RESEARCH ARTICLE

SAMRIDDHI Volume 15, Issue 1, 2023 Print ISSN: 2229-7111 Online ISSN: 2454-5767

Stress and Deflection of Laminated Composite Plates Based on Refined PlateTheory

Ashes Maji[1*], Ganesh Shankar[2], Prashanta Kr Mahato[3]

[1]Asansol Engineering College, Asansol-713305, India.

[2]Dumka Engineering College, Jharkhand-814101, India.

[3]Indian Institute of Technology (ISM) Dhanbad-826001, India.

Abstract

A higher-order refined mathematical model has been formulated to analyze the stress and deflection of laminated composite plates. The displacement functions of the laminated composite satisfy both the upper and lower surfaces of the plate and account for the parabolic variation of transverse shear stress across the plate thickness without applying shear correction factors. The displacement functions are used to derive the strain-displacement relations and incorporate with the equation of motion for simple supported laminated plates based on Hamilton's principle. The Finite element techniques are used through eigenvalue formulation by using Navier's solution technique The analytical solutions of the present refined plate are compared with exact theories. Comparison studies show that the developed refined model achieves accuracy and is computationally efficient.

Keywords: laminated composite plate, Mathematical model, higher-order shear deformation theory, von Karman hypothesis.

SAMRIDDHI : A Journal of Physical Sciences, Engineering and Technology (2023); DOI: 10.18090/samriddhi.v15i01.35

Introduction

The laminated composite materials are widely used in automotive, robotics, marine applications, aerospace, healthcare instruments, and many other industries because of their high strength and stiffness-to-weight ratio and material having excellent for the high structural performance of unidirectional fiber composites. To increase engineering applications, different laminated theories have been developed so far. As classical laminated plate theory (CLPT) proposed by Love [1] and Kirchhoff [2] has an extension of the Love–Kirchhoff classical plate theory (CPT). According to CPT, the midplane normally remains straight after and before the deformation of the plate. Reissner [3] developed an analytical solution of CLPT without using rotary inertia and transverse shear deformation effects and the results underestimate the thin laminate and overestimate the thick laminates. Timoshenko [4] has proposed the thin beam theory based on the shear deformation and rotatory inertia effects to overcome the limitations of CPT. Reissner [5] further extended the Timoshenko work based on the thick plate theory and the results are closely associated with the Timoshenko beam theory. The first-order shear deformation theory of laminated plates including transverse shear deformation and rotary inertia effect of the plates first proposed by Reissner [6]. A detailed overview of different shear deformation theories with their applicability is presented by Zhang and Yang [7]. Reddy [8] proposed higher-order shear deformation theory for symmetric cross-ply composite plates for the cubic variations of in-plane and out-off-plane stresses. Whitney [9] developed the higher-order shear deformation theory for the analysis of cylindrical bending of cross-ply and angle-ply laminated anti-symmetric plates with sinusoidal functions for both in-plane and out-of-plane displacement. Shimpi [10] first developed the two variable refined plate theories for isotropic plates and extended the work by Shimpi and Patil [11] and Kim and Thai [12] for laminated composite orthotropic plates. Guangyu [13] developed the simple third-order shear deformation theory based on three principles methods, first, the kinematics displacement reduced from the higher-order displacement, second, the order of differentiation in terms of three generalized displacements of bending plates, and third, each edge considered five DOFs of plate boundaries. Hirwani et al [14] developed two higher-order kinematic models to calculate the nonlinear bending and stress value of

Corresponding Author: Ashes Maji, Asansol Engineering College, Asansol-713305, India., e-mail: ashes.m12@gmail.com

How to cite this article: Maji, A,. Shankar, G,. Mahato, P.K. (2023). Stress, and Deflection of Laminated Composite Plates Based on Refined PlateTheory. *SAMRIDDHI : A Journal of Physical Sciences, Engineering and Technology,* 15(1), 200-205.

