RESEARCH ARTICLE

SAMRIDDHI Volume 12, Issue 2, 2020 Print ISSN: 2229-7111 Online ISSN: 2454-5767

Nonlinear Dynamic Stability of Laminated Composite Plates Based on Refined Theory

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Abstract

Refined theory is developed for nonlinear dynamic stability of laminated composite plates based on higher order shear deformation theory. The developed refined plate theory is used to analyze the geometric nonlinearity or large amplitude effects on the dynamic stability of the composite laminated plates. Based on the shear deformation theory involving four dependent unknowns and satisfying the vanishing of transverse shear stresses at the top and bottom surfaces of the plate without using shear correction factors. The displacement functions are used to derive the non-linear strain-displacement relations based on the Von-Karman hypothesis. The Finite element analysis are obtained through eigenvalue formulation by using Navier's solution technique. The analytical solutions developed from the present refined model are compared with 3D elasticity theories. The comparison shown that refined model achieves accuracy and efficient.

Keywords: laminated composite plate, Refined Model, higher-order shear deformation theory, Navier's solution technique.

SAMRIDDHI : A Journal of Physical Sciences, Engineering and Technology (2020); DOI: 10.18090/samriddhi.v12i02.13

Introduction

The laminated composite materials are used in robotics, automotive, marine applications, aerospace, healthcare instruments, and many other industries because of their high strength, stiffness, refractive index and thermal conductivity. These types of materials having excellent for the high structural performance of unidirectional fiber composites. To increase engineering applications, different laminated theories have been developed. As classical laminated plate theory (CLPT) proposed by Love [1] and Kirchhoff [2] has an extension of the Love–Kirchhoff classical plate theory (CPT). According to CPT, the midplane normally remains straight after and before the deformation of the plate. Reissner [3] developed an analytical solution of CLPT without using rotary inertia and transverse shear deformation effects and the results underestimate the thin laminate and overestimate the thick laminates. Timoshenko [4] has proposed the thin beam theory based on the shear deformation and rotatory inertia effects to overcome the limitations of CPT. Reissner [5] further extended the Timoshenko work based on the thick plate theory and the results are closely associated with the Timoshenko beam theory. The first-order shear deformation theory of laminated plates including transverse shear deformation and rotary inertia effect of the plates first proposed by Reissner [6]. A detailed overview of different shear deformation theories with their applicability is presented by Zhang and Yang [7]. Reddy [8] proposed higher-order shear deformation theory for symmetric cross-ply composite plates for the cubic variations of in-plane and out-off-plane stresses. Whitney [9] developed the higher-order shear deformation theory for the analysis of cylindrical bending of cross-ply and angle-ply laminated anti-symmetric plates with sinusoidal functions for both in-plane and out-of-plane displacement. Shimpi [10] first developed the two variable refined plate theories for isotropic plates and extended the work by Shimpi and Patil [11] and Kim and Thai [12] for laminated composite orthotropic plates. Guangyu [13] developed the simple third-order shear deformation theory based on three principles methods, first, the kinematics displacement reduced from the higher-order displacement, second, the order of differentiation in terms of three generalized displacements of bending plates, and third, each edge considered five DOFs of plate boundaries.

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How to cite this article: Maji, A,. Shankar, G,. Mahato, P. K. (2020). Nonlinear Dynamic Stability of Laminated Composite Plates Based on Refined Theory. *SAMRIDDHI : A Journal of Physical Sciences, Engineering and Technology,* 12(2), 137-144.

Source of support: Nil

Conflict of interest: None

A new refined plate theory for the analysis of static, bending and free vibration of orthotropic laminated composite plates was developed by Adim et al [14]. Ghadiri and Safarpour [15] investigated the free vibration behavior of magneto-electro-elastic composite materials subjected to thermo-electro-magnetic loading based on first-order shear deformation theory. Hirwani et al [16] discussed the implicit transient response of the shear deformable layered composite plate under mechanical transverse loading. Ghadiri et al [17] discussed the free vibration behavior of functionally graded size-dependent rotating nanobeams based on a nonlocal continuum model. Ghadiri and Safarpour [18] developed the theory on the free vibration behavior of a functionally graded porous cylindrical microshell subjected to the thermal environment based on first order shear deformation shells and modified couple stress theory. Barooti et al [19] have suggested a spinning 3D single-walled carbon nanotubes for the influence of critical speed on free vibration behavior by using modified couple stress theory. Safarpour and Ghadiri [20] proposed a model based on the influence of rotating speed and velocity of viscous fluid flow of spinning single walled carbon nanotubes for the investigation of the free vibration behavior of the materials. Zine et al [21] discussed the bending and free vibration of multilayered plates and shells based on new higher order shear deformation theory. Hirwani et al [22] developed two higher-order kinematic models to calculate the nonlinear bending and stress value of internally damaged composite flat panel layered structures. Fallah and Karimi [23] study the nonlinear bending based on transverse mechanical loading of functionally graded circular sector plates with simply supported radial edges. Reddy [24] proposed the nonlinear theory by using finite element analysis of anti-symmetric angle ply laminated plates for out-off plate deformation.Von Karman [25] proposed the theory based on geometrically nonlinear normal and shear strain displacement analysis. This Von Karman non-linear theory was extended to a geometrically nonlinear analysis by Schmidt [26]. A nonlinear finite element model has been developed by Park et al [27] for the study of natural frequency and critical buckling of composite plates by using the von Karman nonlinearity function based on FSDT. Xia and Shen [28] developed a model based on HSDT by using von Karman nonlinear functions.

