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Transient Vibration Control of Delaminated Composite Plates Using Active Fiber Composites and Velocity Feedback Gain

Ganesh Shankar[a], Ashes Maji[b] , P.K.Mahato[c]

[a]Mechanical Engineering Department , Ramgarh Engineering College (Estd. by Govt. of Jharkhand and run by Techno India under PPP) Jharkhand 814101 India

[b]Mechanical Engineering Department Asansol Engineering College Asansol, West Bengal

[c]Department of Mechanical Engineering, Indian Institute of Technology(Indian School of Mines), Dhanbad 826004, India

Abstract

This article presents an active control strategy to mitigate the undesirable vibration responses in delaminated composite plates. The control system integrates Active Fiber Composite (AFC) patches as actuators and employs a velocity feedback control algorithm with a tunable gain (Gv) to provide active damping. A finite element model, coded in MATLAB, forms the basis for simulating the closed-loop control performance. A parametric study is conducted to evaluate the effectiveness of the controller by analyzing the influence of the velocity feedback gain (Gv) and the optimal placement of the AFC patches. The results demonstrate that the implemented active control strategy successfully attenuates the amplified dynamic displacements caused by Delaminations damage. The study concludes that the integrated AFC system with velocity feedback is a highly effective method for suppressing vibrations and enhancing the performance of damaged composite structures.

Keywords: Active Vibration Control, Active Fiber Composite (AFC), Velocity Feedback, Smart Structures, Composite Plates, Delaminations..

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Introduction

Composite materials find wide applications in the aerospace, marine, sports, and automobile industries owing to their well-recognized properties of light weight and high strength. During service, composite laminates are often subjected to repeated cyclic loading and external impacts, which may lead to delamination between their plies. Such delamination reduces the stiffness and strength of the structure, while simultaneously causing significant deformation along both longitudinal and transverse directions due to increased residual stresses within the laminae. To counteract these undesirable effects, smart structural models are required. Active Fiber Composite (AFC) patches and velocity feedback gain are employed as both actuators and sensors to suppress undesirable vibratory responses in the time domain. A finite element formulation for the AFC patches and the delaminated plate is implemented in MATLAB.

Active vibration control is always a challenging area for researchers. Piezoelectric actuators and sensors having large applications in vibration control and shape control of laminated structures. In 1990 Tzou and Tseng [1] proposed a new structure containing integrated distributed piezoelectric sensor and actuator where the distributed piezoelectric sensing layer monitors the structural oscillation due to direct piezoelectric effect and the distributed actuator layer suppresses the oscillation via the converse piezoelectric effect .Thus, the performance of plate model was evaluated. Woo-Seok et al. [2] suggested a numerical solution and design strategy for a laminated composite plate with piezoelectric sensors or actuator where the vibration is controlled by passive and active control methods. Sun and Tong [3] in 2001 presented a novel approach for vibration control of smart

Corresponding Author: Ganesh Shankar, Mechanical Engineering Department , Ramgarh Engineering College (Estd. by Govt. of Jharkhand and run by Techno India under PPP) Jharkhand 814101 India e-mail: gshankar406@gmail.com

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plates using discretely distributed piezoelectric actuator and sensor and the results obtained using the present optimal criteria. Some extensive dynamic analyses of smart structures are listed in articles [4-7]. AFC is a special type of piezoelectric material with interdigitated electrode embedded on its surface which was originally developed by Hagood and Bent [8]. It has high value of piezoelectric charge constant Mahato and Maiti [9-, 10] investigated flutter control analysis of AFC laminate subjected to hygrothermal environment. Brunner et al. [11] discussed the capability of AFC in structural health monitoring. Guruprasad et al. [12] have modelled the AFC as a sensing device for detecting the damage and delamination in structure. Martinez and Artemev [13] studied the actuation degradation of AFC in presence of fibre damage.