Source of support: Nil

Conflict of interest: None

internally damaged composite flat panel layered structures. Fallah and Karimi [15] study the nonlinear bending based on transverse mechanical loading of functionally graded circular sector plates with simply supported radial edges. Zine et al [16] discussed the bending and free vibration of multilayered plates and shells based on new higher order shear deformation theory. Hirwani et al [17] developed two higher-order kinematic models to calculate the nonlinear bending and stress value of internally damaged composite flat panel layered structures. Fallah and Karimi [18] study the nonlinear bending based on transverse mechanical loading of functionally graded circular sector plates with simply supported radial edges. Reddy [19] proposed the nonlinear theory by using finite element analysis of anti-symmetric angle ply laminated plates for out-off plate deformation.

The present research accounts for the deflection and stresses of in-plane and out-off-plane analysis of laminated plates based on simplified higher order shear deformation theory in. Hamilton's principle is used to derive the equation of motion of the plates. The transverse shear stresses across the plate thickness are parabolic and satisfy zero transverse shear effect at two free surfaces of the plates without considering shear correction factors. The closed-form analytical solutions are obtained through eigenvalue formulation by using Navier's solution technique. Lastly, discuss the accuracy of the present theory based on the 3D elasticity theory proposed by Reddy [19] and Noor [20].

Theoretical formulation.

The displacement field of the HSDT is given by:

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) + z\phi_x(x,y,t) + z^2\zeta_x(x,y,t) + z^3\psi_x(x,y,t) \\ v(x,y,z,t) &= v_0(x,y,t) + z\phi_y(x,y,t) + z^2\zeta_y(x,y,t) + z^3\psi_y(x,y,t) \\ w(x,y,z,t) &= w_0(x,y,t) \end{aligned} \quad (2.1)$$

Where, u_0, v_0 and w_0 are the displacement functions of the coordinate points (x, y, z) on the reference plane and ϕ_x and ϕ_y are the mid-plane rotation about horizontal axes. The functions ζ_x, ζ_y, ψ_x, and ψ_y are to be evaluated by using the condition that the transverse shear stress at the upper and bottom surfaces of the plate is zero.

$$\sigma_4\left(x, y, \pm\frac{h}{2}\right) = 0 \quad \sigma_5\left(x, y, \pm\frac{h}{2}\right) = 0, \quad (2.2)$$

For, laminated plates, these approaches are equivalent to the corresponding shear strain:

$$\begin{aligned} \varepsilon_5 &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \varepsilon_{xz} = \phi_x + 2z\zeta_x + 3z^2\psi_x + \frac{\partial w_0}{\partial x} \\ \varepsilon_4 &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \varepsilon_{yz} = \phi_y + 2z\zeta_y + 3z^2\psi_y + \frac{\partial w_0}{\partial y} \end{aligned} \quad (2.3)$$

Should be zero at z = ± h/2, or

$$\varepsilon_4\left(x, y, \pm\frac{h}{2}\right) = 0 \qquad \varepsilon_5\left(x, y, \pm\frac{h}{2}\right) = 0, \quad (2.4)$$

Then, we obtained,

$$\zeta_y = 0 \quad \zeta_x = 0, \quad (2.5)$$

$$\psi_x = \left[\frac{1}{4} - \frac{5}{3h^2}\left(\frac{\partial w_0}{\partial x} + \phi_x\right)\right], \qquad \psi_y = \left[\frac{1}{4} - \frac{5}{3h^2}\left(\frac{\partial w_0}{\partial y} + \phi_y\right)\right] \quad (2.6)$$

Substituting equation (2.6) in eqn. (2.1), the displacement field becomes:

$$\begin{aligned} u &= u_0 + z\left[\phi_x + \left\{\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right\}\left(\frac{\partial w_0}{\partial x} + \phi_x\right)\right] \\ v &= v_0 + z\left[\phi_y + \left\{\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right\}\left(\frac{\partial w_0}{\partial y} + \phi_y\right)\right] \\ w &= w_0(x,y,t) \end{aligned} \quad (2.7)$$