The present research accounts for the Dynamic Stability of laminated composite plates based on simplified higher order shear deformation theory in conjunction with Von Karman nonlinear strain equations. This paper simplified the transverse displacement field into bending and shears components respectively with reduced to four unknowns. Hamilton's principle is used to derive the equation of motion of the plates. The closed-form analytical solutions are obtained through eigenvalue formulation by using Navier's solution technique. Lastly, discuss the accuracy of the present theory based on the 3D elasticity theory proposed by Noor [29] and Reddy [24].

Theoretical formulation

The displacement field of the HSDT by:

$$u = u_0 + z\phi_x + z^2\zeta_x + z^3\psi_x$$
$$v = v_0 + z\phi_y + z^2\zeta_y + z^3\psi_y \qquad (2.1)$$
$$w = w_0$$

Where, u_0, v_0, and w_0 are the displacement functions of the coordinate points (x, y, z) on the reference plane and ϕ_x and ϕ_y are the mid-plane rotation about horizontal axes. The functions ζ_x, ζ_y, ψ_x, and ψ_y are to be evaluated by using the condition that the transverse shear stress at the upper and bottom surfaces of the plate is zero.

$$\sigma_4\left(x, y, \pm\frac{h}{2}\right) = 0 \qquad \sigma_5\left(x, y, \pm\frac{h}{2}\right) = 0, \qquad (2.2)$$

For, laminated plates, these approaches are equivalent to the corresponding shear strain:

$$\varepsilon_5 = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \varepsilon_{xz} = \phi_x + 2z\zeta_x + 3z^2\psi_x + \frac{\partial w_0}{\partial x}$$
$$\varepsilon_4 = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \varepsilon_{yz} = \phi_y + 2z\zeta_y + 3z^2\psi_y + \frac{\partial w_0}{\partial y} \qquad (2.3)$$

Should be zero at z = ± h/2, or

$$\varepsilon_4\left(x, y, \pm\frac{h}{2}\right) = 0 \qquad \varepsilon_5\left(x, y, \pm\frac{h}{2}\right) = 0, \qquad (2.4)$$

Then, we obtained,

$$\zeta_y = 0 \qquad \zeta_x = 0, \qquad (2.5)$$

$$\psi_x = \left[\frac{1}{4} - \frac{5}{3h^2}\left(\frac{\partial w_0}{\partial x} + \phi_x\right)\right], \qquad \psi_y = \left[\frac{1}{4} - \frac{5}{3h^2}\left(\frac{\partial w_0}{\partial y} + \phi_y\right)\right] \qquad (2.6)$$

Substituting equation (2.6) in eqn. (2.1), the displacement field becomes:

$$u = u_0 + z\left[\phi_x + \left\{\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right\}\left(\frac{\partial w_0}{\partial x} + \phi_x\right)\right]$$
$$v = v_0 + z\left[\phi_y + \left\{\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right\}\left(\frac{\partial w_0}{\partial y} + \phi_y\right)\right] \qquad (2.7)$$
$$w = w_0$$

The mid-plane rotations are assumed to be similar in the x and y directions. Therefore, the expression can be written, $\phi x = -\partial wb/\partial x$ and $\phi y = -\partial wb/\partial y$, The displacement field of eqn. (2.7) based on the above assumptions can be written:

$$u = u_0 - z\left[\frac{\partial w_b}{\partial x} - \left\{\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right\}\frac{\partial w_s}{\partial x}\right]$$
$$v = v_0 - z\left[\frac{\partial w_b}{\partial y} - \left\{\frac{1}{4} - \frac{5}{3}\left(\frac{z}{h}\right)^2\right\}\frac{\partial w_s}{\partial y}\right] \qquad (2.8)$$
$$w = w_b + w_s$$

