Combined Effects of Periodic Suction and Permeability on MHD Oscillatory Flow of Rivlin Ericksen Fluid past a Moving Semi-infinite Porous Plate in the Presence of Thermal Radiation

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Abstract

In this paper the behaviour of unsteady flow of viscous incompressible and electrically conducting Rivlin Ericksen fluid past a semi-infinite vertical porous plate having variable permeability under thermal radiation effects is examined. Further the time dependent suction is assumed at the plate which is moving with constant velocity whereas the free stream velocity is assumed to be oscillating with time. The dimensionless governing equations for the fluid flow under investigation are reduced to set of ordinary differential equations using two term harmonic and non-harmonic functions and solved analytically under relevant boundary conditions. Further the analytical results obtained for velocity, temperature and concentration profiles are evaluated numerically and their variation with different flow parameters are shown graphically. Also, the variation behaviour of Skin friction, Nusselt number and Schmidt number along with their amplitudes and phase angles for pertinent parameters is displayed graphically.

1. INTRODUCTION

The growing interest of MHD flow of viscoelastic fluids through porous medium is due to their vast applicability in varied industrial and engineering process such as food processing, paper manufacturing, petroleum drilling, geo thermal extraction process, drug suffusion through human skin, irrigation process through ground water hydrology, filtration process in chemical industry and many such processes. Walter [1, 2] developed the theory for general linear visco-elastic liquid contained in concentric sphere elastoviscometer and vertical co axial cylinders respectively with the assumption of forcing harmonic angular oscillations on the outer ones about their axis. Further Walter [3] solved some steady flow problems for elastic viscous fluids with the consideration of equations of state to be linear in stresses and second degree in stress and rates of strain. The convective heat transfer phenomenon of incompressible and electrically conducting visco-elastic fluids in the presence of heat source/ sink or thermal radiation has been studied by many scholars and researchers due to its interesting applications in cosmical studies, thermal engineering, Geophysical and Astrophysical Sciences. The main implementation of this phenomenon can be seen through the modelling of packed sphere beds, chemical catalytic reactors, granular insulation, cooling of electronic devices and many more. Harnett [4] presented his studies on heat transfer nature of viscoelastic aqueous polymer solution in channel flow for both laminar and turbulent cases. Magdy [5] studied the velocity and temperature distribution behaviour for the unsteady incompressible and electrically conducting non-Newtonian power law fluid of viscous nature past an infinite porous plate with periodic injection/suction. Chaudhary and Islam [7] provided the closed form solutions to a 2-dim unsteady MHD free convection flow of viscoelastic fluid past an infinite porous plate using perturbation technique. Chaudhary and Jain [8] examined the mass transfer and Hall effect on MHD mixed convection flow of a viscoelastic fluid past an infinite vertical plate under radiation effect. Further Chaudhary and Jain [9] investigated the different flow parameter effects on velocity, temperature and concentration field for MHD free convective flow pastan oscillating plate through porous medium. Das and Mitra [10]gave the numerical solution to the unsteady mixed convective MHD flow past an accelerated infinite vertical porous plate with constant suction. Samad and Mohebujjaman [11] studied the heat generation effects on MHD free convection flow along a vertical stretching sheet in presence of magnetic field. Further Makinde [12] studied the boundary layer flow on MHD free convection past a vertical plate in a porous medium in the presence of constant heat flux. Further Dasand Jana [13] studied heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium. Chaudhary and Debnath [14] analysed the unsteady oscillatory MHD flow of a viscoelastic incompressible fluid past a porous vertical plate having periodic suction along with periodic free stream velocity and temperature.

In above said studies the porosity and permeability of the medium is taken as constant to investigate the velocity or temperature distribution or some stability characteristics of fluid. But the variation in porosity is observed in many industrial processes such as packed bed heat exchangers, dying and fixed bed catalytic reactors etc. where it is found maximum at wall and minimum away from wall. So many researchers worked on the fluid flow problems under different configuration with the consideration of non-uniform porosity. Chandraseckhara and Namboodiri [15] investigated the combined free and forced convection effects about inclined surface in porous medium variable permeability. Further Sreekanth et al. [16] and El-Kabier, [17] presented his studies on hydrodynamic free convective flow through a porous medium with variable permeability. Seddeek and Almushiheh [18] studied heat and mass transfer phenomenon on MHD free convective flow over a stretching sheet in the presence of thermal radiation and chemical reaction with variable viscosity. Pal [19] studied the velocity and temperature distribution profiles of Magnetohydrodynamic non-Darcy mixed convection flow past a vertical heated plate embedded in a porous medium with variable porosity**.** Satya Narayana and Sravanthi [20] concluded the thermal radiation and chemical reaction effects on unsteady free convective flow of viscous incompressible and electrically conducting fluid past a semi inûnite inclined plate with variable permeability. The heat and mass transfer effects on MHD micropolar fluid past a vertical porous plate having non uniform permeability in presence of sink/source was studied by Hareeshand Satya Narayana [21]. Srivastava and Deo [22] presented the magnetic ûeld effects on the viscous ûuid ûow in a porous channel with variable permeability**.** In above said studies, porosity near the surface is observed maximum as compared to the region far away from surface. But we can't access any kind of relation between porosity and distance from wall through some mathematical expression. So, it may say that the porosity and hence permeability have no more constant behaviour in a given medium.

Rivlin - Ericksen fluid model was given by Rivlin - Ericksen to the class of viscoelastic fluids which fails to be characterized by Maxwell's constitutive equation and Oldroyd's constitutive equation. In literature various parametric effects under different imposable conditions on Rivlin Ericksen fluid has been investigated due to its relevant importance in some chemical industry and technology.Noushima et al. [23] studied the unsteady Rivlin Ericksen visco elastic flow of electrically conducting fluid bounded by an infinite vertical porous plate with constant suction and variable permeability. Varshney et al. [24] compiled the effects of constant heat and mass flux on rotatory Rivlin Ericksen visco elastic flow of electrically conducting fluid past an exponentially moving infinite surface having constant suction. Uwanta and Hussain [25] discussed the mass transfer effects on hydromagnetic free convective Rivlin – Erickson flow past a vertical infinite porous plate with variable suction. Banyal and Sharma [26] gave the mathematical analysis to the Rivlin Ericksen viscoelastic fluid acted upon by a uniform vertical rotation using linearized stability theory and normal mode analysis. RaviKumar et al. [27] studied the effects of heat transfer on a heat absorbing electrically conducting the Rivlin Ericksen viscoelastic fluid past a semi-infinite vertical porous plate moving with constant velocity having time dependent suction and free stream velocity. Hussaini et al. [28] studied the velocity profiles of Rivlin Ericksen flow through a porous medium with slip boundary conditions. Recently Reddy et al. [29] investigated numerically the diffusion thermo and thermal diffusion effects on MHD free convection flow of Rivlin- Ericksen fluid past a semi- infinite vertical plate with constant mass flux.

Our present work is motivated through above works done by researchers. In our problem we have investigated the heat and mass transfer effects on well-known non- Newtonian fluid called Rivlin Ericksen fluid which is passing a semi-infinite vertical porous plate having periodic suction and permeability moving with some constant velocity under pressure gradient and thermal radiation. Also, the free stream velocity and pressure gradients are assumed to be oscillating with time. Our study is an extension of Ravikumar et al. [27] work.

