

# Space Time Block Coding and MIMO Channels: Performance Results

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## ABSTRACT

We analyse the Space-time Block coding for wireless communication and present an overview of applying MIMO concept and shows how it improves the SNR without decreasing data rates. It provides a new paradigm for transmission over Rayleigh fading channel using multiple transmit antennas. Data is encoded using a space time block code and the encoded data is split into  $n$  streams which are simultaneously transmitted using  $n$  transmit antennas. Maximum likelihood decoding is achieved in a simple way through decoupling of the signals transmitted from different antennas rather than joint detection. This uses the orthogonal structure of the space time block code and gives a maximum likelihood decoding algorithm which is based only on linear processing at the receiver. We review the encoding and decoding algorithms for various codes and provide simulation results demonstrating their performance. It is shown that using multiple transmit antennas and space-time block coding provides remarkable performance at the expense of almost no extra processing.

**Keywords:** Diversity, MIMO, fading, multipath channels, multiple antennas, space-time block, wireless communication.

## 1. INTRODUCTION

COMMUNICATIONS technologies have become around the world subscribe to existing second and third generation cellular systems supporting data rates of 9.6 kbps to 21.1 Mbps. More recently, IEEE 802.11 wireless LAN networks enable communication at rates of around 54Mbps and have attracted more than 1.6 billion USD in equipment sales. Over the next few years the capabilities of these technologies are expected to move towards 100 Mbps- 1 Gbps Range and to subscriber numbers of over two billion. At the present time, the wireless communication research community and industry discuss standardizations for the fourth mobile generation (4G). The research community has generated a number of promising solutions for significant improvements in system performance. For contributing to this cause we introduce space-time block coding, a new paradigm for communication over Rayleigh fading channels using multiple antenna.

In most situations, the wireless channel suffers attenuation due to destructive addition of multipath in the propagation media and to interference from other users. The channel statistic is significantly often

Rayleigh which makes it difficult for the receiver to reliably determine the transmitted signal unless some less attenuated replica of the signal is provided to the receiver. This technique is called diversity, which can be provided using temporal, frequency, polarization, and spatial resources [1-5]. In many situations, however, the wireless channel is neither significantly time variant nor highly frequency selective. This forces the system engineers to consider the possibility of deploying multiple antennas at both the transmitter and receiver to achieve spatial diversity.

The purpose of this paper is to evaluate the performance of the space-time block codes constructed in [6] to provide the details of the encoding and decoding procedures. We begin by considering encoding and decoding algorithms for some of these codes. We then provide simulation results confirming that with space-time block coding and multiple transmit antennas a significant performance gain can be achieved at almost

No processing expense. We also consider transmission of a continuous amplitude source over a MIMO block Rayleigh fading channel. We are interested in minimizing the end-to-end average

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distortion of the source. The outline of this paper is as follows. In section II We define a MIMO channel Model and analyse how it overcomes the effect of multipath propagation. We review examples of space time block codes constructed and will describe their encoding and decoding algorithm in III. Section IV deals with performance result of space time block codes which shows same performance as observed by maximum ratio combining. Final conclusion and comments are given in section V.

## 2. MIMO CHANNEL MODEL

Wireless communication channels are often characterized by severe multipath [7]. The transmitted signal propagates along L multiple paths created by reflection and scattering from physical objects in the terrain as illustrated in Fig. 1. We derive a far field' signal model for linearly modulated digital communication for a system with  $M_T$  transmitting antennas and  $M_R$  receiving antennas. Considering the propagation geometry.