The nonlinear Von Karman strain based on displacement functions in Eq. (2.8) is:

$$\varepsilon_1 = \frac{\partial u}{\partial x} = \frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right)^2 - z\frac{\partial^2 w_b}{\partial x^2} - f(z)\frac{\partial^2 w_s}{\partial x^2}$$
$$\varepsilon_2 = \frac{\partial v}{\partial y} = \frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y}\right)^2 - z\frac{\partial^2 w_b}{\partial y^2} - f(z)\frac{\partial^2 w_s}{\partial y^2}$$
$$\varepsilon_3 = \frac{\partial w}{\partial z} = 0$$
$$\varepsilon_4 = \frac{5}{4}\frac{\partial w_s}{\partial y} - z^2\frac{5}{h^2}\frac{\partial w_s}{\partial y} \qquad (2.9)$$
$$\varepsilon_5 = \frac{5}{4}\frac{\partial w_s}{\partial x} - z^2\frac{5}{h^2}\frac{\partial w_s}{\partial x}$$
$$\varepsilon_6 = \left(\frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}\right) + \left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right)\left(\frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y}\right) - 2z\frac{\partial^2 w_b}{\partial x \partial y} - 2f(z)\frac{\partial^2 w_s}{\partial x \partial y}$$

Where

$$f(z) = \left[\frac{1}{4}z - z^3\frac{5}{3h^2}\right]$$

Constitutive Equations

The orthotropic plate consists of 'n' lamina layers of x, y, and z coordinate axes as shown in Fig.1. The plate laminate thickness *h* and horizontal sides *a* and *b* are considered. Each lamina in the laminate has a material symmetry with a lamina plane parallel to the coordinate plane.

The governing differential equations for a laminate can be written:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\\ \sigma_4\\ \sigma_5\end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{12} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{66} & 0 & 0\\ 0 & 0 & 0 & Q_{44} & 0\\ 0 & 0 & 0 & 0 & Q_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_6\\ \varepsilon_4\\ \varepsilon_5\end{Bmatrix} \qquad (3.1)$$

Where Q_{ij} is the material constants in the axes of the lamina.

The constitutive relations of the k^{th} layer is given:

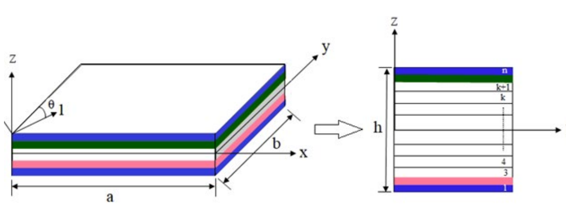

Fig 1: Lamina arrangement of composite plate

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\end{Bmatrix}^k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0\\ \bar{Q}_{21} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0\\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45}\\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55}\end{bmatrix}^k \begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\end{Bmatrix} \qquad (3.2)$$

Where $\bar{Q}_{ij}$ defined laminate constants of materials transformation from by [30].

Equilibrium Equations

The equation of motion by Hamilton's principle is given:

$$0 = \int_0^T (\delta U + \delta W - \delta K)dt \qquad (4.1)$$

Where, δU, δW, and δK are the virtual strain energy, virtual work done, and virtual kinetic energy, respectively.

The strain energy variation can be defined:

$$\delta U = \int_0^a \int_0^b \int_{-h/2}^{+h/2} (\sigma_1 \delta\varepsilon_1 + \sigma_2 \delta\varepsilon_2 + \sigma_6 \delta\varepsilon_6 + \sigma_4 \delta\varepsilon_4 + \sigma_5 \delta\varepsilon_5)\, dzdxdy$$
$$= \int_0^a \int_0^b \left\{\left[N_1 \frac{\partial \delta u_0}{\partial x} + \frac{\partial}{\partial x}\left[N_1\left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right)\right]\right] + M_1\left(-\frac{\partial^2 \delta w_b}{\partial x^2}\right) + P_1\left(-\frac{\partial^2 \delta w_s}{\partial x^2}\right)\right.$$
$$+ N_2 \frac{\partial \delta v_0}{\partial y} + \frac{\partial}{\partial y}\left[N_2\left(\frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y}\right)\right] + M_2\left(-\frac{\partial^2 \delta w_b}{\partial y^2}\right) + P_2\left(-\frac{\partial^2 \delta w_s}{\partial y^2}\right)$$
$$+ N_6\left(\frac{\partial \delta u_0}{\partial y} + \frac{\partial \delta v_0}{\partial x}\right) + \frac{\partial}{\partial y}\left[N_6\left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x}\right)\right] + \frac{\partial}{\partial x}\left[N_6\left(\frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y}\right)\right] \qquad (4.2)$$
$$+ M_6\left(-2\frac{\partial \delta w_b}{\partial x \partial y}\right) + P_6\left(-2\frac{\partial^2 \delta w_s}{\partial x \partial y}\right) + Q_2 \frac{\partial \delta w_s}{\partial y}$$
$$\left. + R_2\left(-\frac{5}{h^2}\frac{\partial \delta w_s}{\partial y}\right) + Q_1 \frac{\partial \delta w_s}{\partial x} + R_1\left(-\frac{5}{h^2}\frac{\partial \delta w_s}{\partial x}\right)\right\} dxdy$$