In context of dynamic control of smart delaminated composite structures few researches are available. Chattopadhyay et al. [14] have discussed the deflection of delaminated composite beam in time history. Piezoelectric sensor are used to delamination detection in beam with finite element formulation is done by Perel et al. [15]. Effect of delamination in dynamic response is carried by Ghoshal et al. [16, 17]. An active control algorithm is used to recover the vibration characteristic from delaminated structures to healthy structures is performed by Sohn and Kim [18].

It has been observed from literature review that very limited research are available in the context of active vibration control of delaminated composite plates. However the present work describe a very confine mathematical model for closed loop vibration control of delaminated composite plate by using Active Fiber composite as an actuators and sensors, which code has developed in the MATLAB. The present model of control strategy have mitigate the undesirable vibration response in time constraint.

Mathematical Modeling

The mathematical modeling is based on First Order Shear deformation theory, where constitutive equation of Electro-Elastic relationship is given in Equation (1). Where, $\{\sigma_{ij}\}$ is the stress vector $[Q_{ij}]$ is the constitutive matrix, $\{\varepsilon_{ij}\}$ is the strain vector due to mechanical loading, $\{D_i\}$ is the electric displacement, $[e_{ij}]$ is the piezoelectric stress coefficient matrix, $[\kappa]$ is the dielectric constant, $\{E_i\}$ is the electric field vector. (1)

If V is the electric potential difference between the two electrodes and h_{et} is the distance between two electrodes and we assuming the electric field is acting along the X-direction so the electric field vector can be written as equation (2)

$$\begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix} = \begin{Bmatrix} -1/h_{et} \\ 0 \\ 0 \end{Bmatrix} V \qquad (2)$$

In above equations (1-2), the constitutive relations are in material co-ordinate systems. Each lamina may have different orientations with respect to global or structural co-ordinate system;

Finite element formulation and stress resultant matrix for healthy plate element

Element for a healthy plate based on the first order displacement theory is applied. An eight nodded serendipity element, with five degrees of freedom at each node is adopted here for element formulation. Displacement fields are interpolated using Lagrangian shape function as follows.

$$u_0 = \sum_{i=1}^{8} N_i u_i \ , \ v_0 = \sum_{i=1}^{8} N_i v_i \ , w_0 = \sum_{i=1}^{8} N_i w_i \ , \theta_x = \sum_{i=1}^{8} N_i \theta_{x_i} \ , \theta_y = \sum_{i=1}^{8} N_i \theta_{y_i} \qquad (3)$$

Where ui, vi, wi are the nodal displacementsθxi, θyi are nodal rotation degree of freedom along mid-plane and Ni is the shape function of corresponding node. Now strain-displacement relation is deriving from above equation.

Finite element procedure for Delamination modeling

The analytical modeling of a delaminated laminated plate involves its partitioning into distinct sub-domains: an integral (healthy) region and a delaminated region. Figure 1 illustrates this concept, where the zone denoted 'abcd' represents the delaminated area. This region is subsequently treated as two discrete, un-bonded sub-laminates—an upper and a lower segment.

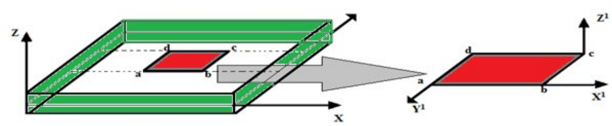

Fig. 1: Isometric view and coordinate system of delaminated plate

For a mid-plane delamination, the upper and lower segments within the delaminated zone are discretized with independent finite element meshes. A local coordinate system (x_1, y_1, z_1) is established for this zone. A key aspect of this modeling approach is maintaining a global z-coordinate reference for both the integral laminate and the delaminated sub-laminates. This convention correctly captures the eccentricity of the sub-laminate mid-planes relative to the mid-plane of the original intact laminate.

