this problem will plays significant role for the development of new structures having wide applicable in real life.

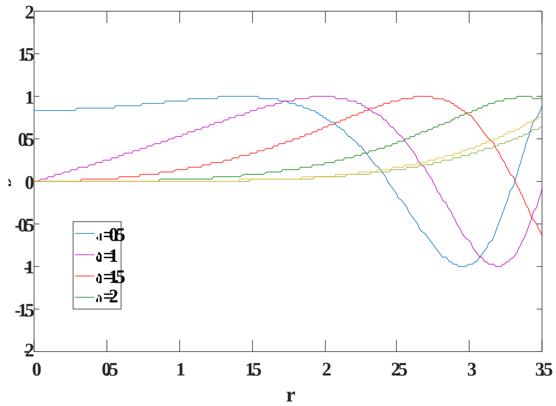

Figure 1: Dimensionless temperature distribution along radial direction for different value of fractional derivative α

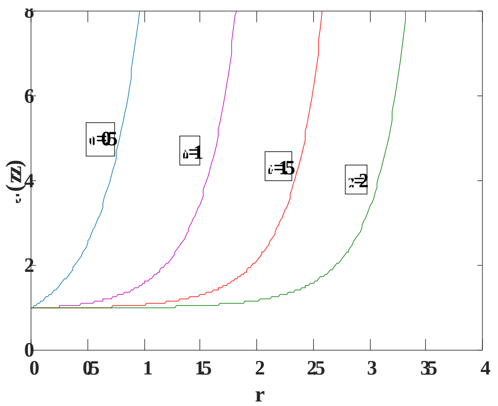

Figure 2: Dimensionless axial stress distribution along radial direction for different value of fractional derivative α

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