RESEARCH ARTICLE

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Thermoelastic Modeling of Time Fractional Heat conduction in Circular Disk with Internal Heat Generation

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ABSTRACT

In this paper, thermoelastic modeling of a circular sector disk is done under the influence of fractional time theory of thermoelasticity to evaluate the changes in temperature flow, displacement and stress behavior subjected to internal heat generation within it. At the fixed edge a time dependent heat flux is applied while thermally insulation type boundary are maintained at upper part of surface as well as the lower part surface is kept at zero temperature. Also certain boundary surfaces at $\varphi = 0,\ \varphi = \varphi_0$ are fixed as a function of temperatures field $f_1,\ f_2$ which is function of (r, z, t) respectively. Furthermore, method of Integral transformations is employed to determine the required solution of equation of heat conduction. The obtained expressions areof Bessel's type and in form of series. For numerical analysis the pure aluminum circular sector disk considered and obtained results are plotted as shown in figures.

Keywords: thermaldisk sector; fractional calculus theory; Method of Integral transformation, stress behavior.

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INTRODUCTION

A differential equation is said to beof fractional order if it's involves derivatives of fractional type. Fractional order theory of thermoelasticity is based on the heat conduction equation of fractional order and is utilized to solve many heat exchange problems in which physical properties are described in terms of fractional order. Various application of fractional order theory is observed in complex media for experimental as well as theoretical studies. Some of the applications found in polymers, glasses, dielectrics and semiconductors, random and disordered materials, porous, etc. Further generalization in heat equations constructs corresponding new generalized thermal stresses theory like Green and Naghdi theory based on thermo elasticity without energy dissipation, whereas Lord and Shulman theory leads telegraph equation by Cattaneo's. Due to memory based affects a lot of application of this theory found in field of science and technology, bioengineering, rheology, robotics etc.

Povstenko [1-6] develops lots of mathematical models with the heat conduction equation of fractional order and determined corresponding thermal stresses. Kamdi et al. [7-9] determined stress behavior due to temperature distribution in finite bodies respectively by fractional approach theory with convection on surfaces. Thakre and Warbhe [10, 11] studied time fractional order theory in non-homogeneous bodies subjected to moving heat source. Lamba and Khobragade [12] determined temperature and stresses in an uncoupled thermoelastic Analysis of a thick cylinder with radiation. Kedar et al. [13] examine thermoelastic state of a thermally sensitive F. G. thick hollow cylinder by integral transformation. Other work related to the

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circular bodies done by different researchers is as reflected in [14-19].

By looking the current scenario and applications of non classical theory especially fractional time order in physical domain, here the author is motivated to study and examine the thermal model of sector disk by considering fractional time order heat conduction equation.

FORMULATION OF THE PROBLEM

A mathematical model for the sector circular disk is constructed by assuming dimensions $0 \le r \le b, 0 \le \varphi_0 \le \varphi \le 2\pi,\ 0 \le z \le h,$ within the effect of internal heat generation $Q(r, \varphi, z, t)$ in it.The heat equation for this sector disk is modeled in the fractional time order derivative $0 \le \alpha \le 2$. At the fixed edge a time dependent heat flux is applied while thermally insulation type boundary are maintained at upper part of surface as well as the lower part surface is kept at zero temperature. Also certain boundary surfaces at $\varphi = 0,\ \varphi = \varphi_0$ are subjectedas a function of temperatures field $f_1,\ f_2$ which is function of (r, z, t)respectively.

Further, mathematical heat equation is presented for conduction in circular sector disk by assuming α order time fraction (Caputo type) [8] as

$$\frac{d^\alpha T(t)}{dt^\alpha} = \begin{cases} \dfrac{1}{\Gamma(p-\alpha)} \displaystyle\int_0^t (t-\tau)^{p-\alpha-1} \frac{d^p T(\tau)}{d\tau^p} d\tau : p-1 < \alpha < p, \\ \dfrac{d^p T(\tau)}{dt^p}, \qquad \alpha = p \end{cases} \tag{1}$$

Laplace transform rule for above fraction derivative with parameter p is defined as

$$L\left\{\frac{d^\alpha T(t)}{dt^\alpha}\right\} = s^\alpha L\{\bar{T}(s)\} - \sum_{k=0}^{p-1} T^{(k)}(0^+) s^{\alpha-1-k},\ \ p-1 < \alpha < p \tag{2}$$