The mid-plane rotations are assumed to be similar in the x and y directions. Therefore, the expression can be written, $\phi_x = -\partial w_b / \partial x$ and $\phi_y = -\partial w_b / \partial y$,

The displacement field of Eqn. (2.7) based on the above assumptions can be written:

$$\begin{aligned} u &= u_0 - z\left[\frac{\partial w_b}{\partial x} - \left\{\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right\}\frac{\partial w_s}{\partial x}\right] \\ v &= v_0 - z\left[\frac{\partial w_b}{\partial y} - \left\{\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right\}\frac{\partial w_s}{\partial y}\right] \\ w &= w_b(x,y,t) + w_s(x,y,t) \end{aligned} \quad (2.8)$$

Constitutive Equations

The orthotropic plate consists of 'n' lamina layers of x, y, and z coordinate axes as shown in Fig.1. The plate laminate thickness *h* and horizontal sides *a* and *b* are considered. Each lamina in the laminate has a material symmetry with a lamina plane parallel to the coordinate plane.

The governing differential equations for a laminate can be written:

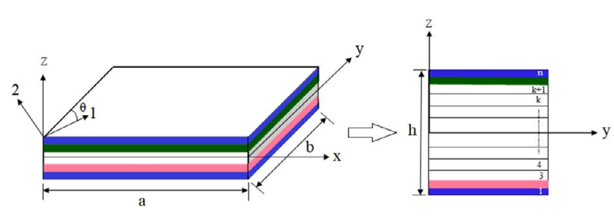

Fig 1: Lamina arrangement of composite plate

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \\ \sigma_4 \\ \sigma_5 \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \\ \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} \quad (3.1)$$

Where Q_{ij} is the material constants in the axes of the lamina. The constitutive relations of the k^{th} layer is given:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix} \quad (3.2)$$

Where`Qij defined laminate constants of materials transformation from by [21].

Equilibrium Equations

The equation of motion by Hamilton's principle is given:

$$0 = \int_0^T (\delta U + \delta W - \delta K) dt \quad (4.1)$$

Where, δU, δW, and δK are the virtual strain energy, virtual work done, and virtual kinetic energy, respectively.

When the transverse shear effect ($w_s = 0$) has been neglected the governing differential equations of composite laminated plates behave as a classical plate theory.

Numerical Results

The Navier solutions have been considered for rectangular simply supported composite plates. The different boundary conditions are assumed to be the closed-form solutions. A MATLAB-based FEM code is developed based on the above formulation. The accuracy of the present theory is verified with the help of various numerical observation.

In all the examples the shear correction factor is considered as 5/6 and the material constants are:

- Material 1. $E_1/E_2 = 40$, $G_{23}/E_2 = 0.5$, $G_{12}/E_2 = G_{13}/E_2 = 0.6$ and $\nu = 0.25$ [12]
- Material 2. $E_1/E_2 = 25$, $G_{23}/E_2 = 0.2$, $G_{12}/E_2 = G_{13}/E_2 = 0.5$ and $\nu = 0.25$ [22]

Where, subscripts used 1, 2, and 3 are the x, y, and z axes of the laminated plates.

The non-dimensional properties are used in this article:

$$w = \frac{100E_2h^3}{qa^4}\bar{w}\left(\frac{a}{2},\frac{b}{2}\right),\ \sigma_x = \frac{h^2}{qa^2}\bar{\sigma}_x\left(\frac{a}{2},\frac{b}{2}\right),\ \sigma_y = \frac{h^2}{qa^2}\bar{\sigma}_y\left(\frac{a}{2},\frac{b}{2}\right),$$
$$\sigma_{xy} = \frac{h^2}{qa^2}\bar{\sigma}_{xy}(0,0), \sigma_{xz} = \frac{h}{qa}\bar{\sigma}_{xz}\left(0,\frac{b}{2}\right),\ \sigma_{yz} = \frac{h}{qa}\bar{\sigma}_{yz}\left(\frac{a}{2},0\right) \quad (5.1)$$