The displacement field within the delaminated region is formulated according to the First-Order Shear Deformation Theory (FSDT), as expressed in Equation (4). The corresponding displacement fields for the individual upper and lower sub-laminates are obtained by applying the appropriate subscript modifications to this governing equation.

$$
\begin{aligned}
u_B(x^1, y^1) &= u_B^0 + z\theta_{x_B} \\
v_B(x^1, y^1) &= v_B^0 + z\theta_{y_B} \\
w_B(x^1, y^1) &= w_B^0
\end{aligned} \qquad (4)
$$

Dynamics analysis and control mechanism

The kinetic energy of delaminated and healthy plate element and the total potential energy due to mechanical, electrical, mechanical external load and AFC (piezo-fiber) external load is given below in equation (5) and (6).

$$
T_{KE} = \frac{1}{2}\sum_{K=1}^{N}\int_A \{\dot{\delta}\}^T[m]\{\dot{\delta}\}dA + \frac{1}{2}\sum_{K=1}^{P}\int_A \{\dot{\delta}_d\}^T[m]\{\dot{\delta}_d\}dA + \frac{1}{2}\sum_{K=P+1}^{N}\int_A \{\dot{\delta}_d\}^T[m]\{\dot{\delta}_d\}dA \qquad (5)
$$

$$
\begin{aligned}
T_p = & \frac{1}{2}\sum_{L=2}^{n-1}\int_\Omega \{\varepsilon^L\}^T\{\sigma^L\}d\Omega + \frac{1}{2}\int_\Omega \{\varepsilon^1\}^T\{\sigma^1\}d\Omega - \frac{1}{2}\int_\Omega \{E^1\}^T\{D^1\}d\Omega \\
& - \frac{1}{2}\int_\Omega \{\varepsilon^n\}^T\{\sigma^n\}d\Omega - \frac{1}{2}\int_\Omega \{E^n\}^T\{D^n\}d\Omega - \int_A \{\delta\}^T\{q\}dA - \int_A V_{z=h_n}\psi(x,y)dA
\end{aligned} \qquad (6)
$$

Now applying the principle of minimum energy approach and substituting above equation in equation (6) we get three set of equilibrium equations with respect to nodal variable which are given as:

$$
\begin{aligned}
&[M^e]\{\ddot{\delta}^e\} + [K_{dd}^e]\{\delta^e\} - [K_{da}^e]\{V^{en}\} - [K_{ds}^e]\{V^{e1}\} = \{F_1^e\} \\
&[K_{ad}^e]\{\delta^e\} + [K_{aa}^e]\{V^{en}\} = \{F_2^e\} \\
&[K_{sd}^e]\{\delta^e\} + [K_{ss}^e]\{V^{e1}\} = \{0\}
\end{aligned} \qquad (7)
$$

Where $[M^e]$ is the elemental mass matrix and $[k^e{}_{dd}]$, $[k^e{}_{aa}]$ and $[k^e{}_{ss}]$ are the elemental stiffness matrices due to mechanical displacements, the actuators and sensors potential respectively. $[k^e{}_{da}]$ and $[k^e{}_{ds}]$ are the electromechanical coupling stiffness matrices where subscript 'a' for actuator and 's' for the sensor. $\{F^e{}_1\}$ and $\{F^e{}_2\}$ are elemental mechanical force vector and elemental electrical force vector respectively. Now the global equilibrium equations are obtained after the assembling the equation (7) with respect to global axis.

$$
\begin{aligned}
&[M]\{\ddot{X}\} + [K_{dd}]\{X\} - [K_{da}]\{V_a\} - [K_{ds}]\{V_s\} = \{F_1\} \\
&[K_{ad}]\{X\} + [K_{aa}]\{V_a\} = \{F_2\} \\
&[K_{sd}]\{X\} + [K_{ss}]\{Vs\} = \{0\}
\end{aligned} \qquad (8)
$$

Where $[M]$ is the global mass matrices and $[K_{dd}]$, $[K_{aa}]$, $[K_{ss}]$, $[K_{da}]$, $[K_{ds}]$, $[K_{ad}]$, $[K_{sd}]$ are the global generalized stiffness matrices $\{X\}$ is the global displacement and $\{F_1\}$ is the mechanical force vector, $\{F_2\}$ is the electric force vector. Eliminating $\{V_a\}$ and $\{V_s\}$ in Equation (8) can be rewritten as:

$$
[M]\{\ddot{X}\} + [K^*]\{X\} = \{F_1\} + \{F_c\} \qquad (9)
$$

Where, $[K^*]$ and $\{Fc\}$ are the controlling stiffness and control feedback force. The detail control mechanism is given in Mahato and Maiti [9].