2. PROBLEM FORMULATION

Consider a two-dimensional laminar flow of Rivlin Ericksen fluid which is of incompressible, electrical conducting and viscous nature in the presence of thermal radiation effects. This is assumed to past a semi- infinite vertical porous plate having periodic suction and permeability which is subjected to a uniform transverse magnetic field. The plate is moving with constant velocity in the flow of direction and free stream velocity is assumed to be of oscillating in time. Under the supposition of above assumptions, the stating equations for conservation of mass, momentum and energy in Cartesian frame of reference can be written as

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \nu \frac{\partial^2 u'}{\partial y'^2} - \beta_1\left(\frac{\partial^3 u'}{\partial t' \partial y'^2} + v' \frac{\partial^3 u'}{\partial y'^3}\right)$$

$$+ g\beta\left(T' - T_\infty\right) + g\beta'\left(C' - C_\infty\right) - \left(\frac{\nu}{K'} + \frac{\sigma B_0^2}{\rho}\right) \tag{2}$$

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial Q'}{\partial y'} \tag{3}$$

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} \tag{4}$$

Boundary conditions are

$$u' = u'_w,\ T' = T_w + \varepsilon\left(T_w + T'_\infty\right)e^{i\omega' t'} \text{ and}$$

$$C' = C_w + \varepsilon\left(C_w + C'_\infty\right)e^{i\omega' t'} \text{ at } y' = 0$$

$$u' \to u'_\infty,\ T' \to T'_\infty,\ C' \to C'_\infty \text{ as } y' \to \infty \quad (5)$$

where $u', v', \rho, \nu, \beta_1, g, \beta, \beta', K', \sigma, B, T',$

$k, C_p, Q', C', \mathrm{D}, \omega', u'_w, T_w, C_w, u'_\infty, T'_\infty, C'_\infty$

are the dimensional velocities in *x'* and *y'* directions, density of fluid, kinematic viscoelasticity, acceleration due to gravity, volumetric coefficient of thermal expansion, volumetric coefficient of concentration expansion, permeability of porous medium, thermal conductivity, strength of applied magnetic field, dimensional temperature, thermal conductivity, specific at constant pressure, dimensional radiative heat flux, dimensional mass diffusivity, dimensional concentration, wall dimensional velocity, wall dimensional temperature, wall dimensional concentration, free stream velocity, temperature and concentration respectively.

From equation (1), it is clear that suction velocity is dependent on time only, therefore assume its form to be

$$v' = -v^0\left(1 + \varepsilon A e^{i\omega' t'}\right) \quad (6)$$

Also, the permeability of the medium and free stream velocity is assumed to be periodic, therefore it can be taken as

$$K' = -K_0\left(1 + \varepsilon d e^{i\omega' t'}\right),\quad u'_\infty = u_0\left(1 + \varepsilon e^{i\omega' t'}\right) \quad (7)$$

Under the assumption of the fluid to be optically thin with relative low density, radiative heat flux is taken as

$$\frac{\partial Q'}{\partial y'} = 4\alpha^2\left(T' - T'_\infty\right) \quad (8)$$

where v^0 represents the non -zero constant suction velocity, ε, A, d are real positive constant such that $\varepsilon, \varepsilon A, \varepsilon d$ are small less than unity and α is the mean radiation absorption coefficient.

The following dimensionless quantities are introduced as

$$\mathrm{u} = \frac{u'}{u_0},\ \mathrm{v} = \frac{v'}{v^0},\ \mathrm{x} = \frac{v^0 x'}{\nu},\ \mathrm{y} = \frac{v^0 y'}{\nu},\ \mathrm{u_w} = \frac{u'_w}{u_0},$$

$$\mathrm{t} = \frac{v^{0\,2} t'}{4\nu},\ \mathrm{T} = \frac{T' - T'_\infty}{T_w - T'_\infty},\ \mathrm{C} = \frac{C' - C'_\infty}{C_w - C'_\infty},$$

$$\omega = \frac{4\nu\omega'}{v^{0\,2}},\ p = \frac{p'}{\rho v^{0\,2}},\ \mathrm{Rm} = \frac{\beta_1 v^{0\,2}}{\nu^2},$$

$$\mathrm{Gr} = \frac{\nu\beta g\left(T' - T'_\infty\right)}{u_0 v^{0\,2}},\ \mathrm{Gm} = \frac{\nu\beta' g\left(C' - C'_\infty\right)}{u_0 v^{0\,2}},\ \mathrm{K} = \frac{\nu^2}{K_0 v^{0\,2}},$$

$$\mathrm{M} = \frac{\sigma B_0^2 \nu}{\rho v^{0\,2}},\ \mathrm{R} = \frac{4\alpha^2\nu}{v^{0\,2}\rho \mathrm{C_p}},\ \mathrm{Pr} = \frac{\nu\rho \mathrm{C_p}}{\mathrm{k}},\ \mathrm{Sc} = \frac{\nu}{D} \quad (9)$$

Equations (2)-(4) with the help of equations (6) -(9) reduces to the following dimensionless form as

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2}$$

$$- R_m\left(\frac{1}{4}\frac{\partial^3 u}{\partial t \partial y^2} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial^3 u}{\partial y^3}\right)$$

$$+ GrT + GmC - \left(\frac{1}{K\left(1 + \varepsilon d e^{i\omega t}\right)} + M\right)u \quad (10)$$

$$\frac{1}{4}\frac{\partial T}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} - RT \quad (11)$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \quad (12)$$

Where Rm, Gr, Gm, K, M, Pr, R and Sc denotes the viscoelastic parameter of Rivlin Ericksen fluid, the thermal Grash of number, the mass Grash of number, permeability parameter, the magnetic

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number, the Prandtl number, dimensionless radiation coefficient and the Schmidt number respectively.

The boundary equations in dimensionless form can be written as

$$u = u_w,\ \ T = 1 + \varepsilon\, e^{i\omega t},\ \ C = 1 + \varepsilon e^{i\omega t'}\ \text{ at } y = 0$$
$$u \to 1 + \varepsilon\, e^{i\omega t},\ \ T \to 0,\ \ C \to 0\ \text{ as } y' \to \infty \quad (13)$$

3. SOLUTION OF THE PROBLEM

The system of non -linear partial differential equations (10)-(12) representing the required fluid flow can be solved analytically by reducing it to the set of ordinary differential equations by assuming the solution for velocity, temperature and concentration field as

$$u(y,t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y) + O(\varepsilon^2) \quad (14)$$

$$T(y,t) = T_0(y) + \varepsilon e^{i\omega t} T_1(y) + O(\varepsilon^2) \quad (15)$$

$$\mathrm{C}(y,t) = C_0(y) + \varepsilon e^{i\omega t} C_1(y) + O(\varepsilon^2) \quad (16)$$

On substituting the equations (14)-(16) into the equations (10)-(12) and equating the harmonic and non-harmonic parts of velocity, temperature and concentration terms with ignorance of higher order terms of *0(ε²)*, we get the following system of equations

$$\mathrm{Rm}\frac{\partial^3 u_0}{\partial y^3} + \frac{\partial^2 u_0}{\partial y^2} + \frac{\partial u_0}{\partial y} - \left(\frac{1}{K} + M\right) u_0$$
$$= -P - GrT_0 - GmC_0 \quad (17)$$

$$\mathrm{Rm}\frac{\partial^3 u_1}{\partial y^3} + \left(1 - \frac{Rmi\omega}{4}\right)\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial u_1}{\partial y} - \left(\frac{1}{K} + M\right) u_1$$
$$= \frac{i\omega}{4} - P - GrT_1 - GmC_1 - Rm\frac{\partial^3 u_0}{\partial y^3} - A\frac{\partial u_0}{\partial y} - \frac{d}{K}u_0 \quad (18)$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2 T_0}{\partial y^2} + \frac{\partial T_0}{\partial y} - RT_0 = 0 \quad (19)$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2 T_1}{\partial y^2} + \frac{\partial T_1}{\partial y} - \left(\mathrm{R} + \frac{i\omega}{4}\right) T_1 = -A\frac{\partial T_0}{\partial y} \quad (20)$$

$$\frac{1}{Sc}\frac{\partial^2 C_0}{\partial y^2} + \frac{\partial C_0}{\partial y} = 0 \quad (21)$$

$$\frac{1}{Sc}\frac{\partial^2 C_1}{\partial y^2} + \frac{\partial C_1}{\partial y} - \frac{i\omega}{4} C_1 = -A\frac{\partial C_0}{\partial y} \quad (22)$$