TEMPERATURE DISTRIBUTION

The following differential heat equation describes the flow of temperature $\theta(r, \varphi, z, t)$ along effect of internal source of heat in context of time-fractional derivate of order α is

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{1}{r^2}\frac{\partial^2\theta}{\partial\varphi^2} + \frac{\partial^2\theta}{\partial z^2} + \frac{Q(r,\varphi,z,t)}{K} = \frac{1}{a}\frac{\partial^\alpha\theta}{\partial t^\alpha} \tag{3}$$

Corresponding boundaries as

$$k \left.\frac{\partial\theta}{\partial r}\right|_{r=b} = g(\varphi, z, t) \tag{4}$$

$$\theta|_{\varphi=0} = f_1(r, z, t) \tag{5}$$

$$\theta|_{\varphi=\varphi_0} = f_2(r, z, t) \tag{6}$$

$$\theta|_{z=0} = 0 \tag{7}$$

$$k \left.\frac{\partial\theta}{\partial z}\right|_{z=h} = 0 \tag{8}$$

$$\theta\Big|_{t=0} = 0, \qquad 0 < \alpha \le 2\ 3 \tag{9}$$

$$\frac{\partial \theta}{\partial t}\Big|_{t=0} = 0 \qquad 1 < \alpha \le 2 \tag{10}$$

For the material of the circular sector disk, K stands for thermal conductivity, a demotes thermal diffusivity and k for heat transfer coefficient. Also $Q(r, \varphi, z, t)$ is the internal heat generation within it.

DISPLACEMENT POTENTIAL AND THERMAL STRESSES

The relation between displacement components U_i and temperature for a thin body in plane stress state is as [2]

$$(1-v)\frac{\partial^2 U_i}{\partial x_k^2} + (1+v)\frac{\partial}{\partial x_i}\frac{\partial U_k}{\partial x_k} = 2(1+v)a_t\frac{\partial \theta}{\partial x_i} \quad ; \quad k, i = 1, 2 \tag{11}$$

Introducing,

$$U_i = \frac{\partial \psi}{\partial x_i} \ ; \ i = 1, 2 \tag{12}$$

We have

$$\frac{\partial^2 \psi}{\partial x_k^2} = (1+v)\, a_t\, \theta \tag{13}$$

$$\sigma_{ij} = 2\mu\left(\frac{\partial^2 \psi}{\partial x_i\, \partial x_j} - \delta_{ij}\frac{\partial^2 \psi}{\partial x_k^2}\right) \ ; \ i, j, k = 1, 2 \tag{14}$$

Here, for sector disk v and a_t respectively denotes Poisson's ratio and the linear coefficient of material thermal expansion, e refers for dilation, further Lame constant is referred as μ and Kronecker symbol by δ_{ij}.

Next, relationship between temperature flow and Displacement potential $\psi(r, \varphi, z, t)$ is as

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} = (1+v)a_t T \tag{15}$$

with

$$\frac{\partial \psi}{\partial r} = 0 \quad at \quad r = b \tag{16}$$

The stresses are associated to displacement potential as,

$$\sigma_{rr} = -\frac{2\mu}{r}\frac{\partial \psi}{\partial r} \tag{17}$$

$$\sigma_{\theta\theta} = -2\mu\frac{\partial^2 \psi}{\partial r^2} \tag{18}$$

For the condition of traction-free in sector disk one have

$$\sigma_{r\varphi} = \sigma_{rr} = 0 \qquad : r = b, 0 \le \varphi_0 \le \varphi, \ t > 0 \tag{19}$$

$$\sigma_{r\varphi} = \sigma_{\varphi\varphi} = 0 \qquad : \varphi = 0, 0 \le r \le b, \ t > 0 \tag{20}$$

$$\sigma_{r\varphi} = \sigma_{\varphi\varphi} = 0 \qquad : \varphi = \varphi_0, 0 \le r \le b,\ t > 0 \tag{21}$$

Above eqn's from (3) to (21) is the mathematical construction formulation of the sector disk under certain prescribed boundaries with effect of heat generation.

