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Generalized Time Fractional Ito System: Solitary Wave Solutions by an Analytical Technique

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Abstract

This study gives an approximate solution to the generalized time-fractional Ito system using fractional variational iteration. The proposed numerical methods are shown to be efficient by convergence analysis and numerical experiment. Diverse fractional values of time-derivative are evaluated to existing approaches and illustrated using figures. This research shows that schemes are appealing, credible and easy to exploit.

Keywords: The time-fractional ITO system, Caputo fractional derivative, Non-linear partial differential equation, and the fractional vibrational iteration method.

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Introduction

Several academics have studied the strict traveling wave and solitary wave resolutions of nonlinear partial differential equations in recent decades. In biology, physics, chemistry, computer science, engineering, geophysics, and others have various applications of nonlinear PDEs[1-7] of solitary wave and the strict traveling wave resolutions. Recent studies on fractional calculus theory and applications have expanded the literature. Fractional order calculus has become a useful means in fluid dynamic traffic, neurophysiology, bioengineering, electromagnetic theory, industrial robotics, control theory, electric technology, mathematical economy, biology, potential theory, and more in recent decades.[8-17] Fractional calculus is most important for the real-world. Heat diffusion (heat flow equals half-temperature) contains Fractional order systems into a semi-infinite solid. In recent progress, many different applications in engineering and applied science used to fractional differential equations (FDEs) and fractional calculus. A integral and differential equations of fractional order system explain most physical and engineering processes more realistically than a conventional system.[18-21] Sun *et al.*[22] presented non-locality-critical applications in several research and engineering fields. Several fractional calculus researchers have written concise descriptions. This guide will assist novice researchers see some of the primary real-world applications. Fernandez et al. considered a transform integral initiated by generalized Mittag-Leffler and multi-parameter functions in.[23] Prakash [24] proposed a new computational method, known as the iterative sumudu transform technique, to numerically tackle a problem. Two dimensional Zakharov-Kuznetsov nonlinear time-fractional equations, Yusuf *et al.*[25] studied the integer order time fractional dispersive long wave equation. Recent methods for solving nonlinear FPDEs include Adomian decomposition,[26] differential transform,[27] homotopy perturbation,[28] fractional variation iteration [29,30] homotopy analysis,[31,32] analysis transform,[33,34] and perturbation transform.[35]

Researchers have recently tackled new directions in fractional nonlinear equations by defining fractional forms as nonlinear evolution systems and linking their discovering to "Memory-index" features of modeling. This pattern continues as we investigate the time-fractional integrable Ito system,[36-39] which reads

$$D_t^{\alpha} u(x,t) = \frac{\partial v}{\partial x}, D_t^{\alpha} v(x,t) = -2\frac{\partial^3 v}{\partial x^3} - 6\frac{\partial (uv)}{\partial x}, \tag{1}$$

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The symbol α is used to denote the fractional derivative in the Caputo sense, which is considered as the memory index in this model. The fractional variational iteration method (FVIM) targets nonlinear FDEs directly, lacking the need to discover specific polynomials for nonlinear factors, and produces an infinite series that rapidly converges to an analytical solution. That approach does not necessitate minor perturbations, discretization, linearization, or any other limiting assumptions. It considerably reduces mathematical computations.

The goal of that study is to acquire an exact numerical solution of the generalized time-fractional Ito system using FVIM. We utilized the Caputo fractional derivative since it has the similar advantage of the integer order differential equations in terms of initial conditions for FDEs.

Preliminaries

Definition 2.1: For $\chi > 0$, assume the real functions $h(\chi)$. In space $C\_\zeta, \zeta \in R$, if $b > \zeta$, s.t. $h(\chi) = \chi b\, h1(\chi), h_1 \in C[0,\infty)$. It is obvious that $C_\zeta \subset C_\gamma$ if $\gamma \leq \zeta$.

Definition: 2.2: We assume the function $h(\chi)$ for $\chi > 0$. The function C_ζ^m *and* $m \in N \cup \{0\}$ is called in space if $h^{(m)} \in C_\zeta$.

