RESEARCH ARTICLE

SAMRIDDHI Volume 13, Issue 2, 2021 Print ISSN : 2229-7111 Online ISSN : 2454-5767

A Pseudo Projective Curvature Tensor on a Kenmotsu Manifold

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ABSTRACT

The paper deals the properties of pseudo projective curvature tensor in Riemannian and Kenmotsu manifolds.

Keywords: Riemannian manifold, Kenmotsu manifold, Einstein manifold, pseudo projective curvature tensor.

SAMRIDDHI : A Journal of Physical Sciences, Engineering and Technology, (2021); DOI : 10.18090/samriddhi.v13i02.11

INTRODUCTION

The almost contact manifold studied by Kenmotsu [9]. This structure is almost similar to the warped product of two Riemannian manifolds and differentiable manifold with this structure is known as Kenmotsu manifold [7] ; for example, hyperbolic space $M^n(-1)$.

Let M^n be an n-dimensional (where $n = 2m + 1$) differentiable manifold of class C^∞. If φ is a tensor field of type (1,1), ξ is a vector field and η is a 1-form on be such that

$$\varphi^2 = -I + \eta \otimes \xi, \qquad \varphi\xi = 0, \tag{1}$$

$$\eta o \varphi = 0, \qquad \eta\xi = 0, \tag{2}$$

then M^n is said to have an almost contact structure (φ, ξ, η) and is called an almost contact manifold [1].

If g is a Riemannian metric tensor field on M^n satisfies the following conditions:

$$g(\varphi X, \varphi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{3}$$

$$g(X,\xi) = \eta(X), \tag{4}$$

for any vector fields X and Y on M^n, then the structure (φ, ξ, η, g) is called an almost contact metric structure and is called almost contact metric manifold.

The almost contact metric manifold M^n is called Kenmotsu manifold if it satisfies

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How to cite this article : Kumar, Rajesh (2021). A Pseudo Projective Curvature Tensor on a Kenmotsu Manifold.

SAMRIDDHI : A Journal of Physical Sciences, Engineering and Technology, Volume 13, Issue (2), 125-134.

Source of support : Nil

Conflict of interest : None

$$(D_X\varphi)Y = g(\varphi X, Y)\xi - \eta(Y)\varphi X, \tag{5}$$

$$D_X \xi = X - \eta(X)\xi, \tag{6}$$

where D_x is the Riemannian connection of the Riemannian metric g and it also follows that

$$(D_X\eta)Y = g(X,Y) - \eta(X)\eta(Y). \tag{7}$$

In Kenmotsu manifold following relations also hold [2, 3, 4, 8],

$$g(R(X,Y)Z,\xi) = \eta(R(X,Y)Z) \tag{8}$$

$$= g(X,Z)\eta(Y) - g(Y,Z)\eta(X),$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{9}$$

$$R(X,\xi)Y = g(X,Y)\xi - \eta(Y)X, \tag{10}$$

$$R(\xi,X)Y = \eta(Y)X - g(X,Y)\xi, \tag{11}$$

$$S(X,\xi) = -(n-1)\eta(X), \tag{12}$$

$$S(\varphi X, \varphi Y) = S(X,Y) - (n-1)\eta(X)\eta(Y), \tag{13}$$

for all vector field X, Y, Z on M^n, where R is the Riemannian curvature tensor of the manifold and S is the Ricci tensor of the type (0, 2)defined as $S(X,Y)= g(QX, Y)$, where Q is the Ricci operator.

A Kenmotsu manifold is said to be Einstein manifold if

$$S(X,Y) = k\, g(X,Y), \tag{14}$$

where k = - (n-1)

Pseudo projective curvature tensor on a Riemannian manifold (M^n, g) $(n > 2)$ is defined as follows [12]

$$\tilde{P}(X,Y)Z \,\&= a\,R(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y] - \frac{r}{n}\left(\frac{a}{n-1} + b\right)[g(Y,Z)X - g(X,Z)Y], \tag{15}$$

where a and b are constant such that $a,b \neq 0$ and r denote the scalar curvature of the manifold. If $a=1$ and $b = -\frac{1}{n-1}$ then (15) takes the form

$$P(X,Y)Z = R(X,Y)Z - \frac{1}{n-1}[S(Y,Z)X - S(X,Z)Y] \tag{16}$$

which is know as Weyl projective curvature tensor [10].