EVALUATION OF TEMPERATURE FLOW

To find the expression for temperature flow, at first one define the Fourier finite transform and its corresponding inversion formula with respect to variable z in the range $0 \le z \le h$ defined in [1] as

$$\bar{f}(\xi_p) = \int_0^h K_1(\xi_p, z') f(z') dz' \tag{22}$$

$$f(z) = \sum_{p=1}^{\infty} K_1(\xi_p, z) \bar{f}(\xi_p) \tag{23}$$

where $K_1(\xi_p, z) = \dfrac{\sin(\xi_p, z)}{\sqrt{h/2}}$

and $\xi_1, \xi_2, \xi_3, \ldots\ldots$ denotes the roots(positive) of the equation

$\sin(\xi_p, h) = 0;\ p = 1, 2, 3, \ldots\ldots;$

i.e. $\xi_p = p\pi / h\ ;\ \ p = 1, 2, 3, \ldots\ldots$

Secondly, the Fourier finite transform with its inversion for the variable φ is defined below for the range $0 \le \varphi \le \varphi_0$

$$\overline{F}(\nu) = \int_0^{\varphi_0} K_2(\nu, \varphi') F(\varphi') d\varphi' \tag{24}$$

$$F(\varphi) = \sum_1^{\infty} K_2(\nu, \varphi) \overline{F}(\nu) \tag{25}$$

Where, $K_2(\nu, \varphi) = \dfrac{\sin(\nu\varphi)}{\sqrt{\varphi_0/2}}$

Also, the Eigen values ν are the roots (positive) of $\sin(\nu\varphi_0) = 0$

i.e. $\nu = n\pi/\varphi_0\ ;\ \ n = 1, 2, 3, \ldots\ldots$

Lastly, Hankel finite transform and its corresponding inversion for r in the range $0 \le r \le b$ are defined in [1] respectively as,

$$\overline{G}(\beta_m) = \int_0^b r' K_3(\beta_m, r') G(r') dr' \tag{26}$$

$$G(r) = \sum_m^{\infty} K_3(\beta_m, r) \overline{G}(\beta_m) \tag{27}$$

Where, $K_3(\beta_m, r) = \dfrac{\sqrt{2}}{a} \dfrac{J_o(\beta_m, r)}{J_o(\beta_m, b)}$ (28)

$\beta_1, \beta_2, \beta_3, \ldots.$ Denotes the root (positive) of equation $J_1(\beta_m, b) = 0.$

Now, on employing the transformation as defined above in equations (22), (24) and (26) one get

$$\frac{\partial^{\alpha}\overline{\overline{\overline{\theta}}}\left(\beta_m,\nu,\xi_p,t\right)}{\partial t^{\alpha}}+a\left(\beta_m^2+\frac{\nu^2}{r^2}+\xi_p^2\right)\overline{\overline{\overline{\theta}}}\left(\beta_m,\nu,\xi_p,t\right)=A\left(\beta_m,\nu,\xi_p,t\right) \tag{28}$$

$$A\left(\beta_m,\nu,\xi_p,t\right)=\frac{a}{K}\overline{\overline{\overline{Q}}}\left(\beta_m,\nu,\xi_p,t\right)+\frac{a}{K}b\,K_3\left(\beta_m,b\right)\overline{\overline{g}}\left(\nu,\xi_p,t\right)$$

$$\left\{\begin{array}{l}\dfrac{dK_0(\nu,\varphi)}{d\varphi}\overline{\overline{f_1}}\left(\beta_m,\xi_p,t\right)\Big|_{\varphi=0}\\ -\dfrac{dK_0(\nu,\varphi)}{d\varphi}\overline{\overline{f_2}}\left(\beta_m,\xi_p,t\right)\Big|_{\varphi=\varphi_0}\end{array}\right\} \tag{29}$$

$$\overline{\overline{\overline{\theta}}}=0, \qquad at \quad t=0\,, \qquad 0<\alpha\le 2 \tag{30}$$

$$\frac{\partial\overline{\overline{\overline{\theta}}}}{\partial t}=0\,, \qquad at \quad t=0\,, \qquad 1<\alpha\le 2 \tag{31}$$

Next, on applying the Laplace transform defined in equations (2) and taking its inversion one obtain

$$L^{-1}\left\{s^{\alpha}+a\left(\beta_m^2+\frac{\nu^2}{r^2}+\xi_p^2\right)\right\}=E_{\alpha}\left[-a\left(\beta_m^2+\frac{\nu^2}{r^2}+\xi_p^2\right)t^{\alpha}\right] \tag{32}$$

$$\overline{\overline{\overline{\theta}}}\left(\beta_m,\nu,\xi_p,t\right)=E_{\alpha}\left[-a\left(\beta_m^2+\frac{\nu^2}{r^2}+\xi_p^2\right)t^{\alpha}\right]A\left(\beta_m,\nu,\xi_p,t\right) \tag{33}$$