The following strains N, M, P, Q and R are defined

$$(Q_1, R_1) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_5\left(\frac{5}{4}, z^2\right)dz, \quad (Q_2, R_2) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_4\left(\frac{5}{4}, z^2\right)dz \qquad (4.3)$$

(4.4)

The variation of work done has been defined:

$$\delta W = -\int_0^a \int_0^b q\delta(w_b + w_s)dxdy \qquad (4.5)$$

Where q is the external load applied transversely.

The Kinetic energy of the laminate can be defined:

$$\delta K = \int_0^a \int_0^b \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(\dot{u}\delta\dot{u} + \dot{v}\delta\dot{v} + \dot{w}\delta\dot{w})\, dzdx$$

$$= \int_0^a \int_0^b \left\{ I_0\left[\dot{u}_0 \delta\dot{u}_0 + \dot{v}_0 \delta\dot{v}_0 + (\dot{w}_b + \dot{w}_s)\delta(\dot{w}_b + \dot{w}_s)\right]\right.$$
$$- I_1\left[\frac{\partial \delta\dot{w}_b}{\partial x}\dot{u}_0 + \frac{\partial \delta\dot{w}_b}{\partial x}\delta\dot{u}_0 + \frac{\partial \delta\dot{w}_b}{\partial y}\dot{v}_0 + \frac{\partial \delta\dot{w}_b}{\partial y}\delta\dot{v}_0\right]$$
$$+ I_2\left[\frac{\partial \dot{w}_b}{\partial x}\frac{\partial \delta\dot{w}_b}{\partial x} + \frac{\partial \dot{w}_b}{\partial y}\frac{\partial \delta\dot{w}_b}{\partial y}\right] - I_3\left[f(z)\left\{\left(\frac{\partial \delta\dot{w}_s}{\partial x}\dot{u}_0 + \frac{\partial \dot{w}_s}{\partial x}\delta\dot{u}_0\right) + \left(\frac{\partial \delta\dot{w}_s}{\partial y}\dot{v}_0 + \frac{\partial \dot{w}_s}{\partial y}\delta\dot{v}_0\right)\right\}\right]$$
$$+ I_4\left[f(z)\left\{\left(\frac{\partial \dot{w}_b}{\partial x}\frac{\partial \delta\dot{w}_s}{\partial x} + \frac{\partial \dot{w}_s}{\partial x}\frac{\partial \delta\dot{w}_b}{\partial x}\right) + \left(\frac{\partial \dot{w}_b}{\partial y}\frac{\partial \delta\dot{w}_s}{\partial y} + \frac{\partial \dot{w}_s}{\partial y}\frac{\partial \delta\dot{w}_b}{\partial y}\right)\right\}\right] \qquad (4.6)$$
$$\left. + I_5\left[2f(z)\left\{\frac{\partial \dot{w}_s}{\partial x}\frac{\partial \delta\dot{w}_s}{\partial x} + \frac{\partial \dot{w}_s}{\partial y}\frac{\partial \delta\dot{w}_s}{\partial y}\right\}\right]\right\} dxdy$$

Where dot- superscript defined the derivatives concering time variable *t*, the mass

$$I_i = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^i \rho dz \quad \text{where, } (i = 0,1,2,3,4,6)$$

$$\tilde{I}_i = -\frac{1}{4} I_i + \frac{5}{3h^2} I_{i+2}$$

density in each layer is ρ and (I0, I1, I2, I3, I4, I5) are the mass moment of inertias: (4.7)