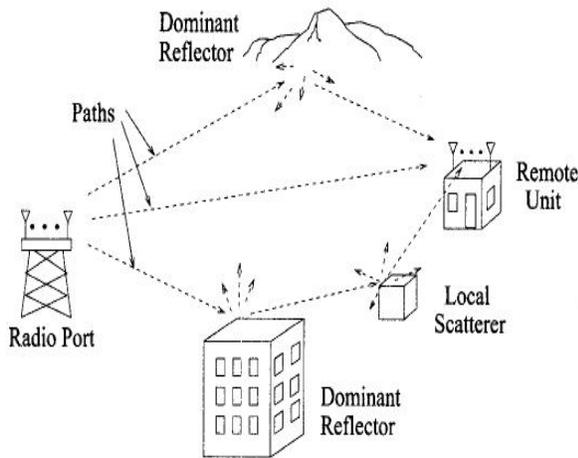


Fig. 1 : Illustration of Physical wireless channel

In the context of a digital signalling scheme, the transmitted baseband signal for the  $j$ th transmitter element is where  $\{Z_3(n)\}$  is the (complex) symbol sequence,  $g(t)$  is the pulse shaping function impulse response and  $T$  is the symbol period. The pulse shaping function is typically the convolution of two separate filters, one at the transmitter and other at the receiver. The optimum receiver filter is a matched filter. In practice, the pulse shape is windowed resulting in

a finite duration impulse response. We assume synchronous complex baseband sampling with symbol period  $T$ . We define  $n_0$  and  $(v + 1)$  to be the maximum lag and length over all  $l$  for the windowed pulse function sequences  $\{g(nT - n)\}$ . To simplify notation, it is assumed that  $n_0 = 0$ , and the discrete-time notation  $g(nT - n) = s_{\sim}(\cdot)$  is adopted.

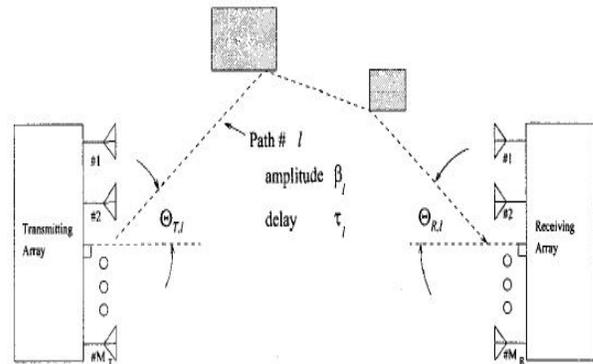


Fig.2 : Propagation path Geometry

When a block of  $N$  data symbols are transmitted,  $N + v$  non-zero output samples result beginning at time sample  $k - N + 1$  and ending with sample  $k + v$ . The composite channel output can now be written as a column vector with all time samples for a given receive antenna appearing in order so that,

$$x(k) = [x_1(k - N + 1) \dots x_1(k + v) \dots x_{M_R}(k - N + 1)] \quad (1)$$

With an identical stacking for the output noise samples  $n(k)$ .

The input symbol vector is written as,

$$[z(k) = (z_1(k - N + 1) \dots z_1(k) \dots z_{M_T}(k - N) \dots z_{M_T}(k))]^T \quad (2)$$

The spatiotemporal channel may then be expressed as a vector equation,

$$x(k) = Hz(k) + n(k) \quad (3)$$

Where the MIMO channel matrix is composed of SISO sub blocks.

$$H = \begin{bmatrix} H_{1,1} & H_{1,M_T} \\ H_{M_R,1} & H_{M_R,M_T} \end{bmatrix} \quad (4)$$

With each sub-block possessing the well known Toeplitz form. To clearly illustrate the effect of multipath, the channel can be written as the sum over multipath components.

$$\sum_{l=1}^L \beta \begin{bmatrix} a_{R,l}(\theta_{R,l}) \\ a_R, M_{R}(\theta_{R,l}) \end{bmatrix} \tag{5}$$

$$G_1 = [a_{T,1}(\theta_{T,1}) \dots a_{T,M_T}(\theta_{T,1})] \tag{6}$$

Where  $G_l$  is the Toeplitz pulse shaping function sample matrix arising from path delay  $T_l$ . This channel description is illustrated in Fig. 3.