With corresponding boundary conditions as

$$u_0 = u_w,\ \ u_1 = 0,\ \ T_0 = 1,\ \ T_1 = 1 \ \text{ and}$$
$$C_0 = 1,\ \ C_1 = 1 \text{ at } y = 0$$
$$u_0 = 1,\ \ u_1 = 1,\ \ T_0 \to 0,\ T_1 \to 0 \ \text{ and}$$
$$C_0 \to 0,\ C_1 \to 0 \ \text{ as } y' \to \infty \quad (23)$$

Since the equations (17) and (18) are of third order differential equations in nature when Rm≠0, so we will use the Bears and Walters rule (1964) for the expansion of u_0 and u_1 which can be written as

$$u_0 = u_{01} + Rm u_{02} + O(Rm^2) \quad (24)$$

$$u_1 = u_{11} + Rm u_{12} + O(Rm^2) \quad (25)$$

Substituting the equations (24) and (25) into (17) and (18), equating different powers of Rm and neglecting higher order of *0 (Rm²)* we get

Zero order equation of Rm

$$\frac{\partial^2 u_{01}}{\partial y^2} + \frac{\partial u_{01}}{\partial y} - \left(\frac{1}{K} + M\right) u_{01} = -P - GrT_0 - GmC_0 \quad (26)$$

$$\frac{\partial^2 u_{11}}{\partial y^2} + \frac{\partial u_{11}}{\partial y} - \left(\frac{1}{K} + M\right) u_{11} = \frac{i\omega}{4} - P - GrT_1 - GmC_1$$
$$- A\frac{\partial u_{01}}{\partial y} - \frac{d}{K}u_{01} \quad (27)$$

First order equation of Rm

$$\frac{\partial^2 u_{02}}{\partial y^2} + \frac{\partial u_{02}}{\partial y} - \left(\frac{1}{K} + M\right) u_{02} = -\frac{\partial^3 u_{01}}{\partial y^3} \quad (28)$$

$$\frac{\partial^2 u_{12}}{\partial y^2} + \frac{\partial u_{12}}{\partial y} - \left(\frac{1}{K} + M\right) u_{12} = -\frac{\partial^3 u_{11}}{\partial y^3} - \frac{i\omega}{4}\frac{\partial^2 u_{11}}{\partial y^2}$$
$$- \frac{\partial^3 u_{01}}{\partial y^3} - A\frac{\partial u_{02}}{\partial y} - \frac{d}{K}u_{02} \quad (29)$$

And the corresponding boundary equations are

$$u_{01} = u_w,\ \ u_{02} = 0,\ u_{11} = 0,\ \ u_{12} = 0 \quad \text{at } y = 0$$

$$u_{01} = 1,\ \ u_{02} = 0,\ \ u_{11} = 1,\ \ u_{12} = 0 \quad \text{as } y' \to \infty \quad (30)$$

In the view of equation (23), solving the equations (19)-(22) and then solving (26) – (29) using boundary conditions (30), finally with the help of (24) and (25) solving the system of differential equations (17) and (18), we get the solutions of velocity, temperature and concentration profiles as

$$u(y,t) = B_5 e^{-m_7 y} - B_4 e^{-m_3 y} - B_3 e^{-Scy} + 1 + Rm\left(B_8 e^{-m_7 y} - B_7 e^{-m_3 y} - B_6 e^{-Scy}\right) + \varepsilon e^{i\omega t}\begin{pmatrix} B_{13} e^{-m_7 y} - B_{12} e^{-m_5 y} + B_{11} e^{-m_3 y} - B_{10} e^{-m_1 y} - B_9 e^{-Scy} + 1 \\ + Rm\left(B_{18} e^{-m_7 y} - B_{17} e^{-m_5 y} + B_{16} e^{-m_3 y} - B_{15} e^{-m_1 y} - B_{14} e^{-Scy}\right)\end{pmatrix} \quad (31)$$

$$\mathrm{T}(y,t) = e^{-m_3 y} + \varepsilon e^{i\omega t}\left((1 - B_2) e^{-m_5 y} + B_2 e^{-m_3 y}\right) + O(\varepsilon^2) \quad (32)$$

$$\mathrm{C}(y,t) = e^{-Scy} + \varepsilon e^{i\omega t}\left((1 - B_1) e^{-m_1 y} + B_1 e^{-Scy}\right) + O(\varepsilon^2) \quad (33)$$

The important parameters studies in such kind of boundary layer flows are skin friction, Nusselt number and Sherwood number which are evaluated as

Skin friction: It represents the drag at the plate and evaluated from the velocity profiles as

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = |U|\cos(\omega t + \phi_1) \quad (34)$$

where $|U| = \sqrt{U_r^2 + U_i^2}$ is the amlitude of skin friction and $\phi_1 = \tan^{-1}\frac{U_i}{U_r}$ such that

$$U_r + iU_i = -m_7 B_5 + m_3 B_4 + ScB_3 + Rm(-m_7 B_8 + m_3 B_7 + ScB_6) + \varepsilon e^{i\omega t}\begin{pmatrix} -m_7 B_{13} + m_5 B_{12} - m_3 B_{11} + m_1 B_{10} + ScB_9 + \\ Rm(-m_7 B_{18} + m_5 B_{17} - m_3 B_{16} + m_1 B_{15} + ScB_{14})\end{pmatrix}$$

Nusselt number: It gives the amount of heat transfer at the plate and derived from temperature profiles as

$$\mathrm{Nu} = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = |V|\cos(\omega t + \phi_2) \quad (35)$$

where $|V| = \sqrt{V_r^2 + V_i^2}$ is the amlitude of skin friction and $\phi_2 = \tan^{-1}\frac{V_i}{V_r}$ such that

$$V_r + iV_i = m_3 + \varepsilon e^{i\omega t}\left(m_5(1 - B_2) + m_3 B_2\right)$$

Sherwood Number: It gives the amount of mass transfer at the plate and derived from concentration profiles as

$$\mathrm{Sh} = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = |W|\cos(\omega t + \phi_3) \quad (36)$$

where $|W| = \sqrt{W_r^2 + W_i^2}$ is the amlitude of skin friction and $\phi_3 = \tan^{-1}\frac{W_i}{W_r}$ such that

$$W_r + iW_i = Sc + \varepsilon e^{i\omega t}\left(m_1(1 - B_1) + ScB_1\right)$$

4. RESULTS AND DISCUSSION

The physical interpretion of the problem under study is described graphically by evaluating various parameter effects such as magnetic Reynold numbner, Grashof number, Prandtl number, suction parameter, permeability parameter, radiation parameter, Schmidt number, scalar constant, permeability constant, magnetic parameter on velocity, temperature and concentratiion profiles. Also the influence of these parameters on skin friction, Nusselt number and Sherwood number is analysed.

The variation behaviour of velocity profiles with different flow parameters are illustrated through the graphs (1) and (12). From the figures (1)- (4), it is found that the increase in A, d, Rm and ε values leads to increase in velocity profiles near the plate. Also the increase in magnetic parameter (M) and viscoelastic parameter of Rivlin Ericksen fluid result in increase in velocity profiles which is presented with figures (5) and (6). However the bahaviour of velocity profiles with the variation in permeability parameter, radiation parameter, Prandtl number and thermal Grashof number is illustrated through figures (7) – (10). It is observed that the velocity profikes shows decreasing behaviour with the increase in values of Pr, K, R and Gr. The variation in mass Grashof number shows the interesting behaviour in velocity distribution as depicted in figure (11). It is noted that the increase in Gm causes reduction in velocity profiles upto some spanwiseco ordinate y but after that it shows increment in velocity distribution. Also the increase in plate moving velocity u_w results in increase in velocity distribution which is illustrated through the figure (12). Figures (13) – (16) depicts the temperature profiles variation with spanwise coordinate y for different flow parameters. It is observed that temperature profiles get lowered with the increase in Prandtl number (Pr), oscillation (ω), radiation parameter (R) and time (t) but the reverse effect is observed with the increase in scalar constant (ε). In figures (17) - (20), the behaviour of concentration profiles with the variation in different physical parameters governing the flow is shown. It is observed the increase in time (t), Schmidt number (Sc), suction constant (A) and oscillation frequency (ω) results in the decrease in concentration profiles.

