Self-Similar Solution of Spherically Symmetrical Discontinuities with Increasing Energy in Generalized Roche Model

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ABSTRACT

By use of the similarity method, the propagation of magnetogasdynamic spherical shock waves are discussed for Roche model in increasing energy medium. The effects of radiation heat flux on the discontinuities are also discussed. We have solve the differential equation by Runge-Kutta method and pattern of flow variables are illustrated by graphs.

Key Words : *Roche Model, Heat Flux, Radiation Pressure, Radiation Energy*

Subject Classification: *(2000) : 76R50*

1. INTRODUCTION

In the earlier investigation, the self-similar solution driven out by a sudden point explosion of core of the generalized Roche model are investigated by Carrus et.al [1]. Rogers [2] has discussed methods for obtaining analytical solution of the same problem. Runga Rao and Purohit [3] has studied the self similar isothermal flow in generalized Roche model. Roseenau [4] has attempted the self similar adiabatic flow behind spherical shock wave in the presence of magnetic field. One of the basic assumption of their work is that the total energy contain behind the shock front is constant. Deb Ray [5] has reviewed the Roche model and obtained the exact non similarity solution taking total energy of the wave non constant. Ray [6] has discussed the problem of point and line explosion and found an exact analytic solution. Analytic solution in the there cases of plane, cylindrically symmetrical, spherically symmetrical flows have also been discussed by Sakurai [7]. Rogers [8] has also studied the similarity solution for all the three cases in uniform atmosphere later on Singh and Vishwakarma [9] have discussed the similarity solution of the flows behind shock waves in a radiative magnetogasdynamics in which total energy increases with time. Vishwakarma and Nath [10] have studied self similar solution of shock propagation in a mixture of a non ideal gas of small solid particles. Michaut and Vincid [11] have been studies the theoretical and exponential studies of radiative shocks. Propagation of shock waves in a dusty gas with exponentially decrease density and temperature has also studies by Vishwakarma [12] Singh and Ram Singh [13] have discussed the propagation of weak shock waves in non ideal gas. Shinde [14] has discussed the propagation of cylindrical shock wave in a non-uniform rotating stellar atmosphere under the action of monochromatic radiation and gravitation. Gtretler and Regenfelder [15] have obtained similarity solution for variable energy shock wave in a dusty gas under isothermal flow field condition Vishwakarma, Yadav [16] have studies self similar analytical solution for blast wave in inhomogeneous atmosphere with frogen in magnetic field. Magnetohydrodynamic cylindrical shock wave in self gravitation gas have been studies by Singh and Singh [17]. Vishwakarma and Vishwakarma [18] have been studies in

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analytical description of converging shock waves in a gas with variable density. Liang and Chen [19] have obtained numerical study of spherical blast-wave propagation and reaction.

In this paper, the similarity solution in the generalized Roche model has been developed, when the radiation heat flux is more important then the radiation pressure and radiation energy. The effect of magnetic field has also taken into account. The unsteady model of Roche consists of a gas distributed with spherical symmetry around a nucleus having a large mass (m). It is assumed that the gravitating effect of gas itself can be neglected compared with the attraction of heavy nucleus.

2. EQUATION OF MOTION AND BOUNDARY CONDITIONS

The equation of continuity momentum, field and energy in the generalized Roche model are,

$$\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}+\rho\frac{\partial u}{\partial r}+\frac{\rho u}{r}=0, \qquad (01)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial P}{\partial r}+\frac{1}{2}\frac{\partial}{\partial r}(h^2)$$

$$+\frac{1}{\rho}\frac{h^2}{r}+\frac{Gm}{r^2}=0, \qquad (02)$$

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial r}+h\frac{\partial u}{\partial r}=0, \qquad (03)$$

$$\frac{\partial P}{\partial t}+u\frac{\partial P}{\partial r}-\frac{\tau P}{\rho}\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)$$

$$+(\tau-1)\frac{1}{r}\frac{\partial}{\partial r}(r.F)=0, \qquad (04)$$

where u, P, ρ, h and F are the velocity, pressure, density, magnetic field and radiation heat flux, at radial distance (r) from the centre of core at time (t); G be the gravitational constant. The magnetic permeability of the medium has taken to be unity through out the problems. The equation of state for ideal gas is given by

$$P=\Gamma\rho T, \qquad (05)$$

where Γ is the gas constant.