Finally, on employing the corresponding inversion formula of transformation using (23), (25) and (27) the temperature flow in sector disk is obtained as

$$\theta(r,\varphi,z,t)=\sum_{m=1}^{\infty}\sum_{\nu}\sum_{p=1}^{\infty}K_3\left(\beta_m,r\right)K_2\left(\nu,\varphi\right)K_1\left(\xi_p,z\right)\times E_{\alpha}\left[-a\left(\beta_m^2+\frac{\nu^2}{r^2}+\xi_p^2\right)t^{\alpha}\right]$$

$$\left\{\frac{a}{K}\int_{r'=0}^{b}\int_{\varphi'=0}^{\varphi_0}\int_{z'=0}^{h}r'K_3\left(\beta_m,r'\right)K_2\left(\nu,\varphi'\right)K_1\left(\xi_p,z'\right)\times Q(r',\varphi',z',t)dr'd\varphi'dz'\right.$$

$$+\frac{a}{K}b\,K_1\left(\beta_m,b\right)\int_{\varphi'=0}^{\varphi_0}\int_{z'=0}^{h}K_2\left(\nu,\varphi'\right)K_1\left(\xi_p,z'\right)g\left(\varphi',z',t\right)d\varphi'dz'$$

$$+a\nu\sqrt{2/\varphi_0}\int_{r'=0}^{b}\int_{z'=0}^{h}r'K_3\left(\beta_m,r'\right)K_1\left(\xi_p,z'\right)f_1(r',z',t)dr'dz'$$

$$\left.-a\nu\sqrt{2/\varphi_0}\cos\left(\nu\varphi_0\right)\int_{r'=0}^{b}\int_{z'=0}^{h}r'K_3\left(\beta_m,r'\right)K_1\left(\xi_p,z'\right)f_2(r',z',t)dr'dz'\right\} \tag{34}$$

DETERMINATION OF DISPLACEMENT POTENTIAL FUNCTION

Using equations (34) in (16), the required equation for distribution of displacement potential is evaluated as below

$$\psi = -(1+\nu)a_t \sum_{m=1}^{\infty}\sum_{\nu}\sum_{p=1}^{\infty}\frac{1}{\beta_m^2} K_3(\beta_m, r) K_2(\nu,\varphi) K_1(\xi_p, z) \times E_\alpha\left[-a\left(\beta_m^2+\frac{\nu^2}{r^2}+\xi_p^2\right)t^\alpha\right]$$

$$\left\{\frac{a}{K}\int_{r'=0}^{b}\int_{\varphi'=0}^{\varphi_0}\int_{z'=0}^{h} r' K_3(\beta_m, r') K_2(\nu,\varphi') K_1(\xi_p, z') \; Q(r',\varphi',z',t)dr'd\varphi'dz'\right.$$

$$+\frac{a}{K} b\, K_3(\beta_m, b)\int_{\varphi'=0}^{\varphi_0}\int_{z'=0}^{h} K_2(\nu,\varphi') K_1(\xi_p, z') g(\varphi',z',t) d\varphi'dz'$$

$$+a\nu\sqrt{2/\varphi_0}\int_{r'=0}^{b}\int_{z'=0}^{h} r' K_3(\beta_m, r')\, K_1(\xi_p, z') \times f_1(r',z',t)dr'dz'$$

$$-a\nu\sqrt{2/\varphi_0}\cos(\nu\varphi_0)\int_{r'=0}^{b}\int_{z'=0}^{h} r' K_3(\beta_m, r')\, K_1(\xi_p, z') \; f_2(r',z',t)dr'dz' \quad (35)$$

DETERMINATION OF THERMAL STRESSES

Using equations (35) in (18) and (19), the expression for stress are obtain as

$$\sigma_{rr} = -2(1+\nu)a_t\,\mu \sum_{m=1}^{\infty}\sum_{\nu}\sum_{p=1}^{\infty}\frac{1}{r\,\beta_m} K_3(\beta_m, r) K_2(\nu,\varphi) \times K_1(\xi_p, z) E_\alpha\left[-a\left(\beta_m^2+\frac{\nu^2}{r^2}+\xi_p^2\right)t^\alpha\right]$$

$$\left\{\frac{a}{K}\int_{r'=0}^{b}\int_{\varphi'=0}^{\varphi_0}\int_{z'=0}^{h} r' K_3(\beta_m, r') K_2(\nu,\varphi') K_1(\xi_p, z') \times Q(r',\varphi',z',t)dr'd\varphi'dz'\right.$$