The energy functions δU, δW and δK are substituted in Eq. (4.1) and assuming the variables of δu_0, δv_0, δw_b and δw_s to be zero. The following motion equations:

$$\delta u_0 : \frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} = I_0 \ddot{u}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial x} - \tilde{I}_1 \frac{\partial \ddot{w}_s}{\partial x}$$

$$\delta v_0 : \frac{\partial N_6}{\partial x} + \frac{\partial N_2}{\partial y} = I_0 \ddot{v}_0 - I_1 \frac{\partial \ddot{w}_b}{\partial y} - \tilde{I}_1 \frac{\partial \ddot{w}_s}{\partial y}$$

$$\delta w_b : \left(\frac{\partial^2 M_1}{\partial x^2} + 2\frac{\partial^2 M_6}{\partial x \partial y} + \frac{\partial^2 M_2}{\partial y^2} \right) + q - N(w) = I_0 (\ddot{w}_b + \ddot{w}_s)$$

$$+ I_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) - I_2 \nabla^2 \ddot{w}_b - \tilde{I}_2 \nabla^2 \ddot{w}_s \qquad (4.8)$$

$$\delta w_s : \left(\frac{\partial^2 P_1}{\partial x^2} + 2\frac{\partial^2 P_6}{\partial x \partial y} + \frac{\partial^2 P_2}{\partial y^2} \right) + \left(\frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} \right) - \left(\frac{\partial R_1}{\partial x} + \frac{\partial R_2}{\partial y} \right) + q - N(w) = I_0 (\ddot{w}_b + \ddot{w}_s)$$

$$+ \tilde{I}_1 \left(\frac{\partial \ddot{u}_0}{\partial x} + \frac{\partial \ddot{v}_0}{\partial y} \right) - \tilde{I}_2 \nabla^2 \ddot{w}_b - k_3 \nabla^2 \ddot{w}_s$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

The two-dimensional Laplacian operators in the Cartesian coordinate system: (4.9)

$$N(\omega) = \frac{\partial}{\partial x} \left[N_1 \left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x} \right) + N_6 \left(\frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y} \right) \right]$$

$$+ \frac{\partial}{\partial y} \left[N_6 \left(\frac{\partial w_b}{\partial x} + \frac{\partial w_s}{\partial x} \right) + N_2 \left(\frac{\partial w_b}{\partial y} + \frac{\partial w_s}{\partial y} \right) \right] \qquad (4.10)$$

$$I_i = \int_{-\frac{h}{2}}^{\frac{h}{2}} z^i \rho dz \quad \text{where, } (i = 0,1,2,3,4,5)$$

$$\tilde{I}_i = -\frac{1}{4} I_i + \frac{5}{3h^2} I_{i+2} \qquad (4.11)$$

$$k_3 = \frac{1}{16} I_2 - \frac{5}{6h^2} I_4 + \frac{25}{9h^2} I_5$$

When the transverse shear effect ($w_s = 0$) has been neglected the governing differential equations of composite laminated plates behave as a classical plate theory.

The finite element formulation

The total solutions described by the finite element techniques of the plate domain have been discretizing in the number of elements:

$$\Omega(u) = \sum_{e=1}^{NE} \Omega^e(u) \qquad (5.1)$$

Where Ω and Ω^e are the system and element total potential energy. The total potential energy has been defined in terms of internal strain energy and work potential for each element "e" and unknown displacement vector u,

$$\Omega^e(u) = U^e - W^e \qquad (5.2)$$

The generalized displacement of the plate is defined:

$$u(\xi, \eta) = \sum_{I=1}^{8} N_I(\xi, \eta) q_I \qquad (5.3)$$

Where $q_I = [u_{0I}, v_{0I}, w_{bI}, w_{sI}]^T$ is the displacement vector nodal degree of freedom (DOFs.) associated with node I and $N_I(\xi, \eta)$ is the shape functions of Lagrange eight-node iso-parametric quadrilateral elements are used,

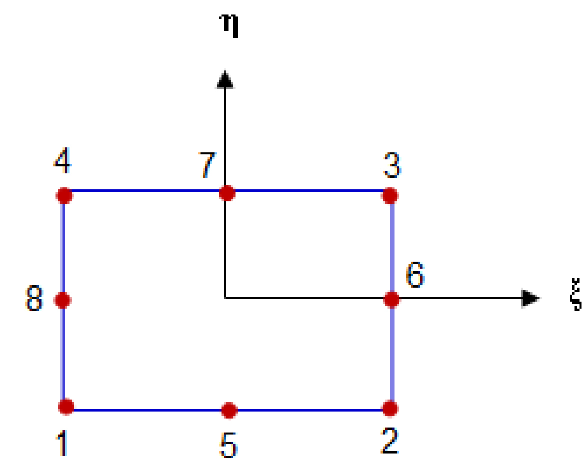

Fig.2: An eight-node quadrilateral iso-parametric element

To describe the weak form solutions of refined nonlinear higher-order theory, the strain and displacement relation can further be simplified,

$$\varepsilon = \varepsilon_0 + zk_1 + f(z)k_2 = \varepsilon_{0,L} + \varepsilon_{0,NL} + zk_1 + f(z)k_2 \qquad (5.4)$$