Also assuming local thermodynamic equilibrium and taking Rosseland's diffusion approximation,

$$F=-\frac{C\mu}{3}\frac{\partial}{\partial r}(\sigma T^4), \qquad (06)$$

where $\frac{\sigma C}{4}$ is the Stefan – Boltzmann constant, C the velocity of light and μ the mean free path of radiation is a function of density and absolute temperature T.

$$\mu=\mu_0\rho^{\alpha}T^{\beta}, \qquad (07)$$

where μ_0, α, β, being constant. In the self similar model the total energy of the wave is dependent of time as,

$$\text{where} \quad E=Bt^q, \quad (q\le 0), \qquad (08)$$

The flow variables immediately ahead of shock denoted by suffix (1) are

$$U_1=0,\ p_1=AR^{-W}, \quad (0<W<2), \qquad (09)$$

where A is a constant and R denotes the radius of the shock surface ahead the shock, the magnetic field distribution is,

$$h=CR^{-\beta}, \quad 2\beta=W+1 \qquad (10)$$

and pressure distribution ahead the shock,

$$P_1=\frac{AGMR^{-(1+W)}}{(W+1)}+\frac{(1-\beta)}{\beta}C^2R^{-2\beta}, \qquad (11)$$

where C, W and β are constant. The Rankine Hugoniot shock conditions headed by an isothermal shock is,

$$\rho_2(V-u_2)=\rho_1 V=m_s, \qquad (12)$$

$$P_2+\frac{h_2^2}{2}-P_1-\frac{h_1^2}{2}=m_s u_2, \qquad (13)$$

$$E_2+\frac{P_2}{\rho_2}+\frac{h_2^2}{\rho_2}+\frac{1}{2}(V-u_2)^2$$

$$-\frac{F}{m_s}=E_1+\frac{P_1}{\rho_1}+\frac{1}{2}V^2+\frac{h_1^2}{\rho_1}, \qquad (14)$$

$$h_2(V-u_2)=h_1, V, \qquad (15)$$

$$T_1=T_2, \qquad (16)$$

where suffix 2 denotes the flow variables just behind the shock and 1 denotes flow variables just ahead the shock, m denotes the mass per unit area across the shock and V be the shock velocity and given by,

$$V = \frac{dR}{dt}. \tag{17}$$

3. TRANSFORMATION OF EQUATIONS OF MOTION

In order to reduce the equation of flow to ordinary differential equation we now introduce the following transformations.

$$\eta = (\alpha MG)^{-1/3} r t^{-\sigma}, \tag{18}$$

where $\sigma = \frac{2}{3} - \frac{2+q}{5-W}, \quad q = \frac{2}{3}(2-W),$ (19)

and the limit of q and W are

$$0 \le q \le \frac{4}{3} \text{ and } 0 \le W \le 2, \tag{20}$$

We see the solution of equation (01) - (04) in the form

$$u = \frac{r}{t}V(\eta),\ \rho = \frac{AMGt^2}{r^{W+3}}R(\eta),\ P = \frac{AMG}{r^{w+1}}P(\eta),\ h = \frac{(AMG)^{1/2}}{r^{w+1}}h(\eta) \tag{21}$$

$$F = \frac{AMG}{r^W t}F(\eta),$$

using equation (18) in the equation (06) with the help of equation (05), we obtain

$$\alpha = \frac{W}{W+1} \text{ and } \beta = \frac{-(5W+7)}{2(W+1)}.$$

The equation (01) - (04) and the equation (05) are then transformed with the help of the relations (18) and (20) to following form,

$$V(\eta) = \frac{\frac{1}{N}\frac{F(\eta)R^{\beta-\alpha+4}}{P(\eta)^{\beta+3}}(V-\sigma) - (V-\sigma)\left[2\eta p + (W+1)(p+H^2) - H^2R - \frac{R}{\alpha\eta^3}\right] + (W+1)Vp - 2p - \frac{(1-W)}{2}VH^2}{[\eta P - \eta R(V-\sigma)^2 + \eta^2 H^2]} \tag{22}$$

where $N = \frac{4}{3}\frac{C\mu O\sigma}{\Gamma^{4+\beta}}(AMG)^{\alpha-1},$

is a dimension less parameter.