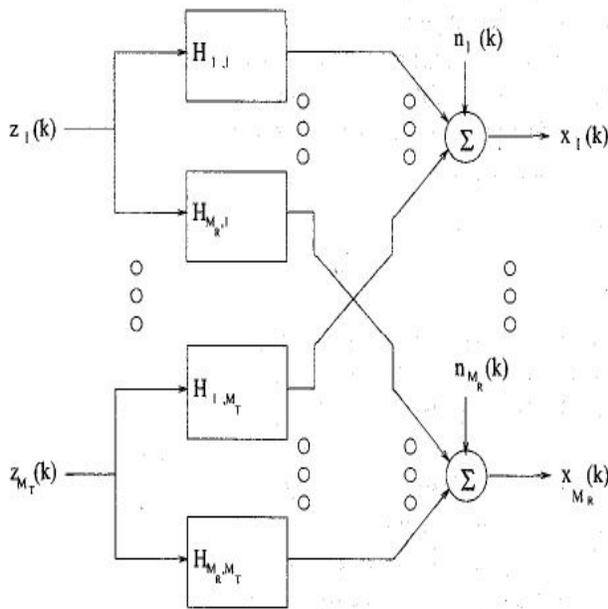


Fig.3 : Discrete time MIMO channel Diagram

### 3. SPACE-TIME BLOCK CODES

Space-time block codes (STBC) are a generalized version of Alamouti scheme. These codes have the same key features. That is, they are orthogonal and can achieve full transmit diversity specified by the number of transmit antennas. In other words, space-time block codes are a complex version of Alamouti's [7] space-time code, where the encoding and decoding schemes are the same as in both the transmitter and receiver sides.

The data are constructed as a matrix which has its rows equal to the number of the transmit antennas and its columns equal to the number of the time slots required to transmit the data. At the receiver side,

when signals are received, they are first combined and then sent to the maximum likelihood detector where the decision rules are applied. Space-time block code was designed to achieve the maximum diversity [8] order for the given number of transmit and receive antennas subject to the constraint of having a simple decoding algorithm.

In addition, space-time block coding provides full diversity advantage but is not optimized for coding gain.

#### A. Encoding Algorithm

Fig.4 shows the structure of space time block encoder for two transmit and one receiver antenna. A space-time block code is defined by a  $P \times n$  transmission matrix, where  $p$  represent the number of transmit antenna and  $n$  represent the number of time slots. The entries of the matrix are linear combinations of the variables and their conjugates. The number of transmission antennas are to separate different codes from each other. For example, to represents a code which utilizes two transmit antennas and is defined by,

$$G_2 = \begin{bmatrix} X_1 & X_2 \\ -\frac{d X_2}{dt} & \frac{d X_1}{dt} \end{bmatrix} \tag{7}$$

We assume that transmission at the baseband employs a signal constellation  $A$  with  $2^b$  elements.

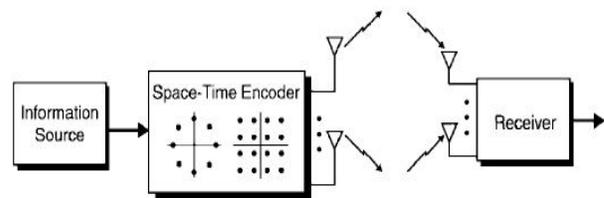


Fig.4 : System Block Diagram

At time slot1 the sample  $x_1$  been send through the antenna 1 and  $x_2$  is being sent through antenna 2. On the time slot 2 the conjugate of  $x_1$  is sent through the antenna 1 and conjugate of  $x_2$  is sent through the antenna 2. In this way at the Encoder the various copies of signal can be send via multiple transmit antenna. We can also generate the transmission matrix for three transmitter antennas and four transmitter antennas. The matrix is represented as  $G_3$  for this case respectively.