$$\gamma = \varepsilon_s + z^2 k_s$$

$$\varepsilon_{0,L} = \left[u_{0,x}, v_{0.y}, (u_{0,x} + v_{0,y}) \right]^T, k_1 = -\left[w^b_{xx}, w^b_{yy}, 2w^b_{xy} \right]^T$$

$$, k_2 = -\frac{5}{3h^2} \left[w^s_{xx}, w^s_{yy}, 2w^s_{xy} \right]^T, \varepsilon_s = \left[w^s_x, w^s_y \right] \& k_s = -\frac{5}{h^2} \left[w^s_x, w^s_y \right] \qquad (5.5)$$

$$\varepsilon_{0,NL} = \frac{1}{2} \left[(w_{b,x} + w_{s,x})^2, (w_{b,y} + w_{s,y})^2, 2(w_{b,x} + w_{s,x})(w_{b,y} + w_{s,y}) \right]^T$$

The nonlinear components can be defined:

$$\varepsilon_{0,NL} = \frac{1}{2} \begin{bmatrix} w_{bs,x} & 0 \\ 0 & w_{bs,y} \\ w_{bs,x} & w_{bs,y} \end{bmatrix} \begin{Bmatrix} w_{bs,x} \\ w_{bs,y} \end{Bmatrix} \qquad (5.6)$$

The in-plane and shear strain can be rewritten in the following form:

Table: 1. The central deflection (w/h) under various load and boundary conditions for anti-symmetric cross-ply and anti-symmetric angle ply.

Plate thickness	Loading condition	Load parameter	Anti-Symmetric cross-ply			Anti-symmetric angle-ply		
			SSSS	CCCC	CCSS	SSSS	CCCC	CCSS
a/h=10	Uniformly distributed load	3	0.04075	0.01468	0.01788	0.03275	0.02653	0.02154
		5	0.06800	0.02410	0.03075	0.05475	0.02846	0.03429
		15	0.20125	0.07208	0.09225	0.18667	0.08795	0.11235
		30	0.37566	0.14354	0.18285	0.32547	0.16875	0.21365
		45	0.55660	0.21275	0.26855	0.45425	0.23451	0.30125
		100	1.03550	0.43950	0.55895	0.92975	0.48974	0.61325
	Sinusoidal load	3	0.02706	0.01145	0.01356	0.02215	0.01214	0.01524
		5	0.04542	0.01875	0.02216	0.03625	0.01957	0.02475
		15	0.13986	0.05314	0.06523	0.11542	0.05615	0.07198
		30	0.26628	0.11423	0.12657	0.21586	0.11996	0.14225
		45	0.38940	0.16475	0.19878	0.31285	0.17496	0.21988
		100	0.75546	0.34056	0.38457	0.61835	0.36174	0.45567
a/ h=100W	Uniformly distributed load	10	0.11987	0.03214	0.03889	0.09847	0.04411	0.05125
		30	0.34688	0.09486	0.12871	0.27856	0.11242	0.15942
		50	0.55320	0.15694	0.19724	0.44100	0.17825	0.25687
		70	0.74465	0.22750	0.27423	0.57662	0.24512	0.35112
		90	0.88654	0.28744	0.35687	0.72115	0.31132	0.44216
		100	0.98025	0.31658	0.38564	0.78723	0.33452	0.48635
	Sinusoidal load	10	0.07625	0.03758	0.03745	0.05889	0.03485	0.04412
		30	0.23285	0.07964	0.09328	0.18356	0.08456	0.11228
		50	0.37386	0.12415	0.23687	0.29452	0.13425	0.17987
		70	0.48785	0.16894	0.27108	0.39965	0.18235	0.24618
		90	0.62822	0.21457	0.25547	0.49696	0.32214	0.40123
		100	0.68596	0.23624	0.28131	0.64395	0.34612	0.43227

$$\tilde{\varepsilon} = [\varepsilon \ \gamma]^T = \sum_{I}^{8}\left(B_I^L + \frac{1}{2}B_I^{NL}\right)q_I \quad (5.7)$$

$$B_I^L = \left[\left(B_I^e\right)^T \left(B_I^{b_1}\right)^T \left(B_I^{b_2}\right)^T \left(B_I^{s_1}\right)^T \left(B_I^{s_2}\right)^T\right] \quad (5.8)$$

$$\left[\varepsilon_0^T \ k_1^T \ k_2^T \ \varepsilon_s^T \ k_s^T\right]^T = \sum_{I=1}^{8}\left[\left(B_I^e\right)^T \left(B_I^{b_1}\right)^T \left(B_I^{b_2}\right)^T \left(B_I^{s_1}\right)^T \left(B_I^{s_2}\right)^T\right]q_I \quad (5.9)$$