$$+\frac{a}{K} b\, K_3(\beta_m, b)\int_{\varphi'=0}^{\varphi_0}\int_{z'=0}^{h} K_2(\nu,\varphi') K_1(\xi_p, z') \times g(\varphi',z',t) d\varphi'dz'$$

$$+a\nu\sqrt{2/\varphi_0}\int_{r'=0}^{b}\int_{z'=0}^{h} r' K_3(\beta_m, r')\, K_1(\xi_p, z') \times f_1(r',z',t)dr'dz'$$

$$\left.-a\nu\sqrt{2/\varphi_0}\cos(\nu\varphi_0)\int_{r'=0}^{b}\int_{z'=0}^{h} r' K_3(\beta_m, r')\, K_1(\xi_p, z') \; f_2(r',z',t)dr'dz'\right\} \quad (36)$$

$$\sigma_{\theta\theta} = -2(1+\nu)a_t\,\mu \sum_{m=1}^{\infty}\sum_{\nu}\sum_{p=1}^{\infty}\frac{1}{\beta_m}(\beta_m K_3(\beta_m, r) - K_3(\beta_m, r)/r)\, K_2(\nu,\varphi) K_1(\xi_p, z)$$

$$\times E_\alpha\left[-a\left(\beta_m^2+\frac{\nu^2}{r^2}+\xi_p^2\right)t^\alpha\right]\left\{\frac{a}{K}\int_{r'=0}^{b}\int_{\varphi'=0}^{\varphi_0}\int_{z'=0}^{h} r' K_3(\beta_m, r')\right.$$

$$\times K_2(\nu,\varphi') K_1(\xi_p, z') Q(r',\varphi',z',t)dr'd\varphi'dz'$$

$$+\frac{a}{K} b\, K_3(\beta_m, b)\int_{\varphi'=0}^{\varphi_0}\int_{z'=0}^{h} K_2(\nu,\varphi') K_1(\xi_p, z') g(\varphi',z',t) d\varphi'dz'$$

$$+a\nu\sqrt{2/\varphi_0}\int_{r'=0}^{b}\int_{z'=0}^{h} r'K_3(\beta_m,r')\,K_1(\xi_p,z')f_1(r',z',t)dr'dz'$$

$$-a\nu\sqrt{2/\varphi_0}\cos(\nu\varphi_0)\int_{r'=0}^{b}\int_{z'=0}^{h} r'K_3(\beta_m,r')\,K_1(\xi_p,z')\times f_2(r',z',t)dr'dz'\Big\} \quad (37)$$

Special Cases And Numerical Calculations

An instantaneous source of heat q_s^i for cylindrical region surface is fixed at the (r',φ',z') with releasing energy given as

$$Q(r,\varphi,z,t)=\frac{q_s^i}{2\pi r}\delta(r'-r_1)\delta(\varphi'-\varphi_1)\delta(z'-z_1)\delta(t)$$

Here, $q_s^i/2\pi r$ represents the source strength per unit area.

$$\theta(r,\varphi,z,t)=\frac{q_s^i}{2\pi r}\sum_{m=1}^{\infty}\sum_{\nu}\sum_{p=1}^{\infty}K_3(\beta_m,r)K_2(\nu,\varphi)K_1(\xi_p,z)\times E_\alpha\left[-a\left(\beta_m^2+\frac{\nu^2}{r^2}+\xi_p^2\right)t^\alpha\right]$$

$$\Big\{\frac{a}{K}r_1K_3(\beta_m,r_1)K_2(\nu,\varphi_1)K_1(\xi_p,z_1)+\frac{a}{K}b\,K_3(\beta_m,b)\int_{\varphi'=0}^{\varphi_0}\int_{z'=0}^{h}K_2(\nu,\varphi')K_1(\xi_p,z')g(\varphi',z',t)d\varphi'dz'$$

$$+a\nu\sqrt{2/\varphi_0}\int_{r'=0}^{b}\int_{z'=0}^{h} r'K_3(\beta_m,r')\,K_1(\xi_p,z')\times f_1(r',z',t)dr'dz'$$

$$-a\nu\sqrt{2/\varphi_0}\cos(\nu\varphi_0)\int_{r'=0}^{b}\int_{z'=0}^{h} r'K_3(\beta_m,r')\,K_1(\xi_p,z')\,f_2(r',z',t)dr'dz'\Big\} \quad (38)$$