The concircular curvature tensor C and conformal curvature tensor V are given by [10]

$$C(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y], \tag{17}$$

$$V(X,Y)Z = R(X,Y)Z - \frac{1}{n-}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY] + \frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y]. \tag{18}$$

Kenmotsu manifold and its properties have been studied by several authors such as De [2], Sinha and Srivastava [13], Jun, et.al.[8], De and Pathak [3], De, et.al. [4], Özgür and De [11] and many others.

The pseudo projective curvature tensor

Theorem 1. *The pseudo projective curvature tensor and Weyl projective curvature tensors of the Riemannian manifold M^n are linearly dependent if and only if M^n is an Einstein manifold.*

Proof. Let us consider

$$\tilde{P}(X,Y)Z = \alpha\, P(X,Y)Z,$$

where α being any non-zero constant.

In view of (15) and (16), above equation becomes

$$(a-\alpha)R(X,Y)Z + \left(b + \frac{\alpha}{n-1}\right)[S(Y,Z)X - S(X,Z)Y] - \frac{r}{n}\left(\frac{a}{n-1} + b\right)[g(Y,Z)X - g(X,Z)Y] = 0.$$

Contracting above with respect to, we get (19)

$$S(Y,Z) = \frac{r}{n}\, g(Y,Z),$$

which shows that the Riemannian manifold M^n is Einstein manifold.

In consequence of (15), (16) and (19), we can obtain the converse part of the theorem.

Theorem 2. *A Riemannian manifold will be an Einstein manifold if and only if the pseudo projective curvature tensor $\tilde{P}$ and concircular curvature tensor C are linearly dependent.*

Proof. Let us consider

$$\tilde{P}(X,Y)Z = \alpha\, C(X,Y)Z,$$

where α being any non-zero constant.

In view of (15) and (17), above equation becomes

$$(a-\alpha)R(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y] - \frac{r}{n}\left(\frac{a-\alpha}{n-1} + b\right)[g(Y,Z)X - g(X,Z)Y] = 0$$

Contracting above with respect to X, we obtain (19).

In consequence of (15), (17) and (19), we can obtain the converse part of the theorem.

Theorem 3. *A Riemannian manifold will be be an Einstein manifold if and only if the pseudo projective curvature tensor $\tilde{P}$ and conformal curvature tensor V are linearly dependent.*

The proof is straight forward as Theorem 2.

Corollary 1. *In an -dimensional Riemannian manifold M^n, the following are equivalent*

(i): M^n is an Einstein manifold,

(ii): pseudo projective curvature tensor and Weyl projective curvature tensor are linearly dependent,

(iii): pseudo projective curvature tensor and concircular curvature tensor are linearly dependent,

(iv): pseudo projective curvature tensor and conformal curvature tensor are linearly dependent.

Pseudo Projectively Flat Kenmotsu Manifold

Let us assume that $\tilde{P}(X,Y)Z = 0$.

Then from (15), we have

$$a\,R(X,Y)Z = -b[S(Y,Z)X - S(X,Z)Y] + \frac{r}{n}\left(\frac{a}{n-1} + b\right)[g(Y,Z)X - g(X,Z)Y]. \quad (20)$$

On taking $Z = \xi$ in (20) and using (2), (4), (9) and (12), we obtain

$$(r + n(n-1))\,[\eta(X)Y - \eta(Y)X] = 0, \text{ provided } a + b(n-1) \neq 0 .$$

Since $[\eta(X)Y - \eta(Y)X] \neq 0$, hence from above, we get

$$r = -n(n-1).$$

Thus, we can state.

Theorem 4. *The scalar curvature r of a pseudo projectively flat Kenmotsu manifold M^n $(n > 2)$ is constant and given by $r = -(n-1)$, provided $a + b(n-1) \neq 0$.*

Einstein Kenmotsu Manifold Satisfying

In this section we assume that $\tilde{P}(X,Y)Z = 0$.