$$
\begin{aligned}
&[K^*] = [K_{dd}] + [K_{da}][K_{aa}]^{-1}[K_{ad}] + [K_{ds}][K_{ss}]^{-1}[K_{sd}] \\
&\{F_c\} = [K_{da}][K_{aa}]^{-1}\{F_2\}
\end{aligned} \qquad (10)
$$

A Proportional- Derivative controller is used here and the voltage applied in actuator is given as below equation.

$$
\{V_a\} = G_v\{\dot{V}_s\} \qquad (11)
$$

Newmark's time integration scheme Fisette et al. [19] is employed here for the calculation of dynamic response in time history. The frequency response is based on Discrete Fourier Transformation analysis.

Boundary condition:

Three types of boundary condition are considered here for the numerical simulation in present article, which are S-S-S-S, C-F-F-F and C-C-C-C. Here 'S' stands for the simply supported edge, 'C' stand for clamped edge and 'F' for free edge.

Results and Discussion

The numerical analysis presented in this section is carried out using a finite element code developed in MATLAB. AFC patches are embedded on the top and bottom surfaces of the plate, with electro-mechanical coupling incorporated into the formulation for the AFC patches. A detailed parametric study is performed to examine the influence of velocity feedback

Table 1. Material properties of graphite/epoxy and AFC -50% fiber volume fraction.

Elastic module	graphite/epoxy [23]	AFC layer [23]
E11 (Gpa)	128	119.7
E22 (Gpa)	6.12	129.1
µ12	0.3	0.35
µ13	---	0.38
G12 (Gpa)	5.0	39.14
G13 (Gpa)	5.0	32.35
G23 (Gpa)	2.5	32.35
e11(c/m2)	---	14.14
e21(c/m2)	---	-3.34
e24(c/m2)	---	10.79
κ11(F/m)	---	8.599×10-9
κ33(F/m)	---	6.485×10-9
Density (Kg/m3)	1600	6700

gain (Gv), AFC patch placement, patch dimensions, presence of small midplane delamination, and different boundary conditions. The material properties of the graphite/epoxy composite and the AFC layer are summarized in Table 1.

An asymmetric laminated square plate of dimension 0.6*m*×0.6*m*×0.006*m* and orientation of each ply is (90/0/90/0) is taken for the analysis. A square delamination of very small sizes 0.14697*m*×0.14697*m* (6% of the total area), 0.15874 *m*×0.15874*m* (7% of the total area), 0.16970 *m*×0.16970*m* (8% of the total area), 0.18*m*×0.18*m* (9% of the total area) and 0.18974*m*×0.18974*m* (10% of the total area), are present in middle of the laminate. Space between two electrodes is 0.00025*m*. The plate is divided into 8×8 mesh element.

Figure 2 shows the AFC patches position at different location on both side of plate. Here, two different location of patches position is discussed for the dynamics analysis of delaminated composite plate. Dimension of AFC patches which is shown in Figure 2 (a), 2 (b), are discussed in Table 2. Figure 2(a) shows the AFC patch position 1 which is away from the delaminated region and Figure 2(b) shows the AFC patch position 2 which is likely same dimension as the size of delamination details are given in Table

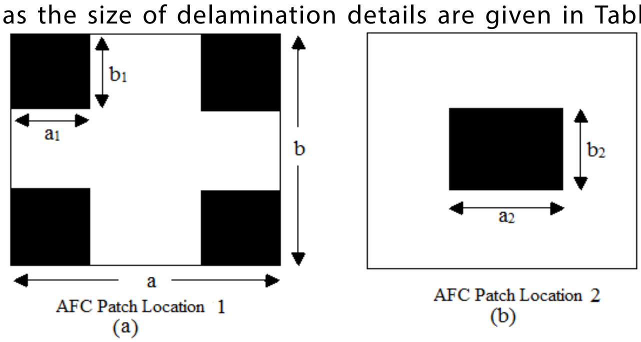

Fig. 2 AFC patches location at different places on plate

Active control mechanism of delaminated plate through AFC patches in various boundary conditions.