Bending analysis

The deflection and stress bending of laminated composite

Table 1. Non-dimensionalized deflections and stresses in 2-ply (0/90)1 square laminate.

a/h	*Theory*	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\sigma}_{xy}$
2	Pagano,1970	4.9362	-0.9070	1.4480	-0.0964
	Reddy,1984	4.5619	-1.4277	1.4277	-0.0719
	Present	4.5618	-1.4276	1.4276	-0.0719
5	Pagano,1970	1.7287	-0.7723	0.8036	-0.0586
	Reddy,1984	1.6670	-0.8385	0.8385	-0.0558
	Present	1.6671	-0.8383	0.8382	-0.0557
10	Pagano,1970	1.2318	-0.7317	0.7353	-0.0540
	Reddy,1984	1.2161	-0.7468	0.7468	-0.0533
	Present	1.2161	-0.7466	0.7466	-0.0532
20	Pagano,1970	1.1060	-0.7200	0.7206	-0.0529
	Reddy,1984	1.1018	-0.7235	0.7235	-0.0527
	Present	1.1016	-0.7234	0.7234	-0.0526
100	Pagano,1970	1.0742	-0.7219	0.7219	-0.0529
	Reddy,1984	1.0651	-0.7161	0.7161	-0.0525
	Present	1.0652	-0.7158	0.7158	-0.0524

Table 2:Non-dimensionalized deflections and stresses in 4-ply (0/90)2 square laminate.

a/h	Theory	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\sigma}_{xy}$	$\bar{\sigma}_{xz}$	$\bar{\sigma}_{yz}$
2	Pagano,1970	11.767	1.388	0.835	-0.0863	0.153	0.295
	Reddy,1984	5.1286	1.3112	0.5876	-0.0889	-	-
	Present	5.1285	1.3121	0.5864	-0.0878	0.1434	0.2885
4	Pagano,1970	4.491	0.720	0.663	-0.0467	0.219	0.292
	Reddy,1984	1.9218	0.7344	0.5028	-0.0497	-	-
	Present	1.9203	0.7320	0.5017	-0.0487	0.2073	0.2892
10	Pagano,1970	-	0.559	0.401	-0.0275	0.301	0.196
	Reddy,1984	0.7125	0.5684	0.2690	-0.0277	-	-
	Present	0.7123	0.5678	0.2864	-0.0276	0.3032	0.1897
20	Pagano,1970	-	0.543	0.308	-0.023	0.328	0.156
	Reddy,1984	0.5041	0.5460	0.2043	-0.0230	-	-
	Present	0.5032	0.5452	0.2180	-0.0221	0.3264	0.1652
50	Pagano,1970	-	0.539	0.276	-0.0216	0.337	0.141
	Reddy,1984	0.4430	0.5399	0.1836	-0.0216	-	-
	Present	0.4418	0.5397	0.1886	-0.0214	0.3367	0.1412
100	Pagano,1970	-	0.539	0.271	-0.0214	0.339	0.139
	Reddy,1984	0.4342	0.5390	0.1806	-0.0214	-	-
	Present	0.4322	0.5388	0.1816	-0.0213	0.3387	0.1368