Hence from (15), we get

$$a\,'R(X,Y,Z,W) = -b[S(Y,Z)g(X,W) - S(X,Z)g(Y,W)]$$
$$+\frac{r}{n}\left(\frac{a}{n-1} + b\right)[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)],$$

where $'R(X,Y,Z,W) = g(R(X,Y)Z,W)$.

Above equation can be replaced in the following form by putting $X = W = \xi$,

$$a\,'R(\xi,Y,Z,\xi) = -b[S(Y,Z) - S(\xi,Z)\eta(Y)] + \frac{r}{n}\left(\frac{a}{n-1} + b\right)[g(Y,Z) - \eta(Y)\eta(Z)]. \quad (21)$$

In view of (8), (12) and (14), (21) becomes

$$[a + b(n-1)][r + n(n-1)]g(\varphi Y,\varphi Z) = 0. \quad (22)$$

Since $g(\varphi Y,\varphi Z) \neq 0$, hence from (22), we get

$r = -n(n-1)$, provided $a + b(n-1) \neq 0$.

Hence we have the following theorem.

Theorem 5. *The scalar curvature of a pseudo projectively flat Einstein Kenmotsu manifold M^n $(n > 2)$ is constant and given by $r = -n(n-1)$, provided $a + b(n-1) \neq 0$.*

Contracting (15) with respect to X, we get

$$(C_1^1\tilde{P})(Y,Z) = [a + b(n-1)][S(Y,Z) - \frac{r}{n}g(Y,Z)], \quad (23)$$

where $(C_1^1\tilde{P})(Y,Z)$ denotes the contraction $\tilde{P}(X,Y)Z$ of with respect to X.

Let us consider that in a Kenmotsu manifold

$$(C_1^1\tilde{P})(Y,Z) = 0. \quad (24)$$

Then from (23) and (24), we get

$$S(Y,Z) = \frac{r}{n}g(Y,Z), \text{ provided } a + b(n-1) \neq 0, \quad (25)$$

which shows that M^n is an Einstein manifold.

Replacing Z by ξ in (25) and then using (21), we get

$$[r + n(n-1)]\eta(Y) = 0$$

Since $\eta(Y) \neq 0$, hence we get $r = -n(n-1)$ provided $a + b(n-1) \neq 0$.

Hence we have the following theorem.

Theorem 6. *In a Kenmotsu manifold $M^n(n>2)$ if the relation $(C_1^1\tilde{P})(Y,Z) = 0$ hold, then M^n is an Einstein manifold and its scalar curvature is given by $r = -n(n-1)$ provided $a + b(n-1) \neq 0$.*

An Einstein Kenmotsu Manifold Satisfying $(div\,\tilde{P})(X,Y)Z = 0$

Definition 1. *A manifold $(M^n, g)(n > 2)$ is called pseudo projective conservative if $div\,\tilde{P} = 0$* [6].

In this section we assume that

$$div\,\tilde{P} = 0 \tag{26}$$

where div denotes divergence.

Now, differentiating (15) covariantly, we get

$$(D_U\tilde{P})(X,Y)Z = a(D_U R)(X,Y)Z + b[(D_U S)(Y,Z)X - (D_U S)(X,Z)Y]$$
$$-\frac{D_U r}{n}\left(\frac{a}{n-1}+b\right)[g(Y,Z)X - g(X,Z)Y] \tag{27}$$

On contracting (27), we get

$$(div\tilde{P})(X,Y)Z = a(div R)(X,Y)Z + b[(D_X S)(Y,Z) - (D_Y S)(X,Z)]$$
$$-\frac{1}{n}\left(\frac{a}{n-1}+b\right)[g(Y,Z)(dr\,X) - g(X,Z)(dr\,Y)] \tag{28}$$

But from [5], we have

$$(div R)(X,Y)Z = (D_X S)(Y,Z) - (D_Y S)(X,Z) \tag{29}$$

Taking Kenmotsu manifold as an Einstein manifold, and using (14) in (29), we get

$$(div R)(X,Y)Z = (D_X S)(Y,Z) - (D_Y S)(X,Z) = 0 \tag{30}$$

Using (30) in (28), we get

$$(div\tilde{P})(X,Y)Z = -\frac{1}{n}\left(\frac{a}{n-1}+b\right)[g(Y,Z)(dr\,X) - g(X,Z)(dr\,Y)] \tag{31}$$

In consequence of (26), (31) becomes

$g(Y,Z)(dr\,X) - g(X,Z)(dr\,Y) = 0$ provided $a + b(n-1) \neq 0$,

which shows that r is constant.