Definition: 2.3: The left-sided aspect The Caputo fractional derivative of function h, $h \in C_{-1}^m,\ m \in N \cup \{0\}$,

$$D_t^\beta h(t) = \{[I^{m-\beta} h^{(m)}(t)], m-1<\beta<m, m \in N, \frac{d^m}{dt^m} h(t),\ \beta = m,$$

- $I_t^\zeta h(x,t) = \frac{1}{\Gamma\zeta}\int_0^t (t-s)^{\zeta-1} h(x,s)\,ds;\ \zeta, t > 0.$
- $D_\tau^\nu V(x,\tau) = I_\tau^{m-\nu} \frac{\partial^m V(x,\tau)}{\partial t^m}, m-1 < \nu \leq m$.
- $D_t^\zeta I_t^\zeta h(t) = h(t), m-1 < \zeta \leq m, m \in N.$
- $I_t^\zeta D_t^\zeta h(t) = h(t) - \Sigma_1^{m-1} h^k(0+)\frac{t^k}{k!}, m-1 < \zeta \leq m,, m \in N.$
- $I^\nu t^\zeta = \frac{\Gamma(\zeta+1)}{\Gamma(\nu+\zeta+1)} t^{\nu+\zeta}.$

FVIM basic strategy for the generalized time-fractional Ito system

The mathematical model provided by equations (3) can be utilized for consideration.

$$\frac{d^\alpha u(x,t)}{d\tau^\alpha} = \frac{\partial v}{\partial x}, \frac{d^\alpha v(x,t)}{d\tau^\alpha} = -2\frac{\partial^3 v}{\partial x^3} - 6\frac{\partial uv}{\partial x}, \tag{2}$$

Where $0 < \alpha \leq 1$ with the initial condition,

$$u(x,0) = \frac{1}{6} - \frac{1}{2}\tanh^2\left(\frac{x}{2}\right) \text{ and } v(x,0) = \frac{1}{3} - \tanh^2\left(\frac{x}{2}\right).$$

In this study correction functional is formulated for equations (2).

$$u_{(n+1)}(t) = u_n + \int_0^t \lambda\left(\frac{d^\alpha u_n(x,t)}{d\tau^\alpha} - \frac{\partial \tilde{v_n}}{\partial x}\right)(d\tau)^\alpha,$$

$$v_{(n+1)}(t) = v_n + \int_0^t \lambda\left(\frac{d^\alpha v_n(x,t)}{d\tau^\alpha} + 2\frac{\partial^3 \tilde{v_n}}{\partial x^3} + 6\frac{\partial \widetilde{u_n v_n}}{\partial x}\right)(d\tau)^\alpha, \tag{3}$$

In variational theory, the Lagrangian multiplier λ must satisfy the following conditions.

$\frac{d^\alpha \lambda}{d\tau^\alpha}|_{\tau=t} = 0$ and $1 + \lambda|_{\tau=t} = 0.$

We quickly get, $\lambda = -1$. Then, using this value in equations (3), we get

$$u_{(n+1)}(t) = u_n - \int_0^t \left(\frac{d^\alpha u_n(x,t)}{d\tau^\alpha} - \frac{\partial v_n}{\partial x}\right)(d\tau)^\alpha,$$

$$v_{(n+1)}(t) = v_n - \int_0^t \left(\frac{d^\alpha v_n(x,t)}{d\tau^\alpha} + 2\frac{\partial^3 v_n}{\partial x^3} + 6\frac{\partial u_n v_n}{\partial x}\right)(d\tau)^\alpha, \tag{4}$$

Constructing successive approximations $u(x,t)$ and $v(x,t), n > 0$, is possible. $u(x,t)$ and $v(x,t)$ have limited variations with $\delta u = 0$ and $\delta v = 0$. Finally, we acquired sequences $u_{n+1}(x,t)$ and $v_{n+1}(x,t)$ for $n \geq 0$, obtaining the exact respond.