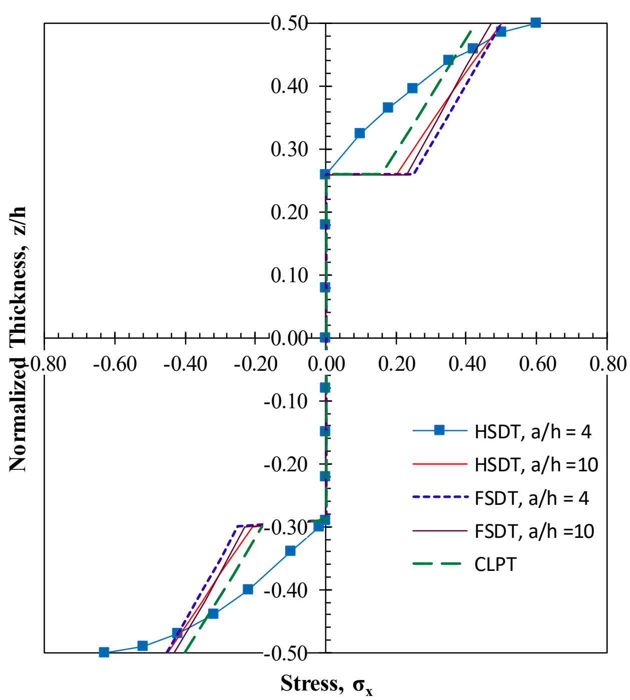

Fig.2: The in-plane normal stress σx vs. normalized thickness of simply supported square

plates rest on simply supported anti-symmetric cross-ply laminated composite plates are analyzed and compared with the higher order solution theories. The verification problems by considering different observations.

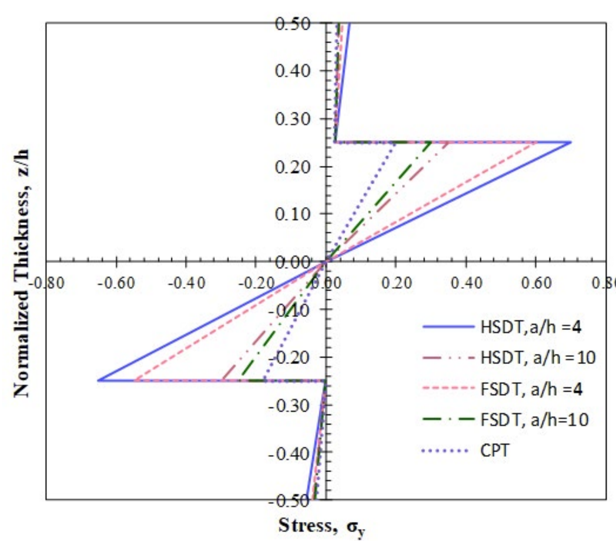

Fig.3: The in-plane normal stress σy vs. normalized thickness of simply supported square plate.

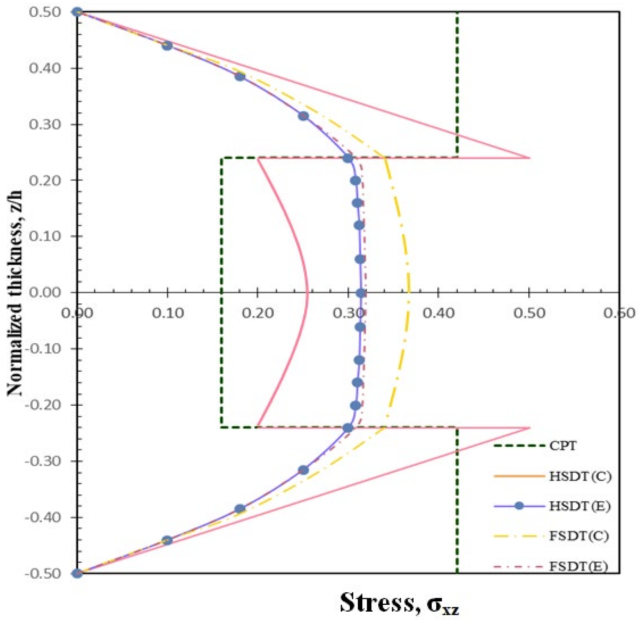

Fig.4: The transverse shear stress σxz vs. normalizedthickness for simply supported square plate for constitutive and equilibrium equations.