$$\begin{bmatrix} X_1 & X_2 & X_3 \\ -X_2 & X_1 & -X_4 \\ -X_3 & X_4 & X_1 \\ -X_4 & -X_2 & X_2 \\ \frac{dX_1}{dt} & \frac{dX_2}{dt} & \frac{dX_3}{dt} \\ -\frac{dX_2}{dt} & \frac{dX_1}{dt} & \frac{dX_2}{dt} \\ -\frac{dX_2}{dt} & \frac{dX_4}{dt} & \frac{dX_1}{dt} \\ -\frac{dX_4}{dt} & -\frac{dX_2}{dt} & \frac{dX_2}{dt} \end{bmatrix} \quad (8)$$

These matrices provides encoding concept for three and four transmitter antennas with the help of Space time coding and helps in achieving improved signal to noise ratio and minimise the effect of fading. In the preceding section we review the decoding algorithm for these codes.

### B. The Decoding Algorithm

Maximum likelihood decoding of any space-time block code can be achieved using only linear processing at the receiver, and we illustrate this by some examples. The space-time block code (first proposed by ) uses the transmission matrix in (8). Suppose that there are signals in the constellation. At the first time slot 2b bits arrive at the encoder and select two complex symbols and . These symbols are transmitted simultaneously from antennas one and two, respectively. At the second time slot, signals and are transmitted simultaneously from antennas one and two, respectively. Then maximum likelihood detection amounts to minimizing the decision metric

$$\sum_{j=1}^m \left( \left| r^j_1 - a_{1,j} x_1 - a_{2,j} x_2 \right|^2 + \left| r^j_2 + a_{1,j} \frac{dx_2}{dt} - a_{2,j} \frac{dx_1}{dt} \right|^2 \right) \quad (9)$$

over all possible values of and . Note that due to the quasi-static nature of the channel, the path gains are constant over two transmissions. The minimizing values are the receiver estimates of and , respectively. We expand the above metric and delete the terms that are independent of the codeword's and observe that the above minimization is equivalent to minimizing

$$\begin{bmatrix} r^j_1 a_{1,j} \frac{dx_1}{dt} + \frac{d}{dt} (r^j_1) a_{1,j} x_1 + r^j_1 \frac{da_2}{dt} j \frac{dx_2}{dt} + \\ -\sum_{j=1}^m \left[ \frac{d}{dt} (r^j_2) a_{2,j} x_2 - r^j_2 \frac{da_1}{dt} j x_2 - \frac{d}{dt} (r^j_2) a_{1,j} j \frac{dx_2}{dt} \right. \\ \left. + r^j_2 a_{2,j} j \frac{dx_1}{dt} \right] + (|x_1|^2 + |x_2|^2) + \sum_{j=1}^m \sum_{i=1}^2 |a_{i,j}|^2 \end{bmatrix} \quad (10)$$

The above metric decomposes into two parts one of which is only a function of  $x_1$  and other is only a function of  $x_2$ . Thus the minimization of (6) is equivalent to minimizing these two parts separately. This in turn is equivalent to minimizing the decision metric (11) for detecting  $x_1$  and (12) for detecting  $x_2$ .

$$\left[ \sum_{j=1}^m \left( r^j_1 \frac{da_2}{dt} j - (r^j_2 a_{1,j}) \right) \right] - x_1 + \left( -1 + \sum_{j=1}^m \sum_{i=1}^2 |a_{i,j}|^2 \right) |x_1|^2 \quad (11)$$