Again, if r is constant then from (31), we get $(div\tilde{P})(X,Y)Z$.

Hence we have the following theorem.

Theorem 7. *A Kenmotsu manifold $(M^n, g)(n > 2)$ is pseudo projectively conservative if and only the scalar curvature is constant, provided $a + b(n-1) \neq 0$.*

AN EINSTEIN KENMOTSU MANIFOLD SATISFYING $R(X,Y)\cdot\tilde{P} = 0$

Let us consider that

$$R(X,Y)\cdot\tilde{P}(U,V)W = 0 \tag{32}$$

and the Kenmotsu manifold M^n be an Einstein manifold, then in view of (14) and (15), we have

$$\tilde{P}(X,Y)Z = a\,R(X,Y)Z + \left(bk - \frac{r}{n}\left(\frac{a}{n-1}+b\right)\right)[g(Y,Z)X - g(X,Z)Y]$$

It can be written as

$$`\tilde{P}(X,Y,Z,W) = a\,`R(X,Y,Z,W) + \left(bk - \frac{r}{n}\left(\frac{a}{n-1}+b\right)\right)[g(Y,Z)g(X,W) - g(X,Z)g(Y,W)] \tag{33}$$

where $'\tilde{P}(X,Y,Z,W)=g(\tilde{P}(X,Y)Z,W)$

Using (8) in (33), we get

$$\eta(\tilde{P}(X,Y)Z)=\left[-a+bk-\frac{r}{n}\left(\frac{a}{n-1}+b\right)\right][g(Y,Z)\eta(X)-g(X,Z)\eta(Y)] \tag{34}$$

Taking $X=\xi$ in (34) and using (4), we get

$$\eta(\tilde{P}(\xi,Y)Z)=\left[-a+bk-\frac{r}{n}\left(\frac{a}{n-1}+b\right)\right][g(Y,Z)-\eta(Y)\eta(Z)] \tag{35}$$

Again taking $Z=\xi$ in (34) and using (4), we get

$$\eta(\tilde{P}(X,Y)\xi)=0 \tag{36}$$

Now,

$$\begin{aligned}R(X,Y)\cdot\tilde{P}(U,V)W&=R(X,Y)\tilde{P}(U,V)W-\tilde{P}(R(X,Y)U,V)W\\&-\tilde{P}(U,R(X,Y)V)W-\tilde{P}(U,V)R(X,Y)W\end{aligned} \tag{37}$$

In consequence of (32), equation (37) becomes

$$R(X,Y)\tilde{P}(U,V)W-\tilde{P}(R(X,Y)U,V)W-\tilde{P}(U,R(X,Y)V)W-\tilde{P}(U,V)R(X,Y)W=0$$

Therefore

$$\begin{aligned}&g(R(\xi,Y)\tilde{P}(U,V)W,\xi)-g(\tilde{P}(R(X,Y)U,V)W,\xi)\\&-g(\tilde{P}(U,R(\xi,Y)V)W,\xi)-g(\tilde{P}(U,V)R(\xi,Y)W,\xi)=0\end{aligned}$$

From this it follows that

$$\begin{aligned}&-'\tilde{P}(U,V,W,Y)+\eta(Y)\eta(\tilde{P}(U,V)W)+g(Y,U)\eta(\tilde{P}(\xi,V)W)\\&-\eta(U)\eta(\tilde{P}(Y,V)W)+g(Y,V)\eta(\tilde{P}(U,\xi)W)-\eta(V)\eta(\tilde{P}(U,Y)W)\\&+g(Y,W)\eta(\tilde{P}(U,V)\xi)-\eta(W)\eta(\tilde{P}(U,V)Y)=0\end{aligned} \tag{38}$$

Using (34), (35) and (36), it follows from (38) that

$$'\tilde{P}(U,V,W,Y)=\left(-a+bk-\frac{r}{n}\left(\frac{a}{n-1}+b\right)\right)[g(V,W)g(Y,U)-g(Y,V)g(U,W)] \tag{39}$$

In consequence of (33) and (39), we get

$'R(U,V,W,Y)=g(Y,V)g(U,W)-g(V,W)g(Y,U)$, provided $a\neq 0$.