$$B_I^{b_1} = \begin{bmatrix} 0 & 0 & N_{I,xx} & 0 & 0 \\ 0 & 0 & N_{I,yy} & 0 & 0 \\ 0 & 0 & 2N_{I,xy} & 0 & 0 \end{bmatrix}$$

$$B_I^e = \begin{bmatrix} N_{I,x} & 0 & 0 & 0 & 0 \\ 0 & N_{I,y} & 0 & 0 & 0 \\ N_{I,y} & N_{I,x} & 0 & 0 & 0 \end{bmatrix},$$

$$B_I^{b_2} = -\frac{5}{3h^2}\begin{bmatrix} 0 & 0 & N_{I,xx} & 0 & 0 \\ 0 & 0 & N_{I,yy} & 0 & 0 \\ 0 & 0 & 2N_{I,xy} & 0 & 0 \end{bmatrix} \quad (5.10)$$

$$B_I^{s_1} = \begin{bmatrix} 0 & 0 & 0 & N_I & 0 \\ 0 & 0 & 0 & 0 & N_I \end{bmatrix}$$

Table: 2: Central deflection of laminated composite plate versus time based on Linear and Nonlinear analysis

Time t(x10-3 s)	Linear Analysis			Nonlinear Analysis		
	FSDT (Lee&Reddy)	HSDT (Lee&Reddy)	Present	FSDT (Lee&Reddy)	HSDT (Lee&Reddy)	Present
0.5	0.3179	0.2842	0.2845	0.3179	0.2841	0.2955
1.0	1.3249	1.3480	1.3495	1.2848	1.3210	1.3322
1.5	2.1768	2.1916	2.2845	1.8638	1.9535	1.8784
2.0	2.2983	2.5440	2.5644	1.4674	1.8584	1.9564
4.0	0.5573	0.4342	0.5217	1.2969	0.9988	0.8879
6.0	1.6739	2.0589	2.0688	0.5528	0.7597	0.8779
8.0	1.1432	0.8399	0.8269	1.2449	1.6265	1.6457
10.0	1.2147	1.7084	1.7546	1.2592	0.8573	0.8897
20.0	1.4209	1.4162	1.4255	1.0943	1.1821	1.2824
30.0	1.3103	1.2874	1.2978	1.0664	1.2147	1.3215
40.0	1.2636	1.3822	1.3624	1.0970	1.1899	1.2789
60.0	1.2750	1.3566	1.3567	1.0882	1.1735	1.1856
80.0	1.2772	1.3602	1.3544	1.0895	1.1747	1.1892
100.0	1.2849	1.3603	1.3712	1.0960	1.1749	1.1654

$$B_I^{s_2} = -\frac{5}{h^2}\begin{bmatrix} 0 & 0 & 0 & N_I & 0 \\ 0 & 0 & 0 & 0 & N_I \end{bmatrix}$$

$$B_I^{NL} = \begin{bmatrix} w_{bs,x} & 0 \\ 0 & w_{bs,y} \\ w_{bs,x} & w_{bs,y} \end{bmatrix}\begin{bmatrix} 0 & 0 & N_{I,x} & 0 & 0 \\ 0 & 0 & N_{I,y} & 0 & 0 \end{bmatrix} \quad (5.11)$$

The Newmark's time integration method is employed for the calculation of dynamic response in time history. The solution for the dynamic response in terms of displacement is considered zero at the initial time t = 0 and, In the second step, the iteration of displacement at (n+1) Δt is expended:

$$\begin{aligned} \ddot{q}_{n+1} &= \frac{1}{\beta\Delta t^2}(q_{n+1}-q_n) - \frac{1}{\beta\Delta t}\dot{q}_n - \left(\frac{1}{2\beta}-1\right)\ddot{q}_n \\ \dot{q}_{n+1} &= \dot{q}_n + (1-\gamma)\Delta t\ddot{q}_n + \gamma\Delta t\ddot{q}_{n+1} \\ q_{n+1} &= q_n + \Delta t\dot{q}_n + \left(\frac{1}{2}-\beta\right)\Delta t^2\ddot{q}_n + \beta\Delta t^2\ddot{q}_{n-1} \end{aligned} \quad (5.12)$$

Where $b = 0.25$ and $\mathbf{g} = 0.5$ were taken from by Fisette et al [31].

Numerical Results

A MATLAB-based FEM code is developed based on the above formulation. The accuracy of the present theory is verified with the help of various numerical examples.

In all the examples the shear correction factor is considered as 5/6 and the material constants are:

Material 1

$E_1/E_2 = 40$, $G_{23}/E_2 = 0.5$, $G_{12}/E_2 = G_{13}/E_2 = 0.6$ and $\nu = 0.25$ [12]

Material 2

$E_1/E_2 = 25$, $G_{23}/E_2 = 0.2$, $G_{12}/E_2 = G_{13}/E_2 = 0.5$ and $\nu = 0.25$ [32]

Where, subscripts used 1, 2, and 3 are the x, y, and z axes of the laminated plates.