The variation in skin friction coefficeint, Nusselt number and Sherwood number for different flow parameter is calculated along with their amplitude and phase angle which is represented in the form tables 1 and 2. It is observed that the modulus of skin friction increases with increase in Pr, R, Gr, Gm, Rm, M and d whereas it decreaseas with increase in A, K, u_w and ω. The phase angle of skin friction coefficient is observed to be increasing with the increase in flow parameters Pr, R, K, Gm, Rm, u_w, d and ω but it shows decreasing with increment in A, Gr and M. Further the skin friction coefficient behaviour is assesed which results that the surface skin friction coefficient increases with radiation parameter, suction parameter, permeability parameter and oscillations whereas it decreases with increment in values of Prandtl number, thermal and mass Frashoff numbers, viscoelatic parameter, magnetic number and permeability constant. The rate of heat and mass transfer calculated in terms of Nusselet number and Sherwood number along with their amplitude and phase angle are placed in table 2. It is noticed that amplitude of Nusselet number increases with the increment in the values of Pr, R\and A whereas it decreases with increase in oscillations frequency. Further the phase angle of Nusselt number shows the increrasing behaviour with increase in values of Pr and ù but it is reduced with the increase in values of R and A. The overall behaviour of Nusselet number assesment depicts that the increase in values of Pr and R, the Nusselt number increases whereas it shows reverse behaviour with the increase in values of A and ω. Further the Sherwood number is observed to be decreasing with increase in Schmidt number and oscillation frequency.