Active vibration control analysis of delaminated plate through AFC patches is studied in this section. A point load of 1000*N* is applied at middle of the plate in simply supported and clamped-clamped boundary, whereas it will be act at midpoint of free end in cantilever boundary condition. After giving an initial displacement, the load is suddenly removed and the plate is kept at vibration. A feedback system is activated at the mean time. The output voltage of the sensor layer is amplified by the amplifier and feedback to the actuator layer. Geometry and closed control loop system of the delaminated plate is shown in Figure 3.

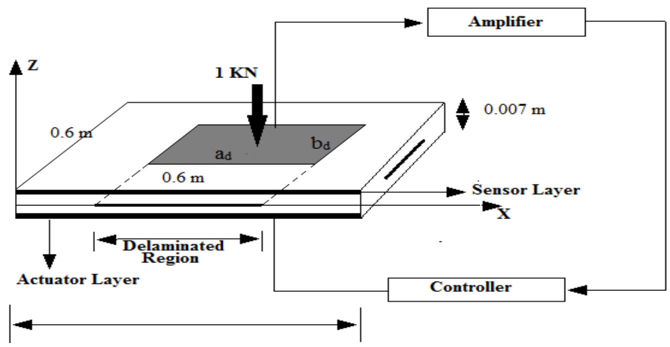

Fig. 3 Delaminated composite plate with feedback control loop system

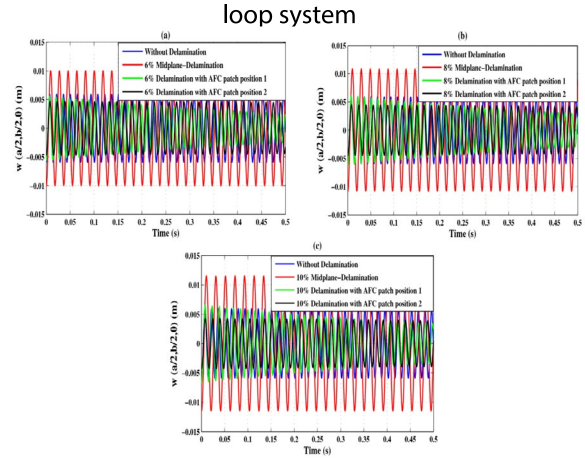

Fig. 4 Active control of 6%, 8%, and 10% delaminated plate under S-S-S-S

Figure 4 (a), 4(b) and 4(c) have showed the active control of dynamic displacement amplitude by using AFC patches in S-S-S-S boundary condition. The analysis is focused on 6 % 8% and 10% midplane delaminated plate. AFC patch position

Table 2: Dimensions of delaminated region and AFC patches.

Delamination along midplane (% of total area)	Size of delamination adx bd (m) Refer figure 2	AFC patch location 1 a1× b1 (m) Refer figure 3(a)	AFC patch location 2 a2× b2 (m) Refer figure 3(b)
6% delamination	0.14697×0.14697	0.2265× 0.2265	0.14697×0.14697
7% delamination	0.15874 ×0.15874	0.2206×0.2206	0.15874 ×0.15874
8% delamination	0.16970 ×0.16970	0.2151×0.2151	0.16970 ×0.16970
9% delamination	0.18×0.18	0.21 ×0.21	0.18×0.18
10% delamination	0.18974×0.18974	0.2051×0.2051	0.18974×0.18974

1 and AFC patch position 2 is used here which detailed shown in Figure 2.