The nonlinear dynamic stability of laminated composite plates rest on simply supported anti-symmetric cross-ply and angle-ply laminates are analyzed and compared with the higher order solution theories and validation of the present theory has been carried out.

Observation

This section, discussed the central deflection of laminated plates based on various load parameters from 3 to 100 for two plate thicknesses a/h = 10 & 100 with material 1 is considered. Table 1 compared the deflection results of three different boundary conditions such as all four sides simply supported (ssss), all four sides clamped (cccc) and two sides simple supported and two sides clamped (ccss) based on anti-symmetric cross-ply and anti-symmetric angle-ply laminated

Table: 3. Central deflection of laminated composite thin plate versus time based on **Linear and Nonlinear analysis.**

Time t (x10-3 s)	Linear Analysis			Nonlinear Analysis		
	FSDT (Lee&Reddy)	HSDT (Lee&Reddy)	Present	FSDT (Lee&Reddy)	HSDT (Lee&Reddy)	Present
0.5	0.0015	0.0016	0.0016	0.0015	0.0016	0.0018
1.0	0.0072	0.0079	0.0080	0.0072	0.0079	0.0085
1.5	0.0186	0.0207	0.0212	0.0186	0.0207	0.0218
2.0	0.0385	0.0408	0.0415	0.0385	0.0408	0.0510
6.0	0.5865	0.5499	0.5588	0.5813	0.5464	0.5554
10.0	1.3558	1.3376	1.3485	1.2308	1.2582	1.2685
30.0	0.5591	0.6741	0.7712	0.6966	0.3694	0.3736
50.0	1.7905	1.8194	1.8432	0.4843	0.6128	0.6232
70.0	0.7000	0.6324	0.6525	0.5808	1.1899	1.1988
100.0	0.8036	0.9235	0.9500	0.7754	0.9260	0.9564
200.0	1.0487	1.1903	1.1878	0.8840	0.9938	0.9857
300.0	1.1218	1.2202	1.3212	0.8439	0.9344	0.9544
400.0	1.1434	1.2074	1.2275	0.8404	0.9187	0.9325
500.0	1.1483	1.2084	1.2156	0.8414	0.9194	0.9188

plates under uniformly distributed load and sinusoidal load conditions. Irrespective of plate thickness and boundary condition the ssss show a larger amplitude value compared to cccc and ccss for two conditions anti-symmetric cross-ply and anti-symmetric angle-ply laminates. It is also shown from the table that the amplitude increases as the load parameter increases.

Observation

A time step is used to determine the linear and nonlinear response of central deflection of thick and thin laminated plate based on the finite element method of mesh size 4 x 4 is discussed in Table 2 and Table 3. The intensity of transverse load for a thick plate is considered $q = 5.0 \times 10^7$ and the transverse load intensity for a thin plate is considered $q = 1.0 \times 10^4$. It can also be observed that the effect of nonlinearity for the transverse deflection in the dynamic analysis is a small difference compared to HSDT proposed by Lee & Reddy [2004] for time responses 6.0 s and 10.0 s. Similarly, for thin laminated plates, the transverse deflections in the dynamic analysis are small differences compared to HSDT invented by Lee & Reddy [2004]. It can also be observed that the other values are closely in agreement with the values of HSDT and FSDT.

Conclusion

A refined nonlinear plate theory based on higher-order shear deformation was developed. The derived nonlinear formulation has been solved by using FEM. Furthermore, the theory developed based on some simplifying assumptions has reduced the unknown variables by one.

The accuracy and effectiveness of the developed theory discussed in the various problems have been concluded as:

- The refined model is used to analyze the Linear and Nonlinear dynamic behavior of the laminated composite plates based on the geometry, material properties, boundary condition and types of loading.
- For nonlinear analysis, the Newmark time integration iteration technique is used in conjunction with the Newton-Raphson method. The numerical results have shown very good agreement with that of the present model.
- The effect of the bending-extension coupling in cross-ply square laminated plates has been more predominant in the case of nonlinear bending. The central deflection is found to be higher for the lower value of a number of layers due to the low coupling between bending and extension but the central deflection decreases as the number of layers increases due to the high coupling effect between bending and extension.
- The central deflection under various loads and boundry condition: SSSS, CCCC and CCSS for anti-symmetric cross-ply and anti-symmetric angle-ply show similar behavior in nonlinear analysis.

Due to the large geometrical nonlinearity response the nonlinear transient of HSDT and FSDT is apparent with increasing the load intensity.

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