Hence we can state the following theorem.

Theorem 8. *If in an Einstein Kenmotsu manifold $(M^n,g)(n>2)$ the relation $R(X,Y)\cdot\tilde{P}=0$ holds then M^n is locally isometric to the hyperbolic space $H^n(-1)$.*

THE IRROTATIONAL PSEUDO PROJECTIVE CURVATURE TENSOR

Definition 2. *Let D be a Riemannian connection, then the rotation (curl) of pseudo projective curvature tensor $\tilde{P}$ on a Riemannian manifold M^n is defined by*

$$Rot\,\tilde{P} = (D_U\tilde{P})(X,Y)Z + (D_X\tilde{P})(U,Y)Z + (D_Y\tilde{P})(X,U)Z - (D_Z\tilde{P})(X,Y)U \tag{40}$$

By the use of Bianchi's second identity in (40), we have

$$Rot\,\tilde{P} = -(D_U\tilde{P})(X,Y)U \tag{41}$$

For the irrotational pseudo projective curvature tensor, we have

$$Rot\,\tilde{P} = 0 \qquad \text{i.e.} \qquad (D_Z\tilde{P})(X,Y)U = 0,$$

which gives

$$D_Z\left(\tilde{P}(X,Y)U\right) = \tilde{P}(D_Z X,Y)U + \tilde{P}(X,D_Z Y)U + \tilde{P}(X,Y)D_Z U \tag{42}$$

Replacing U by ξ in (42), we have

$$D_Z\left(\tilde{P}(X,Y)\xi\right) = \tilde{P}(D_Z X,Y)\xi + \tilde{P}(X,D_Z Y)\xi + \tilde{P}(X,Y)D_Z\xi \tag{43}$$

Now, substituting $Z = \xi$ in (15) and using (2), (4), (9) and (12), we obtain

$$\tilde{P}(X,Y)\xi = \rho[\eta(X)Y - \eta(Y)X] \tag{44}$$

where

$$\rho = \left[a + (n-1)b + \frac{r}{n}\left(\frac{a}{n-1} + b\right)\right] \tag{45}$$

Using (44) in (43), we obtain

$$\tilde{P}(X,Y)Z = \rho[g(X,Z)Y - g(Y,Z)X] \tag{46}$$

By virtue of (15) and (46), we get

$$\rho[g(X,Z)Y - g(Y,Z)X] = a\,R(X,Y)Z + b[S(Y,Z)X - S(X,Z)Y]$$

$$-\frac{r}{n}\left(\frac{a}{n-1} + b\right)[g(X,Z)Y - g(Y,Z)X].$$

Contracting above equation with respect to the vector and using (45), we get

$$S(Y,Z) = -(n-1)g(Y,Z) \tag{47}$$

which gives

$$r = -n(n-1) \tag{48}$$

In consequence of (15), (45), (46), (47) and (48), we can find

$$R(X.Y)Z = g(X,Z)Y - g(Y,Z)X$$

Thus we can state the following

Theorem 9. *The pseudo projective curvature tensor in an Einstein Kenmotsu manifold $M^n\,(n>2)$ will be locally isometric to the hyperbolic space $H^n(-1)$ if and only if it is irrotational.*

Theorems 8 together with Theorem 9 lead to

Corollary 2. *In an Einstein Kenmotsu manifold* $M^n\ (n>2)$ *pseudo projective curvature tensor will be irrotational if and only if* $R(X,Y)\cdot\tilde{P}=0$.

EXAMPLE

In this section we consider coordinate space $\Re^3$ (with coordinate x, y, z) and calculate the components of concircular curvature tensor, conformal curvature tensor and pseudo projective curvature tensor. Then we verify the theorems of pseudo projective curvature tensor.