$$\frac{R'(\eta)}{R(\eta)} = \frac{1}{(V-\sigma)}\left[\frac{(W+1)}{\eta}V(\eta) - V'(\eta) - \frac{2}{\eta}\right], \tag{23}$$

$$P'(\eta) = \frac{(W+1)}{\eta}(P+H^2) - \frac{H^2R}{\eta} - \frac{R}{\alpha\eta^4} - \frac{VR(V-1)}{\eta} - HH', \tag{24}$$

$$F'(\eta) = \frac{1}{\tau-1}\left[\frac{\eta R'P}{R} - P'\right](V-\sigma) + \frac{2\tau P}{(\tau-1)\eta}\frac{Pv}{\eta(\tau-1)}[W(\tau-1)+3\tau-1] - \frac{(1-W)}{\eta}F(\eta). \tag{25}$$

The transformed jump condition at the shock front is given by

$$V(1)=\frac{2}{(4+W)}\left[1-\frac{1}{\tau m^2}-\frac{1}{\tau m_A^2}\right], \quad (26)$$

$$R(1)=\frac{\tau M^2 M_A^2}{(M^2+M_A^2)}, \quad (27)$$

$$P(1)=\frac{4}{(4+W)}\frac{M_A^2}{(M^2+M_A^2)}, \quad (28)$$

$$F(1)=\frac{-4}{(4+W)}\left[\frac{M_A^2+M^2}{\tau M_A+M^4}-1\right], \quad (29)$$

$$H(1)=\frac{4}{(4+W)^2}\left[\frac{3}{\tau M^2}+\frac{2\tau M_A^4}{\tau M^2(M_A^2+M^2)}\right], (30)$$

4. RESULT AND DISCUSSIONS

For exhibiting the numerical solution it is convenient to write the field variables in non dimensional form,

$$\frac{u}{u_2}=\eta\frac{V(\eta)}{V(1)}, \quad (31)$$

$$\frac{\rho}{\rho_2}=\frac{1}{\eta^{W+3}}\frac{R(\eta)}{R(1)}, \quad (32)$$

$$\frac{P}{P_2}=\frac{1}{\eta^{W+1}}\frac{P(\eta)}{P(1)}, \quad (33)$$

$$\frac{h}{h_2}=\frac{1}{\eta^{(W+1)/2}}\frac{H(\eta)}{H(1)}, \quad (34)$$

$$\frac{f}{f_2}=\frac{1}{\eta^{W}}\frac{F(\eta)}{F(1)}, \quad (35)$$

Distribution of flow variables in the flow-field behind the shock front are obtained by numerical integration of equations (22) to (25) by Range-Kutta method of fourth order. For the purpose of numerical integration the values of are W = 1, 1.5. And the other parameters are,

$$M^2=15, M_A^2=25, \alpha=\frac{1}{3}, \beta=\frac{-(7+5W)}{2(1+W)}, N=10$$

From Fig. 1 to 5, the nature of flow and field variables for adiabatic case are illustrated, it is observed from Fig.1, 2 & 4 that that velocity, density and pressure are minimum at shock front but increases rapidly towards the center of explosion in adiabatic case and from Fig. 3 & 5 it is observed that magnetic field and radiation heat flux is maximum at shock front but decreases towards the centre of explosion.