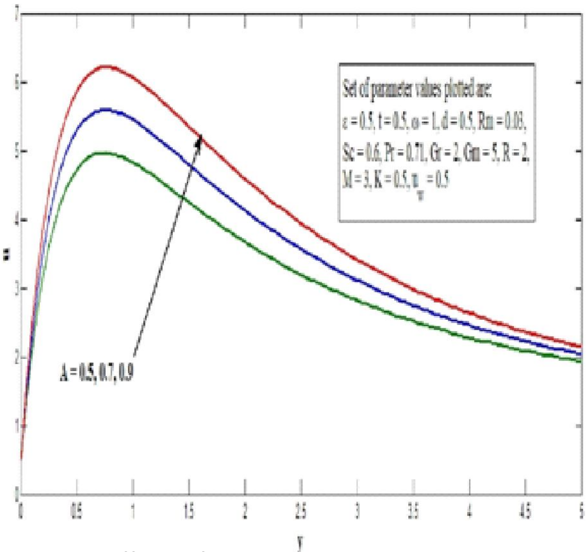

Fig.1: Effects of suction velocity parameter (A) on velocity profiles

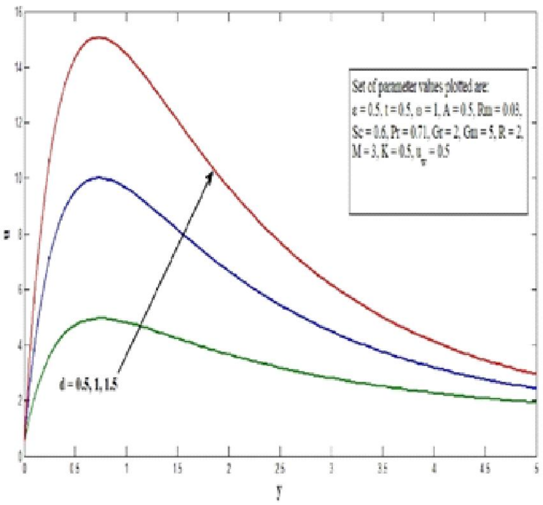

Fig.2: Effects of permeability constant (d) on velocity profiles

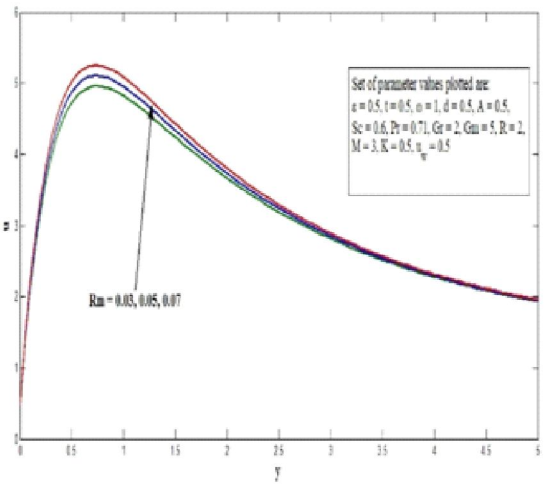

Fig.3: Effects of viscoelastic parameter (Rm) on velocity profiles

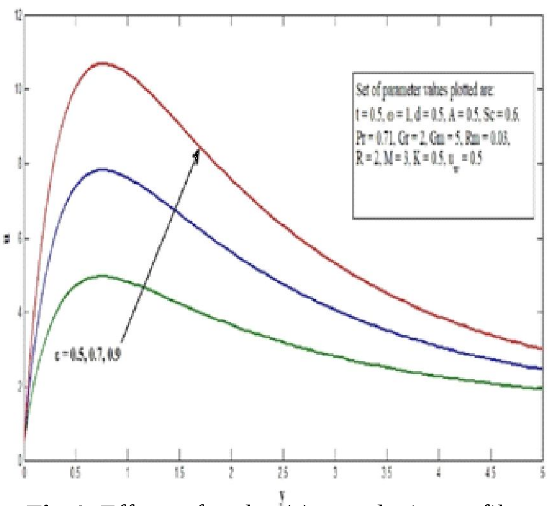

Fig.4: Effects of scalar (ε) on velocity profiles

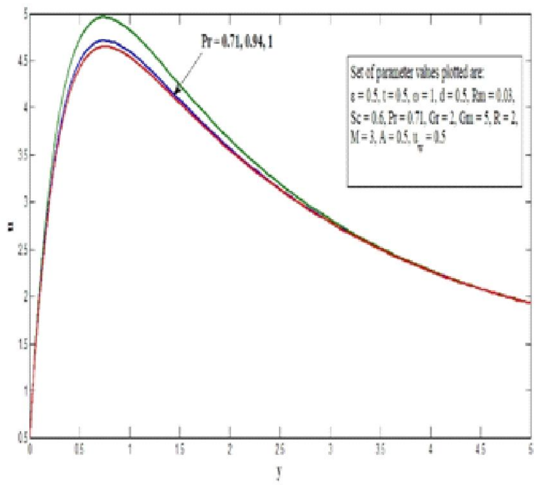

Fig.5 : Effects of Prandtl number (Pr) on velocity profiles

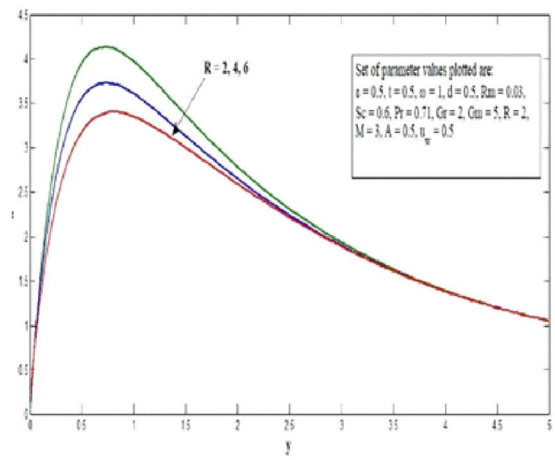

Fig.6: Effects of radiation parameter (R) on velocity profiles

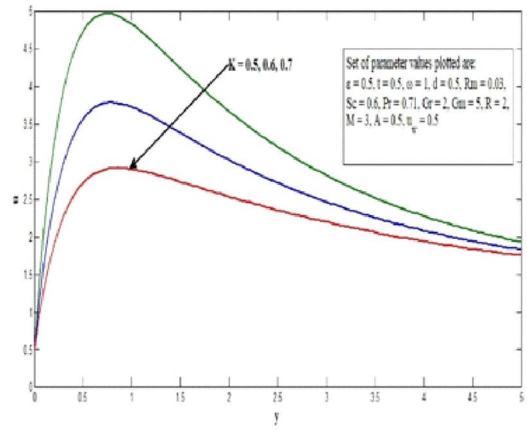

Fig.7: Effects of permeability parameter (K) on velocity profiles

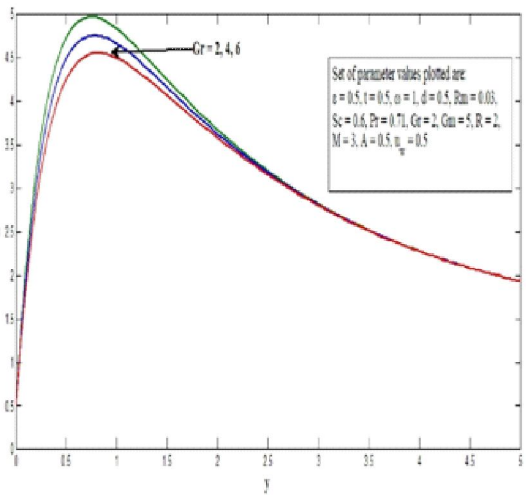

Fig.8: Effects of thermal Grashof number(Gr) on velocity profiles

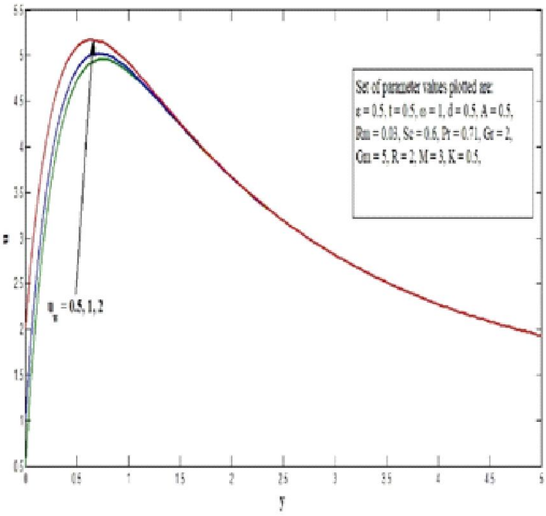

Fig.11: Effects of plate velocity (u_w) velocity profiles

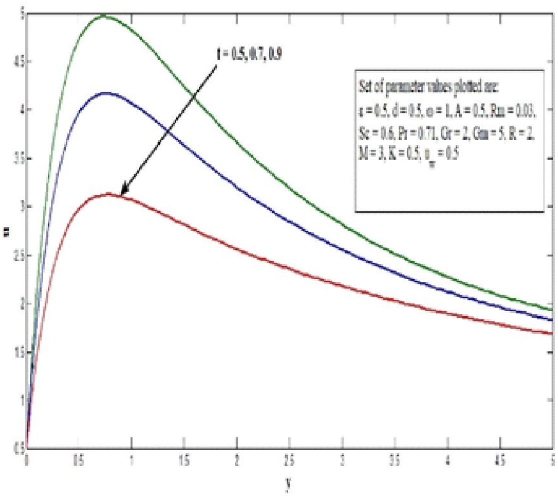

Fig.9: Effects of time (t) on velocity profiles

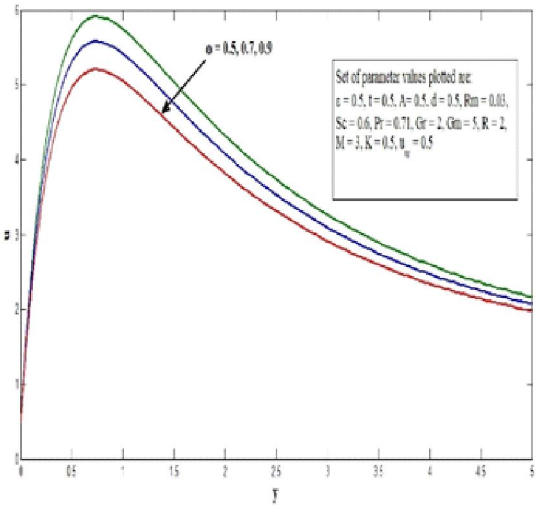