Let us consider the 3-dimensional manifold $M=\{(x,y,z)\in\Re^3 : x\neq 0\}$ where (x,y,z) are the standard coordinate in $\Re^3$. Let the vector fields

$$e_1 = x\frac{\partial}{\partial z},\qquad e_1 = x\frac{\partial}{\partial y},\qquad e_1 = -x\frac{\partial}{\partial x}$$

are linearly independent at each point of M.
Let the Riemannian metric g be defined as

$$g(e_{1,}e_1)=g(e_{2,}e_2)=g(e_{3,}e_3)=1,$$

$$g(e_{1,}e_2)=g(e_{1,}e_3)=g(e_{2,}e_3)=0$$

Let η be a 1-form defined by $\eta(U)=g(U,e_3)$ for any vector field U on M and φ be the (1, 1)-tensor field defined as

$$\varphi e_1 = e_2,\quad \varphi e_2 = -e_1 \quad \text{and} \quad \varphi e_3 = 0.$$

Then using the linearity of φ on g, we have

$$\eta(e_3)=1,\qquad \varphi^2U=-U+\eta(U)e_3,$$

and $g(\varphi U,\varphi W)=g(U,W)-\eta(U)\eta(W)$,

for any vector fields U and W on M Thus for $e_3=\xi$ the structure (φ,ξ,η,g) defines an almost contact metric structure on M .

Let D be the Riemannian connection of g, then we have

$$[e_1,e_2]=0,\quad [e_1,e_3]=e_1 \quad\text{and}\quad [e_2,e_3]=e_2.$$

For the Riemannian metric g, Koszul formula is given by

$$2g(D_XY,Z)=X(g(Y,Z))+Y(g(Z,X))-Z(g(X,Y))$$
$$-g(X,[Y,Z])-g(Y,[X,Z])+g(Z,[X,Y])$$

Using above Koszul formula by taking $e_3=\xi$ for the Riemannian metric g, we have

$$D_{e_1}e_1=-e_3,\quad D_{e_1}e_2=0,\quad D_{e_1}e_3=e_1,$$

$$D_{e_2}e_1=0,\quad D_{e_2}e_2=-e_3,\quad D_{e_2}e_3=e_2$$

$$D_{e_3}e_1=0,\quad D_{e_3}e_2=0,\quad D_{e_3}e_3=0$$

From the above it is clear that the manifold $M(\varphi,\xi,\eta,g)$ satisfies the condition

$$D_X\xi = X - \eta(X)\xi \text{ , for } e_3 = \xi \text{ .}$$

Hence the manifold $M(\varphi,\xi,\eta,g)$ is a *3*-dimensional Kenmotsu manifold.

By using above results we can easily obtain the component of the curvature tensors are

$$R(e_1,e_2)e_2 = -e_1, \quad R(e_1,e_3)e_3 = -e_1, \quad R(e_2,e_1)e_1 = -e_2,$$

$$R(e_1,e_3)e_3 = -e_2, \quad R(e_3,e_1)e_1 = -e_3, \quad R(e_3,e_2)e_2 = -e_3,$$

$$R(e_1,e_2)e_3 = 0, \quad R(e_3,e_2)e_3 = e_2, \quad R(e_3,e_1)e_2 = 0,$$

The Ricci tensor in 3-dimensional manifold is defined as

$$S(U,V) = \sum_{i=1}^{3} g(R(e_i,U)V,e_i) \tag{49}$$

Using the components of the curvature tensor in (49), we get the following results:

$$S(e_1,e_1) = -2, \quad S(e_2,e_2) = -2, \quad S(e_3,e_3) = -2,$$

$$S(e_1,e_2) = 0, \quad S(e_1,e_3) = 0, \quad S(e_2,e_3) = 0,$$

The scalar curvature in *3*-dimensional manifold is given by

$$r = \sum_{i=1}^{3} S(e_i,e_i)$$

which yield

$$r = -6 \cdot$$

In view of above relation (19) gives

$$S(U,V) = -2g(U,V),$$

which shows that M is an Einstein manifold and the only non-vanishing components of pseudo projective curvature tensor and Weyl projective curvature tensor related as

$$\tilde{P}(e_1,e_2)e_3 = a\,P(e_1,e_2)e_3,$$

which shows that pseudo projective curvature tensor and Weyl projective curvature tensor are linearly dependent. So this example verifies Theorem 1. As the above manifold is an Einstein manifold, this example also verifies the Theorem 2 and 3, for 3-dimensional manifolds.

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