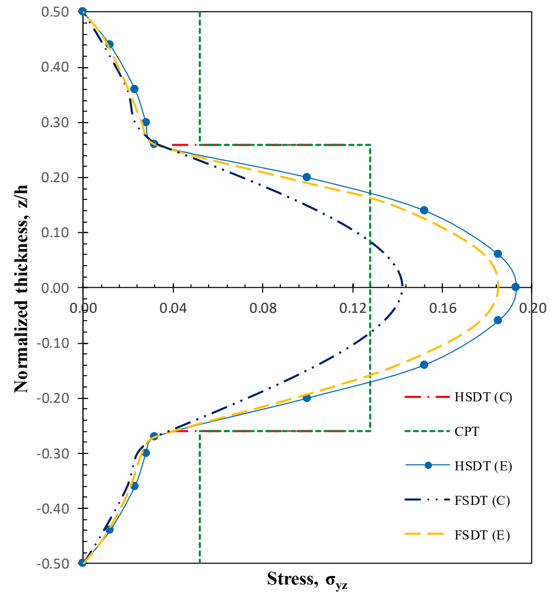

Fig.5: The transverse shear stress normalized thickness for simply sup square plate for constitutive and equations.

Obdervation

A two-ply $[0/90]_1$ simply supported anti-symmetric cross-ply laminated plate is considered for the non-dimensional deflection and stresses. The laminate has been subjected to a sinusoidal transverse load on the top of the plate. Material 1 is used for this analysis. The different numerical values based on various thickness ratio (a/h) of in-plane stresses, transverse shear stresses and transverse deflection are obtained from the present refined model as given in Table 1. The maximum in-plane stress and transverse stress are compared with the 3D elasticity solution proposed by Pagano [1970] and Reddy theory [1984]. The result obtained from the present theory is in close agreement with Reddy theory of all a/h ratio. For thick plates, the result from the present theory has a considerable difference from the 3D elasticity theory by Pagano [1970] therefore; the present results are closely associated with the result of Reddy theory [1984].

Obdervation

In this observation, discussed the non-dimensional deflection and stresses of anti-symmetric cross-ply four-layer $[0/90]_2$ plates based on the present refined model for considering Material 2. The laminate has been subjected to a sinusoidal transverse load on the top of the plate. The transverse deflection, plane stress and transverse shear stress are discussed based on various thickness ratio (a/h) given in Table 2. Also observed that the numerical values indicate the percentage errors of the present model for the calculations of normal stresses, transverse shear stresses and transverse deflections are very less concerning Reddy theory [1984] and minimal difference in the 3D elasticity theory by Pagano [1970]. The numerical values based on thickness ratio (a/h) are 4 and 10 of in-plane stresses are obtained from the present theory is given in Fig.2 & Fig.3, and in transverse shear stresses in Fig.4 & Fig.5, discussed the constitutive and equilibrium equation of the higher order theory compared with FSDT and CPT theory. The present results are compared with the results of Reddy theory [1984]. The thick and moderately thick composite laminated plates for the calculation of in-plane normal stresses and transverse shear stresses are in excellent agreement with the higher-order shear deformation theory by Reddy [1984].

Conclusions

A refined plate theory based on higher-order shear deformation was developed. The derived nonlinear formulation has been solved by using FEM and the deflection and bending behavior of composite plates have been performed. Furthermore, the theory developed based on some simplifying assumptions has reduced the unknown variables by one i.e. only four DOFs per node are considered.

The accuracy and effectiveness of the developed theory discussed in the various observation have been concluded as:

- The present theory used to predict the deflection and stresses in compression with 3D elasticity solution theory gives very minimal differences and more accurate results than Reddy's theory.
- The effect of the bending-extension coupling in cross-ply

square laminated plates has been more predominant in the case of nonlinear bending. The central deflection is found to be higher for the lower value of a number of layers due to the low coupling between bending and extension but the central deflection decreases as the number of layers increases due to the high coupling effect between bending and extension.