Fig.12: Effects of oscillation frequency (ω) on velocity profiles

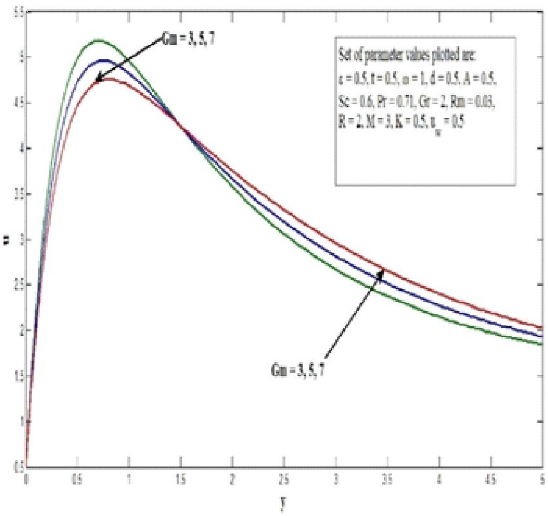

Fig.10: Effects of mass Grashof number (Gm) on velocity profiles

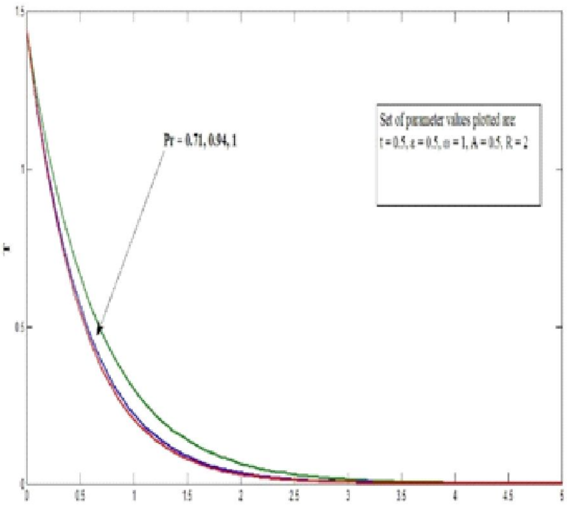

Fig.13: . Effects of Prandtl number (Pr) on temperature profiles

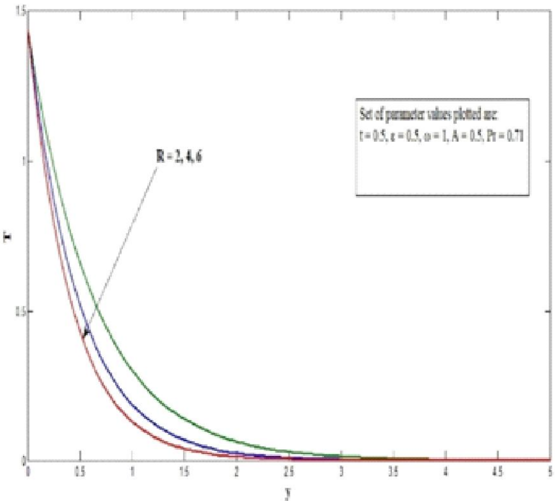

Fig.14 : Effects of radiation parameter (R) on temperature profiles

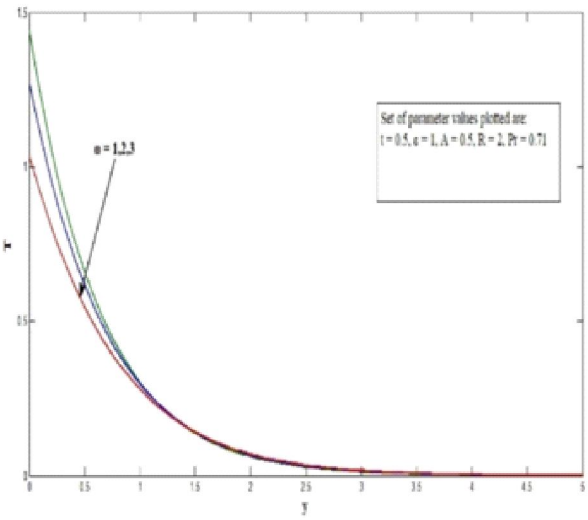

Fig.15 : Effects of oscillation frequency (ω) on temperature profiles

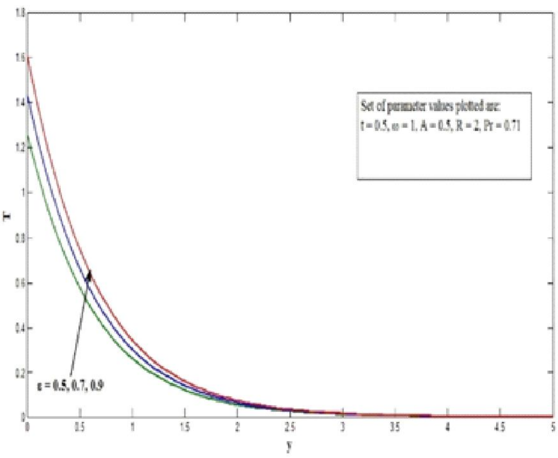

Fig.16. Effects of scalar constant (ε) on temperature profiles

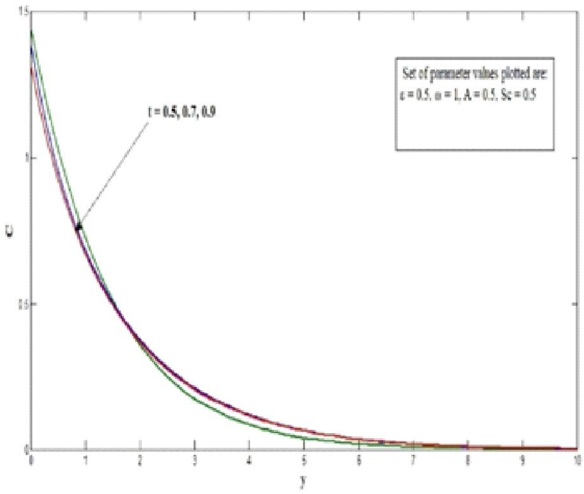

Fig.17 : Effects of time (t) on concentration profiles

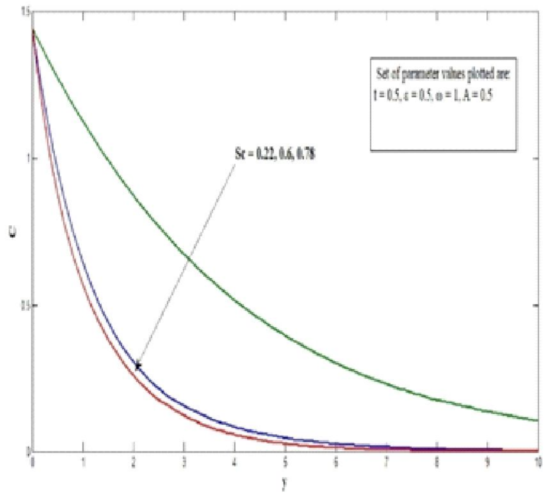

Fig.18: Effects of Schmidt number(Sc) on concentration profiles

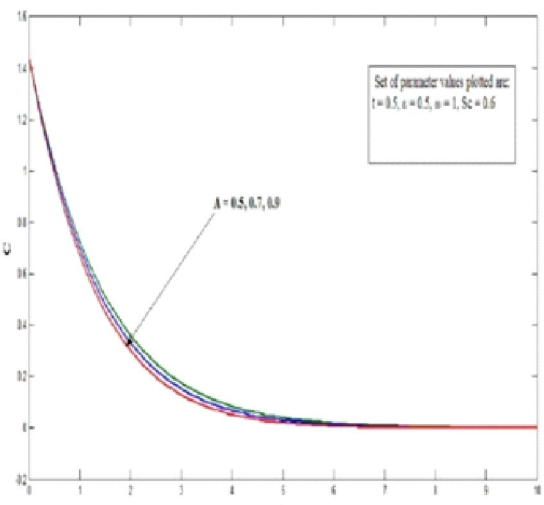

Fig.19: Effects of suction velocity parameter on concentration profiles

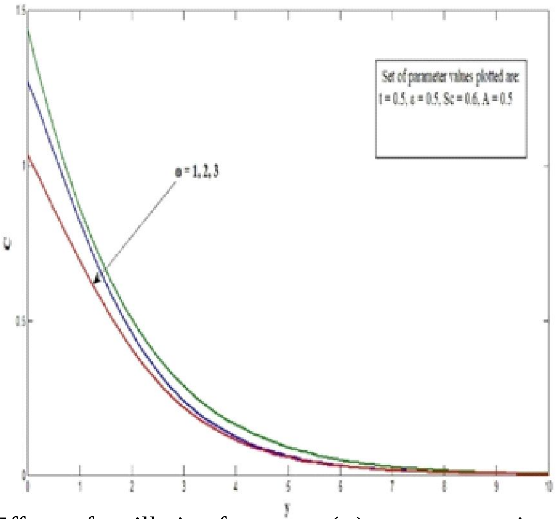

Fig.20: Effects of oscillation frequency (ω) on concentration profiles

Table-1: Effects of flow parameters on Skin friction for $\varepsilon = 0.5,\ \omega t = \frac{\pi}{2}$