Here AFC patches are used as actuator as well as sensor. A 0.001 volt voltage is given to the actuator layer and a PD controller and amplifier is used so that a damping is inducing in the structure which diminishes the amplitude. Figure 4(a) shows the active control of 6% delaminated plate, it is seen that displacement amplitude is increases 69.5% under delamination, which is decreases by 44% under AFC patch position 1 and 53 % under AFC patch position 2, similarly in 8% delaminated plate the displacement amplitude is decreased by 43.52% and 58.33% under AFC patch position 1 and AFC patch position 2 respectively which is shown in Figure 4(b). Finally when 10% delamination occurred in midplane of the plate the displacement amplitude diminished 42.61% and 63.48% under AFC patch position 1 and AFC patch position 2 respectively detailed shown in Figure 4(c)

.Figure 5 shows the active dynamic displacement control in time history of 6%, 8%, and 10% delaminated plate in C-C-C-C boundary condition. In this boundary condition the dynamic displacement is very less due to all edge are restrict to move. It is seen from Figure 5(a),(b) and (c) that when AFC patches are above and below the delaminated region i.e AFC patch position 2 the maximum amplitude of dynamic displacement response is reduced more than AFC patch position 1. The maximum amplitude of dynamic displacement reduces 58.88%, 62.61% and 66.44% under AFC patch position 2 in 6% 8% and 10% delamination respectively, whereas it is averagely 20% reduces under AFC patch position 1 in all three delamination condition.

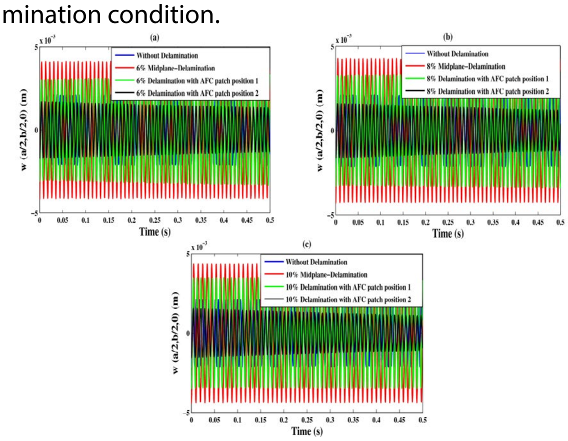

Fig. 5 Active control of 6%, 8%, and 10% delaminated plate under C-C-C-C

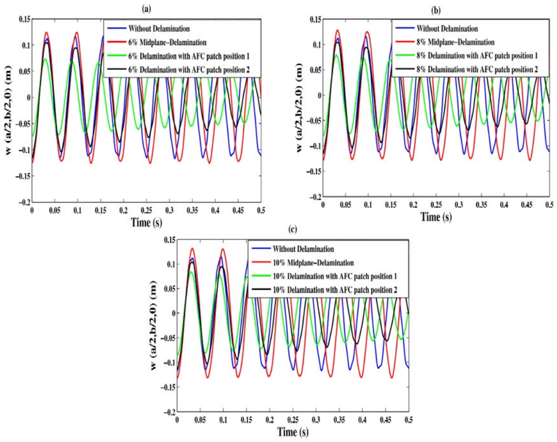

Figure 6 Active control of 6%, 8%, and 10% delaminated plate under C-F-F-F

Similarly the analysis is done for C-F-F-F boundary condition for 6%, 8% and 10% delaminated plate shown in Figure 6. It is observed that amplitude is slightly increases as the delamination introduces in the structure and it keeps increases as the size of delamination increases. The delaminated amplitude is reduced by AFC patches. In 6% delamination the amplitude reduces 41% and 16% under AFC patch position 1 and AFC patch position 2 respectively, while in 8% delamination it is reduced to 36% and 21% under AFC patch position 1 and AFC patch position 2 shown in Figure 6(a) and Figure 6(c). It has been noted that AFC patch position 1 is best suited for C-F-F-F boundary condition because displacement amplitude is reduces more.

Effect of velocity feedback gain on delaminated composite plate at various boundary conditions.

Velocity feedback gain (G_v) is used to control the transverse deflection response in the following section. The dynamic displacement response with varying velocity feedback gain (G_v) value in S-S-S-S boundary condition is shown in Figure 7. AFC patches position 2 and 10% delamination is taken for the analysis. The time and frequency response of the delaminated plate is carried out for $G_v = 0$, $G_v = 0.005$, $G_v = 0.015$, and $G_v = 0.050$ in Figure 7(a) and 7(b).