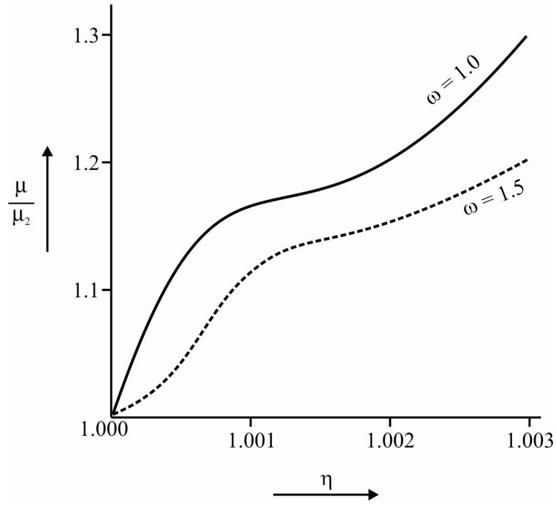

Fig.1: Velocity Distribution

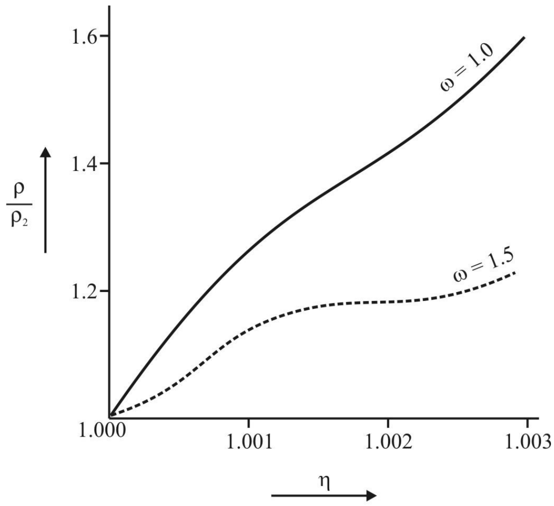

Fig.2: Density Distribution

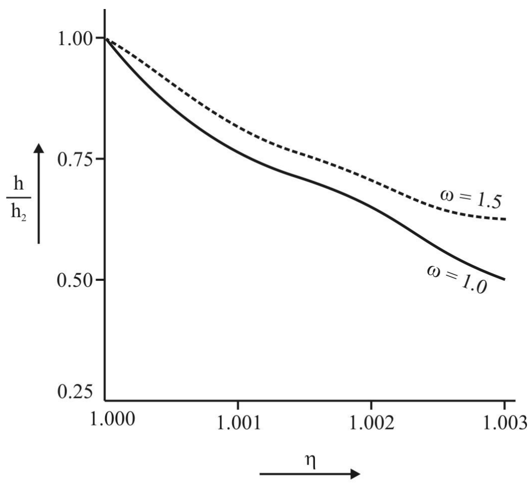

Fig.3: Magnetic Field Distribution

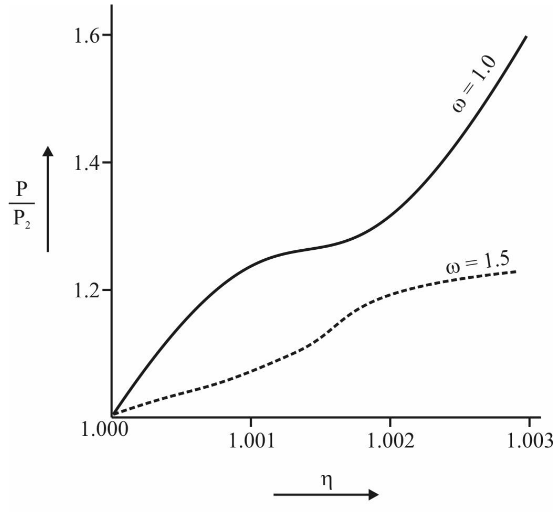

Fig.4: Pressure Distribution

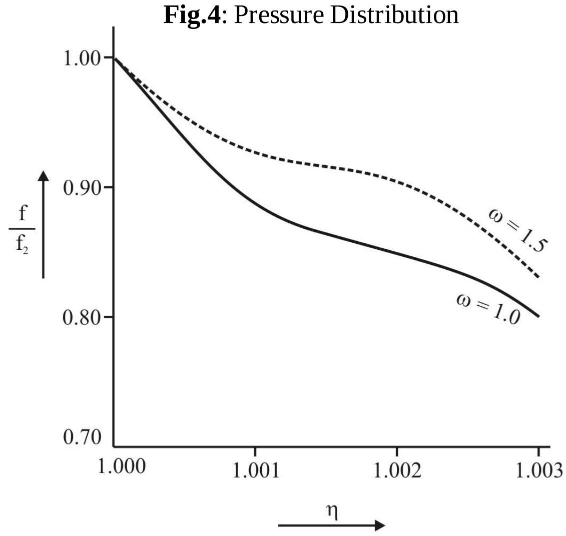

Fig.5: Radiation Distribution

5. CONCLUSION

In present investigation, problem concerning the propagation of spherically symmetrical discontinuities in generalized Roche Model is theoretically investigated,the effect of radiation heat flux is also discussed . The nature of flow and field variables for adiabatic case are illustrated through graphs, it is clear from the graphs that velocity, density and pressure are minimum at shock front .Distribution of magnetic field and radiation heat flux is maximum at shock front but decreases towards the centre of explosion.

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NOMENCLATURE

u- Velocity
P- Pressure
ρ - Density
m - Mass
T - Temperature
α - Density ratio
η- Similarity Variable
F- Radiation Heat Flux
r- Radial Distance from the surface
R- Shock distance
t- Time
G- Gravitational Constant
M_A- Alfven Mach number
E- Total energy
Γ- Gas constant