Pr	R	A	K	Sc	Gr	Gm	Rm	M	Up	d	w	$\lvert U\rvert$	φ_1	τ
0.71	4	0.5	0.5	0.22	2	5	0.03	3	0.5	0.5	1	2.364775	1.684132	-2.349603
0.94	4	0.5	0.5	0.22	2	5	0.03	3	0.5	0.5	1	2.365179	1.685198	-2.349718
1	**4**	0.5	0.5	0.22	2	5	0.03	3	0.5	0.5	1	2.365270	1.685482	-2.349732
1	**5**	0.5	0.5	0.22	2	5	0.03	3	0.5	0.5	1	2.365397	1.686264	-2.349646
1	**6**	**0.5**	0.5	0.22	2	5	0.03	3	0.5	0.5	1	2.365496	1.687036	-2.349533
1	6	**0.7**	0.5	0.22	2	5	0.03	3	0.5	0.5	1	2.256668	1.684309	-2.242145
1	6	**0.9**	**0.5**	0.22	2	5	0.03	3	0.5	0.5	1	2.147858	1.681305	-2.134756
1	6	0.9	**0.6**	0.22	2	5	0.03	3	0.5	0.5	1	1.667305	1.708175	-1.651596
1	6	0.9	**0.7**	**0.22**	2	5	0.03	3	0.5	0.5	1	1.337717	1.737626	-1.319144
1	6	0.9	0.7	**0.6**	2	5	0.03	3	0.5	0.5	1	25.852949	2.061380	-22.803798
1	6	0.9	0.7	**0.78**	**2**	5	0.03	3	0.5	0.5	1	16.632186	1.705887	-16.480651
1	6	0.9	0.7	0.78	**4**	5	0.03	3	0.5	0.5	1	16.726263	1.700391	-16.586001
1	6	0.9	0.7	0.78	**6**	**5**	0.03	3	0.5	0.5	1	16.820840	1.694957	-16.691351
1	6	0.9	0.7	0.78	6	**7**	0.03	3	0.5	0.5	1	19.956136	1.916351	-18.776480
1	6	0.9	0.7	0.78	6	**9**	**0.03**	3	0.5	0.5	1	23.790476	2.072240	-20.861609
1	6	0.9	0.7	0.78	6	9	**0.05**	3	0.5	0.5	1	24.109216	2.080131	-21.049013
1	6	0.9	0.7	0.78	6	9	**0.07**	**3**	0.5	0.5	1	24.429418	2.087814	-21.236417
1	6	0.9	0.7	0.78	6	9	0.07	**5**	0.5	0.5	1	31.508735	1.747579	-31.017655
1	6	0.9	0.7	0.78	6	9	0.07	**7**	**0.5**	0.5	1	42.417507	1.469323	-42.199313
1	6	0.9	0.7	0.78	6	9	0.07	7	**1**	0.5	1	42.277735	1.509878	-42.199313
1	6	0.9	0.7	0.78	6	9	0.07	7	**1.5**	**0.5**	1	42.207891	1.550635	-42.199313
1	6	0.9	0.7	0.78	6	9	0.07	7	1.5	**1**	1	50.604392	1.556937	-50.599532
1	6	0.9	0.7	0.78	6	9	0.07	7	1.5	**1.5**	**1**	59.002330	1.561446	-58.999751
1	6	0.9	0.7	0.78	6	9	0.07	7	1.5	1.5	**2**	52.655213	1.587464	-52.647899
1	6	0.9	0.7	0.78	6	9	0.07	7	1.5	1.5	**3**	50.310382	1.628064	-50.227904

Table-2: Effects of flow parameters on Nusselt number and Sherwood number for $\varepsilon = 0.5, \omega t = \frac{\pi}{2}$

Pr	R	A	Sc	ω	|V|	|W|	φ_2	φ_3	Nu	Sh
0.71	2	0.5	0.22	1	1.680138	0.242403	0.493851	0.838049	-0.796271	-0.180187
0.94	2	0.5	0.22	1	1.984537	0.242403	0.502176	0.838049	-0.955226	-0.180187
1	**2**	0.5	0.22	1	2.058576	0.242403	0.504512	0.838049	-0.995075	-0.180187
1	**4**	0.5	0.22	1	2.663952	0.242403	0.499706	0.838049	-1.276481	-0.180187
1	**6**	**0.5**	0.22	1	3.134326	0.242403	0.497650	0.838049	-1.496208	-0.180187
1	6	**0.7**	0.22	1	3.135782	0.242403	0.497391	0.838049	-1.496191	-0.180187
1	6	**0.9**	**0.22**	1	3.137239	**0.242403**	0.497133	**0.838049**	-1.496175	**-0.180187**
1	6	0.9	**0.6**	1	3.137239	**0.702977**	0.497133	**0.705124**	-1.496175	**-0.455619**
1	6	0.9	**0.78**	**1**	3.137239	**0.926334**	0.497133	**0.688564**	-1.496175	**-0.588619**
1	6	0.9	0.78	**2**	2.920444	**0.890343**	0.533304	**0.746272**	-1.484701	**-0.604459**
1	6	0.9	0.78	**3**	2.703785	**0.867730**	0.572871	**0.806138**	-1.465578	**-0.626171**

5. CONCLUSION

The studied behaviour of unsteady flow of viscous incompressible and electrically conducting Rivlin Ericksen fluid past a semi-infinite vertical porous plate having variable permeability and suction under thermal radiation with variable temperature and concentration is concluded as

- The suction at the plate raises the fluid velocity but it has reverse
- effects on fluid temperature and species concentration.
- The presence of radiation causes the reduction in fluid temperature and this results in decrease in thermal buoyancy forces which in turn lower down the fluid velocity.
- The fluid velocity is increased with increase in dimensionless viscoelastic parameter of Rivlin Ericksen fluid, magnetic number and plate moving velocity.
- The fluid velocity, temperature and species concentration is decreased with the progress in time and for small oscillations but they increases with the increment in scalar constant (ε).
- The surface friction is increased due to suction and permeability in the presence of radiation effect whereas it is reduced with the increment in
- viscoelastic parameter of Rivlin Ericksin fluid and in the presencre of magnetic field.
- The rate of heat transfer is increased in the presence of thermal radiation whereas the suction and oscillations has reverse effect on it.
- The rate of mass transfer is decreased with increase in oscillation frequency and Schmidt number which shows that mass diffusion accelerates the mass transfer at the surface of plate.

6. APPENDIX

$$m_1 = \frac{\mathrm{Sc} + \sqrt{\mathrm{Sc}^2 + i\omega Sc}}{2}$$

$$m_2 = \frac{-\mathrm{Sc} + \sqrt{\mathrm{Sc}^2 + i\omega Sc}}{2}$$

$$m_3 = \frac{\mathrm{Pr} + \sqrt{\mathrm{Pr}^2 + 4R\,\mathrm{Pr}}}{2}$$

$$m_4 = \frac{-\mathrm{Pr} + \sqrt{\mathrm{Pr}^2 + 4R\,\mathrm{Pr}}}{2}$$

$$m_5 = \frac{\mathrm{Pr} + \sqrt{\mathrm{Pr}^2 + (4R + i\omega)\mathrm{Pr}}}{2}$$

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$$m_6 = \frac{-\Pr + \sqrt{\Pr^2 + (4R + i\omega)\Pr}}{2}$$

$$m_7 = \frac{1 + \sqrt{1 + 4\left(\frac{1}{K} + M\right)}}{2}$$

$$m_8 = \frac{-1 + \sqrt{1 + 4\left(\frac{1}{K} + M\right)}}{2}$$

$$B_1 = \frac{4\text{ASci}}{\omega}$$

$$B_2 = \frac{A\Pr m_3}{m_3^{\ 2} - \Pr m_3 - \frac{1}{4}(4R + i\omega)}$$

$$B_3 = \frac{Gm}{Sc^2 - Sc - \left(\frac{1}{K} + M\right)}$$

$$B_4 = \frac{\text{Gr}}{m_3^{\ 2} - m_3 - \left(\frac{1}{K} + M\right)}$$

$$B_5 = U_P + B_3 + B_4 - 1$$

$$B_6 = \frac{Sc^3 B_3}{Sc^2 - Sc - \left(\frac{1}{K} + M\right)}$$

$$B_7 = \frac{m_3^{\ 3} B_4}{m_3^{\ 2} - m_3 - \left(\frac{1}{K} + M\right)}$$

$$B_8 = B_6 + B_7$$

$$B_9 = \frac{\left(ASc - \frac{d}{K}\right)B_3 + GmB_1}{Sc^2 - Sc - \left(\frac{1}{K} + M\right)}$$

$$B_{10} = \frac{\text{G}m(1 - B_1)}{m_1^{\ 2} - m_1 - \left(\frac{1}{K} + M\right)}$$

$$B_{11} = \frac{-Am_3B_4 + \frac{d}{K}B_4 - GrB_2}{m_3^{\ 2} - m_3 - \left(\frac{1}{K} + M\right)}$$

$$B_{12} = \frac{\text{G}r(1 - B_2)}{m_5^{\ 2} - m_5 - \left(\frac{1}{K} + M\right)}$$

$$B_{13} = B_{12} - B_{11} + B_{10} + B_9 - 1$$

$$B_{14} = \frac{(B_9 + B_3)Sc^3 + \frac{i\omega}{4}B_9Sc^2 + \left(ASc - \frac{d}{K}\right)B_6}{Sc^2 - Sc - \left(\frac{1}{K} + M\right)}$$

$$B_{15} = \frac{\left(m_1^{\ 3} + \frac{i\omega}{4}m_1^{\ 2}\right)B_{10}}{m_1^{\ 2} - m_1 - \left(\frac{1}{K} + M\right)}$$

$$B_{16} = \frac{\left(m_3^{\ 3} + \frac{i\omega}{4}m_3^{\ 2}\right)B_{11} - B_4m_3^{\ 3} + \left(\frac{d}{K} - Am_3\right)B_7}{m_3^{\ 2} - m_3 - \left(\frac{1}{K} + M\right)}$$

$$B_{17} = \frac{\left(m_5^{\ 3} + \frac{i\omega}{4}m_5^{\ 2}\right)B_{12}}{m_5^{\ 2} - m_5 - \left(\frac{1}{K} + M\right)}$$

$$B_{18} = B_{17} - B_{16} + B_{15} + B$$

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REFERENCES

[1] K. Walter," The motion of an elastic-viscous liquid contained between concentric spheres", Quarterly Journal of Applied Mechanics, vol. 13, no.3, pp. 325-333, 1960.