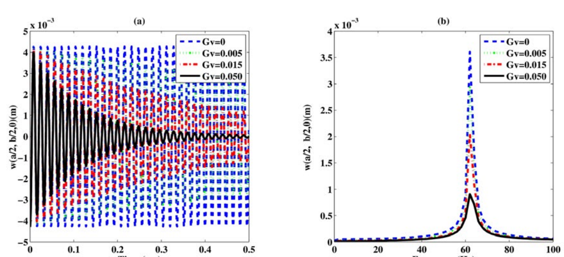

Fig. 7 Time and frequency response of 10% delaminated plate with varying Gv) in S-S-S-S boundary condition

The frequency response is illustrated here by the help of Fast Fourier Transformation (FFT). The higher peak has obtained for lower value of G_v shown in figure 7(b).When $G_v = 0$, AFC patches are not activated and so the response are remain unchanged throughout the time. The damping is induced by increasing the gain (G_v) from 0 to 0.05 so that the response comes to static state. In dynamic control system, the velocity feedback gain is always activating the damping of the system. The values of damping ratio are 0.00, 0.00098, 0.035 and 0.0112 for $G_v = 0$, $G_v = 0.005$, $G_v = 0.015$, and $G_v = 0.050$ respectively in S-S-S-S boundary condition of 10% delaminated plate. The damping ratios are calculated by Logarithmic decrement method. In similar fashion dynamics displacement time response and frequency response in C-C-C-C boundary condition is shown in Figure 8(a) and 8(b),

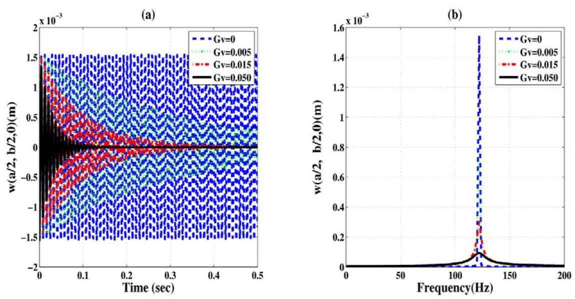

Fig. 8 Time and frequency response of delaminated composite plate with varying Gv in C-C-C-C boundary condition

In cantilever boundary condition the delaminated plate response with respect to time and frequency is shown in Figure 8(a) and 8(b), Here AFC patch position 2 is selected for the analysis. The analysis is carried out for Gv = 0, Gv = 0.001, Gv = 0.005 and Gv = 0.015. It is observed that in C-F-F-F boundary condition, G_v value is kept lower to carrying the response in static state as compare to S-S-S-S boundary condition. Damping induced due to velocity feedback gain is calculated by Logarithmic decrement method which are 0.00, 0.0075, 0.0353 and 0.1019 for Gv = 0, Gv = 0.001, Gv = 0.005 and Gv = 0.015 respectively.

Conclusion

The present study successfully demonstrates the effectiveness of Active Fiber Composite (AFC) patches c
the transient vibrations of delaminated composite plates. The finite element model developed in MATLAB provides an accurate framework for analyzing the interaction between delamination damage, boundary conditions, AFC patch placement, and feedback gain. Results reveal that the presence of mid-plane delamination significantly amplifies the dynamic displacement of the plate, while the integration of AFC patches as both actuators and sensors substantially suppresses these vibrations.

It is observed that the location of AFC patches plays a crucial role, with patches placed directly over the delaminated region offering the highest vibration suppression in simply supported and clamped boundary conditions, whereas patches positioned away from the delamination are more effective in cantilever configurations. Furthermore, increasing the velocity feedback gain (Gv) enhances damping in all boundary conditions, leading to rapid decay of vibration amplitudes.

Overall, the proposed active control strategy proves to be a reliable and efficient solution for restoring the vibration characteristics of damaged composite structures. The integration of AFC patches with velocity feedback control can be extended to the design of smart aerospace, automotive, and marine components where structural integrity and vibration suppression are critical.

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