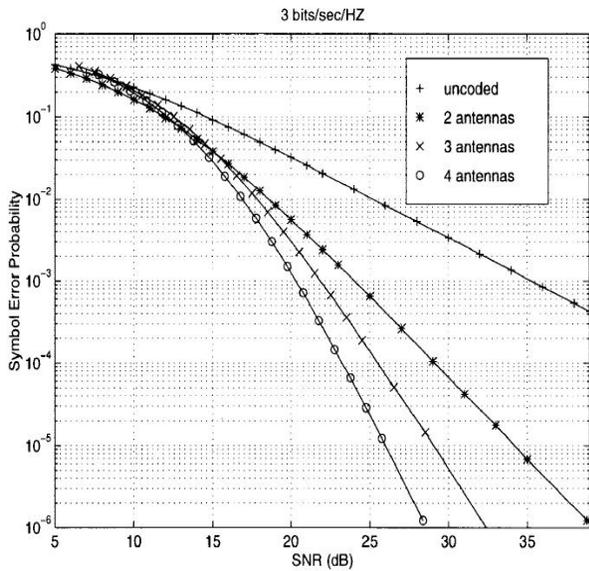
$$\left[ \sum_{j=1}^m \left( r^j_1 \frac{da_2}{dt} j - (r^j_2 a_{1,j}) \right) \right] - x_2 + \left( -1 + \sum_{j=1}^m \sum_{i=1}^2 |a_{i,j}|^2 \right) |x_2|^2 \quad (12)$$

This is the simple decoding scheme described in , and there is no performance sacrifice for using it. Similarly, the decoders for and can be derived. Next we discuss the performance and simulation Results for these codes in preceding section.

## 4. PERFORMANCE ANALYSIS

The performance of space-time block codes depends on the type of modulation and the number of transmit and receive antennas used. Complex modulations give better bit-error-rate performance than real modulations and it is especially true when the number of transmit antennas is larger than two. As an example, if space-time block codes with four transmit antennas and complex modulation scheme are used, then a four by eight (rate of 1/2) transmission matrices will be used.

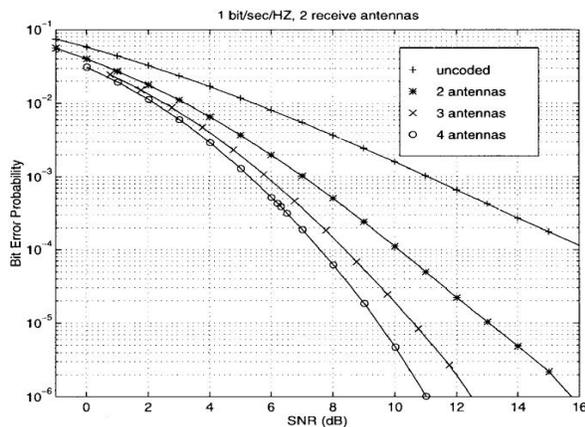
This would give a better performance than the same space-time block code with real modulation of rate of one. However, space-time block code with real modulation would have better bandwidth efficiency performance than complex modulation. This is because space-time block codes with real modulation require transmitting less data than space-time block codes with complex modulation. On the other hand, space-time block codes with larger number of transmit antennas always give better performance than space-time block codes with lower number of transmit antennas.



**Fig. 5 :** SEP vs. SNR for STBC using one Receiver Antenna

This is true because larger number of transmit antennas means larger transmission matrices which means transmitting more data. This would give the receiver the ability to recover the transmitted data as in fig.5. Moreover, with larger number of receive antennas, the same transmitted data would be received by more than one receive antenna. This is an advantage because if one receive antenna did not recover the transmitted data correctly, the second receive antenna could. The chance that at least one out of two receive antennas would receive the transmitted data uncorrupted is always higher than if there is only one receive antenna.

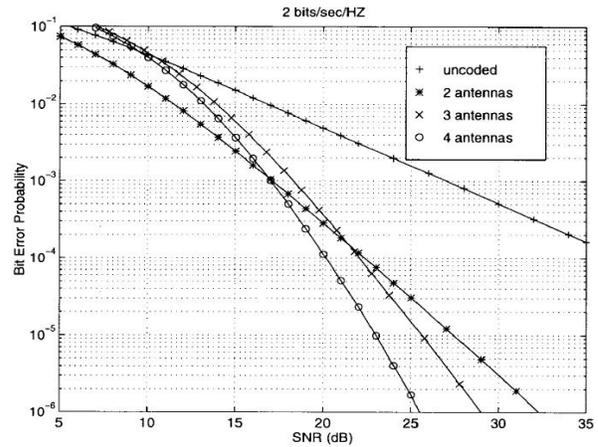
This illustration is shown in fig.6 where we use two transmitter Antennas which provides better results than using only one receiver Antenna.