Dimensions And Properties Of Material

For the purpose of numerical computation, here dimensions and properties of pure aluminum material in sector circular disk are assumed as

$$K=204W/mK,\ c_p=896\ J/Kg.K,$$

$$\mu=26.67\,GPa,\quad \rho=2707\,kg/m^3,\qquad \nu=0.35,$$

$$a=84.18\,m^2/s,\ a_t=22.2\times10^{-6}\frac{1}{K},$$

$$E=70\,GPa,\ b=2\,m,\ h=0.3\,m,\ \varphi_0=270^0,$$

$$r_1=1m,\ z_1=0.2\,m,\ \varphi_1=135^0$$

Graphical Presentation

The resultant analysis of temperature flow and stress variation for different conductivity that is weaker, normal and stronger are expressed graphically by employing Mathematica software incase of circular sector disk.

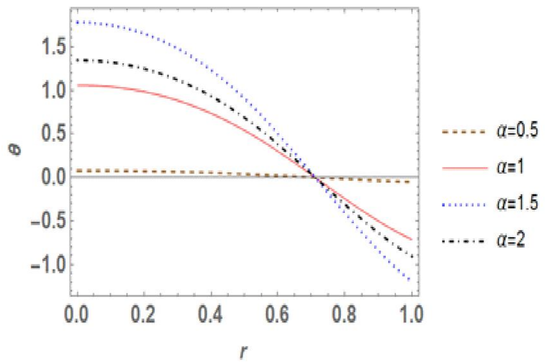

Figure 1: Temperature flow along radial direction for different α parameter

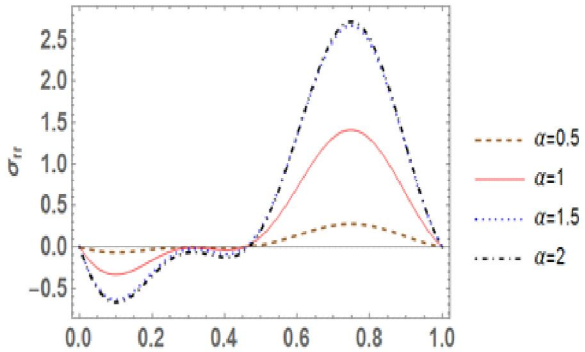

Figure 2: Radial stress flows along radial direction versus for different α parameter

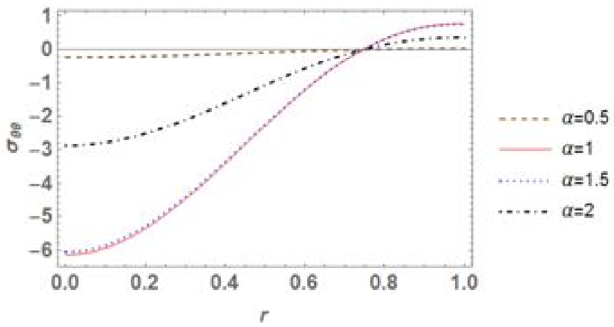

Figure 3: Tangential stress flows along radial direction versus for different α parameter

Figure 1 to 3, denotes the temperature and stress distribution function along radial direction for different value of α . Within the context of internal heat generation the temperature flow found tensile in nature initially and then flows non-uniformly also it becomes zero near to centre (*r* = 0.7) as well as becomes compressive near outer circular edge (*r* = 1). Also, σ_{rr} distribution found compressive till centre and becomes tensile towards outer edge. In Figure 3; variation of $\sigma_{\theta\theta}$ increases initially non-uniformly and becomes steady at outer edge. Further, from the graphical analysis it is observed that propagation of the thermo physical quantities is varying proportional to the fractional parameter α . So,the different fractional parameter variation play significant role in design and development of new structural materials which is highly applicable to physical process.

Conclusion

In this article, quasi-static mathematical modelling of fractional time order thermalproblem is done for a circular sector disk in context of internal heat source. Expressions for temperature flow and thermal variation of stresses are obtained by employing Hankel and the Fourier transformation method. On computing numerically by considering material properties of pure aluminum circular sector disk the following observations are noted as:

(a) Finite propagations speed is obtained in graphical plotting for temperature and thermal stresses.

(b) Fractional parameter significantly effects the thermal distribution.

(c) Fractional parameter makes materials able to conduct heat.

(d) The thermal wave propagation depends upon fractional parameter.

Thus,the study indicates that fractional parameter affects the thermal wave propagation directly and describes particle behavior more realistic than classical approach as well as this plays significant role for the design and development of new structural materials having wide applicable in physical process.

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