- The nonlinearity effect on the transverse direction response is to decrease the amplitude and increase the frequency.
- Due to the large geometrical nonlinearity response the nonlinear transient of HSDT and FSDT is apparent with increasing the load intensity.

In conclusion, the developed new refined nonlinear plate theory has been very effective and also computationally very efficient for the analysis of laminated composite anti-symmetric angle-ply plates.

References

[1] Love A E H. Small free vibrations and deformations of thin elastic shells. *Philos. Trans Roy Society*. A. 1888; 179: 491-549.
[2] Kirchhoff G R. Uber das Gleichgewicht und die Bewegung einer Elastischen Scheibe. *J Reine Angew Math*. 1850; 40: 51–88.
[3] Reissner E. On the theory of bending of elastic plates. *J. Math Phys* 1944; 23:184.
[4] Timoshenko S P. On Transverse Vibrations of Bars of Uniform Cross Section. *Phil. Mag*. 1922; 43(6): 125–131.
[5] Reissner E. The effect of transverse shear deformation on the bending of elastic plates. *ASME J Appl Mech*. 1945; 12: 69–77.
[6] Reissner E. On a variational theorem in elasticity. *J Math and phy*. 1950; 29: 90-95.
[7] Zhang Y X, Yang C H. Recent developments in finite element analysis for laminated composite plates. *Compos Struct*. 2009;88:147-157.
[8] Reddy J N. A simple higher-order theory for laminated composite plates. *ASME J Appl. Mech*. 1984; 51: 745–752.
[9] Whitney J M. The effect of transverse shear deformation in the bending of laminated plates. *J Compo Mat*. 1969; 3: 534–47.
[10] Shimpi R P. Refined plate theory and its variants. *AIAA J*. 2002; 40(1): 137–46.
[11] Shimpi R P, Patel H G. A two variable refined plate theory for orthotropic plate analysis. *Int J Solids Struct*. 2006; 43(22): 6783–6799.
[12] Kim S E, Thai H T. Free vibration of laminated composite plates using two variable refined plate theories. *Int J Mech Sci*, 2010; 52: 626-633.
[13] Guangyu S. A new simple third-order shear deformation theory of plates. *Int J solid and Struct,*2007;44(13);4399-4417.
[14] Hirwani C K, Panda S K, Patle B K. Theoretical and experimental validation of nonlinear deflection and stress responses of an internally debonded layer structure using different higher-order theories. *Acta Mechanica*. 2018; 229(8): 3453-3473.
[15] Fallah F, Karimi M H. Non-Linear Analysis of Functionally Graded Sector Plates with Simply Supported Radial Edges Under Transverse Loading. *Mech of Adv Compo Struct*. 2019; 6: 65-74.
[16] Zine A, Tounsi A, Draiche K, Sekkal M, Mahmoud S R. A novel higherorder shear deformation theory for bending and free vibration analysis of isotropic and multilayered plates and shells. *J Steel and Compo Struct.* 2018; 26(2): 125–137.
[17] Hirwani C K, Panda S K, Patle B K. Theoretical and experimental validation of nonlinear deflection and stress responses of an internally debonded layer structure using different higher-order theories. *Acta Mechanica*. 2018; 229(8): 3453-3473.
[18] Fallah F, Karimi M H. Non-Linear Analysis of Functionally Graded Sector Plates with Simply Supported Radial Edges Under Transverse Loading. *Mech of Adv Compo Struct*. 2019; 6: 65-74.
[19] Reddy J N. A refined nonlinear theory of plates with transverse shear deformation, *Int J Solid Struct*. 1984; 20: 881-890.
[20] Noor A K. Free vibrations of multilayered composite plates. *AIAA J* 1973;11:1038–1039.
[21] Mukhopadhyay M. Mechanics of composite materials and structures. India: *Universities Press*, 2004.
[22] Pagano N J. Exact Solutions for Rectangular Bidirectional Composite and Sandwich Plates, *J Comp Mat*, 1970; 4: 20-34.