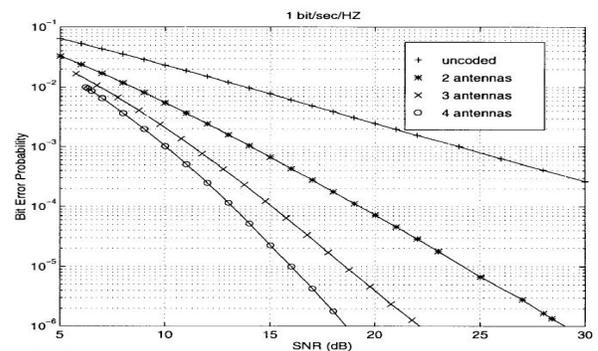
**Fig. 6 :** BER vs. SNR for STBC using two receive Antennas.

Many simulations have been done on the performance of different space-time block codes using different types of modulation schemes and different numbers of transmit and receive antennas. In our simulation on the different implementations of space-time block codes, the channel coefficients are always assumed flat rayleigh.

We provide simulation results for the performance of the codes given in the previous sections. The information source is encoded using a space-time block code, and the constellation symbols are transmitted from different antennas. The receiver estimates the transmitted bits by using the signals of the received antennas. We have seen the performance of space time block coding of the modulation order of 3 bits, Now when we use lower order modulation as seen from the fig.7 and fig.8 it can be understood that by using lower order modulation of 2 bits and 1 bit respectively it always gave low bit-error-rate when compared with space-time block codes that employ higher order modulation methods.

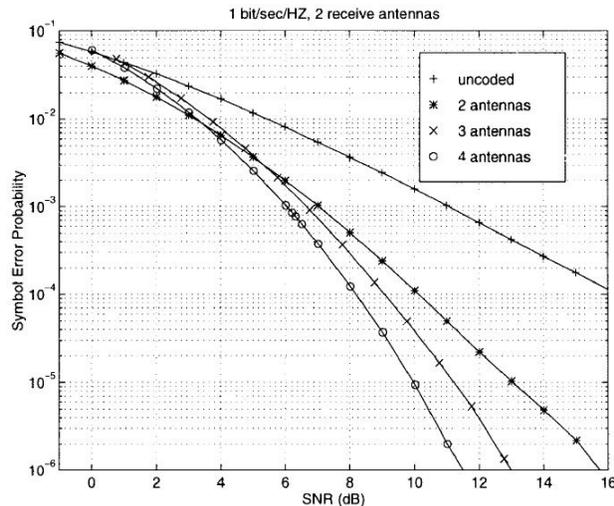


**Fig.7 :** BER vs. SN plot for STBC codes at 2 bits/s/Hz.



**Fig.8 :** BER vs. SNR plot for STBC codes at 1 bit/s/Hz

receivers antenna instead of one receiver antenna, here also it can be seen that when we use lower modulation it would give better SNR or lower bit error rate as compared to higher order modulation as in Fig.9.



**Fig. 9** : SEP vs. SNR plot using 2 receivers Antennas.

Thus in this way we have simulated the results of space time block coding using MIMO concept and found that this STBC together with MIMO have done a remarkable performance in achieving higher SNR and maintain low bit error rate and thus improves the quality of wireless communication

## 5. CONCLUSION

We provided examples of space-time block codes for transmission using multiple transmit antennas. We described both their encoding and decoding algorithms. The encoding and decoding of these codes have very little complexity. Results showed that space-time block codes could achieve better bit-error-rate performance when more antennas were employed at each end. In addition, space-time block codes with lower modulation order always gave low bit-error-rate when compared with space-time block codes that employ higher order modulation methods. In this paper all simulation results are based on perfect knowledge of the channel coefficients at the receiver. We also discuss the MIMO channel model and how this concept helps in achieving better SNR and provide significant gain.

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