[2] K. Walter, "The motion of an elastic-viscous liquid contained between coaxial cylinders", Quarterly Journal of Applied Mechanics, vol. 13, no. 4, pp. 444-461, 1960.

[3] K. Walter,"Non-Newtonian effects in some elastic-viscous liquids whose behaviour at small rates of shear is characterized by a general linear equation of state", Quarterly Journal of Mechanics and Applied Mathematics, vol. 15, no. 1, pp. 63-76, 1962.

[4] J.P. Harnett, "Viscoelastic Fluids: A new challenge in heat transfer", Transactions of ASME, pp. 296-303, 1992.

[5] EzzatA. Magdy, "Magnetohydrodynamic unsteady flow of non-Newtonian fluid past an infinite porous plate, Indian" J. Pure Appl. Math., vol. 25, no. 6, pp. 655-664, 1994.

[6] Dekker, Mercel, "Handbook of Porous Media", New York. Basel, Switzerland, 2000.

[7] M.K. Chaudhary and M.N. Islam,"MHD free convection flow of viscoelastic fluid past an infinite vertical porous plate", Heat and Mass Transfer, vol. 36, pp. 439-447, 2000.

[8] R.C. Chaudharyand P. Jain, "Hall effect on MHD mixed convection flow of a viscoelastic fluid past an infinite vertical plate with mass transfer and radiation", Theoretical and Applied Mechanics, vol. 33, no. 4, pp. 281-309, 2006.

[9] R.C.Chaudhary and A. Jain, "Combined Heat and Mass Transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium", Rom. Journal Phys., vol. 52, no. 5 – 7, pp. 505-524, 2007.

[10] S.S Das and M. Mitra, "Unsteady mixed convective MHD flow and mass transfer past an accelerated infinite vertical porous plate with suction, Indian Journal of Science and Technology", vol. 2, no. 5, pp.18 – 22, 2009.

[11] M.A Samad and M. Mohebujjaman, "MHD Heat and Mass Transfer free convection flow along a vertical stretching sheet in presence of magnetic field with heat generation", Research J. of Applied Sciences Engineering and Technology, vol. 1, no. 3, pp. 98 – 106, 2009.

[12] O. D. Makinde, "On MHD Boundary layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux", International Journal of Num. Methods for Heat and Fluid Flow,vol. 19, pp. 3-4, 546 – 554, 2009.

[13] K. Das and S. Jana, "Heat and Mass Transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium", Bull. Soc. Math. Banja Luka, vol. 17, no. 10, pp. 15-32, 2010.

[14] R. Chaudhary and K. Debnath, "Unsteady oscillatory MHD flow of a viscoelastic fluid past a porous vertical plate with periodic suction", International Journal of Mathematical Archive, vol. 3, no. 5, pp. 1764-1771, 2012.

[15] B.C. Chandraseckharaand P.M. Namboodiri," Inûuence of variable permeability on combined free and forced convection about inclined surface in porous medium", International Journal of Heat and Mass Transfer, vol. 28, pp. 199–206, 1985.

[16] S. Sreekanth, S. Venkataramanaand S. Ramakrishna, Hydrodynamic free convective flow through a porous medium with variable permeability, Acta Ciencia Indica, Vol. XX IIM, no. 3, pp. 267 – 273, 1996.

[17] S.M.M El-Kabier andA. M Rashad," Influence of variable permeability on free convection over vertical flat plate embedded in a porous medium", International Journal of Applied Mathematics and Mechanics, 2(3), pp.12 – 23,2006.

[18] M.A. Seddeekand A. A. Almushigeh, "Effects of Radiation and Variable viscosity on MHD free convective flow and mass transfer over a stretching sheet with chemical reaction". International Journal A. A. M., vol. 5, no.1, pp. 181 – 197, 2010.

[19] D. Pal, "Magnetohydrodynamic non-Darcy mixed convection heat transfer from a vertical heated plate embedded in a porous medium with variable

porosity", Commun. Nonlinear Sci. Numer. Simulat., vol. 15, pp. 3974–3987, 2010.

[20] P.V. Satya Narayana and P.V.S. Sravanthi, "Inûuence of variable permeability on unsteady MHD convection ûow past a semi inûnite inclined plate with thermal radiation and chemical reaction", Journal of Energy, Heat and Mass Transfer, vol. 34, pp. 143–161, 2012.

[21] Hareesh D. and P.V. Satya Narayana,"Inûuence of variable permeability and radiation absorption on the heat and mass transfer in MHD micropolar ûow over a vertical moving porous plate", ISRN Thermodynamics, Article ID 953536, 17 pages. http://dx.doi.org/10.1155/2013/953536, 2013.

[22] B.G. Srivastava and S. Deo,"Effect of magnetic ûeld on the viscous ûuid ûow in a channel ûlled with porous medium of variable permeability", Applied Mathematics and Computation, vol. 219, pp. 8959–8964, 2013.

[23] G. Noushima, M. Human, M. V. Ramana, M. C. Reddy, Ramu A. Rafiuddin and S. Rajender, "Hydro Magnetic free convective Rivlin – Erickson flow through a porous medium with variable permeability", International Journal of Comp. and Applied Mathematics, vol. 5, no. 3, pp. 267 – 275, 2010.

[24] N.K.Varshney, S. Singh and J. Singh, "Effect of rotatory Rivlin Ericksen fluid on MHD free convection and mass transfer flow through porous medium with constant heat and mass flux across moving plate", International Organization of Scientific Research, vol. 1, no.1, pp. 10-17, 2011.

[25] J. Uwanta and A. Hussain, Effects of mass transfer on hydromagnetic free convective Rivlin – Erickson flow through a porous medium with time dependent suction. Ind. J. of Sci., vol. 1, no. 2, pp. 120-124, 2012.

[26] A.S. Banyal and D.K. Sharma, "Thermal Convection of Rivlin Ericksen fluid in the presence of vertical rotation", Journal of Mechanical Engineering and Sciences, vol. 4, pp. 462-471, 2013.

[27] V. RaviKumar, M. C. Raju and G. S. S. Raju, "Combined effects of heat absorption and MHD on convective past a semi-infinite vertical porous plate with variable temperature and suction",Ains Shams Engineering Journal, vol. 5, pp.867-875, 2014.

[28] H. A. Hussaini, M.V.R.Murthy and Rafiuddin, "Magnetohydrodynamics Rivlin- Ericksen flow through porous medium in slip flow regime", Int. J. Sci., Engg. and Tech. Research, vol. 4, no.7, pp. 2568-2572, 2015.

[29] P. C. Reddy, M.C.Raju, G.S.S. Raju and M. C. Reddy,"Diffusion thermo and thermal diffusion effects on MHD free convection flow of Rivlin-Ericksen fluid past a semi- infinite vertical plate". Bulletin of Pure and Applied Sciences, vol. 36E, no. 2, pp. 266-284, 2017.