$$u(x,t) = u_n(x,t),$$

$$v(x,t) = v_n(x,t). \tag{5}$$

FVIM is implemented numerically

Through the utilization of conditions, it is possible to initiate $u_0 = \frac{1}{6} - \frac{1}{2}\tanh^2\left(\frac{x}{2}\right), v_0 = \frac{1}{3} - \tanh^2\left(\frac{x}{2}\right)$ and by using FVIM to Eqs. (2), we get

$$u_1(t) = u_0 - \int_0^t \left(\frac{d^\alpha M_{x(n)}(t)}{d\tau^\alpha} - \frac{\partial v}{\partial x}\right)(d\tau)^\alpha,$$

$$= \frac{1}{6} - \frac{1}{2}\tanh^2\left(\frac{x}{2}\right) - \frac{t^\alpha \operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)}{\Gamma(1+\alpha)}$$

$$v_1(t) = v_0 - \int_0^t \left(\frac{d^\alpha v(x,t)}{d\tau^\alpha} + 2\frac{\partial^3 v}{\partial x^3} + 6\frac{\partial uv}{\partial x}\right)(d\tau)^\alpha,$$

$$= -\frac{2}{3} + \operatorname{sech}^2\left(\frac{x}{2}\right) - \frac{16\, t^\alpha \operatorname{csch}^2(x)\sinh^4\left(\frac{x}{2}\right)}{\Gamma(1+\alpha)}$$

$$u_2(t) = \frac{1}{6} - \frac{1}{2}\tanh^2\left(\frac{x}{2}\right) - \frac{1}{2}\operatorname{sech}^4\left(\frac{x}{2}\right)\left(-\frac{2\,t^\alpha(-2+\cosh(x))}{\Gamma(1+\alpha)} + \frac{\sinh\sinh(x)}{\Gamma(1+\alpha)}\right),$$

$$v_2(t) = -\frac{2}{3} + \operatorname{sech}^2\left(\frac{x}{2}\right) +$$

$$4t^\alpha(x)\tanh^2\left(\frac{x}{2}\right)\left(\frac{2\,t^\alpha(-2+\cosh\cosh(x))(x)}{\Gamma(1+2\alpha)} - \frac{\Gamma(1+\alpha) + \frac{6\,t^{2\alpha}(-2+\cosh\cosh(x))\Gamma(1+2\alpha)\operatorname{sech}^4\left(\frac{x}{2}\right)}{\Gamma(1+3\alpha)}}{\Gamma(1+\alpha)^2}\right).$$

Following this procedure, the next iteration's fundamental elements can be determined with assist of Maple package has a solution as:

$$u_{(n)}(x,t),$$

$$v_{(n)}(x,t), \tag{8}$$

When $\alpha = 1$, Eqn (2) have an exact solution.

$$u(x,t) = \frac{1}{6} - \frac{1}{2}\tanh^2\left(\frac{x}{2} + t\right)$$

$$v(x,t) = \frac{1}{3} - \tanh^2\left(\frac{x}{2} + t\right). \tag{9}$$

Iteration formula for problem (2) can be created from (6) and (7).

$$v_{0(1)} = \frac{1}{6} - \frac{1}{2}\tanh^2\left(\frac{x}{2}\right), v_{0(2)} = \frac{1}{3} - \tanh^2\left(\frac{x}{2}\right),$$

$$v_{1(1)} = -\frac{t^\alpha \operatorname{sech}^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)}{\Gamma(1+\alpha)},$$

$$v_{1(2)} = -\frac{2}{3} + \operatorname{sech}^2\left(\frac{x}{2}\right) - \frac{16\,t^\alpha \operatorname{csch}^2(x)\sinh^4\left(\frac{x}{2}\right)}{\Gamma(1+\alpha)},$$

and so on.

Obtain by computing $\chi_i's$ for this issue, we get

$$\chi_i = \frac{\|v_{i+1}\|}{\|v_i\|} = \|x^\alpha \frac{\Gamma(1+i\alpha)}{\Gamma(1+(i+1)\alpha)}\| < 1,$$

Example: for $i \geq 1, 0 < t \leq 1$ and $0 < \alpha \leq 1$. This proves that variational technique for problem (2) provides a positive, bounded, exact solution.

Statistical results and analysis

Figure's 1 to 4 show that FVIM's generalized time-fractional Ito system solution $u(x,t)$ and $v(x,t)$ are almost equal to the exact result. Figure 5 and 6 show the behavior of $u(x,t)$ and $v(x,t)$ Vs.t at $\alpha = 1$ and compare it to the numerical solution from FVIM. The numerical findings for various α situations are shown in Figure 7, 8. Table 1 compares $u(x,t)$ values and Table 2 compares $v(x,t)$ values at $\alpha = 1$ and

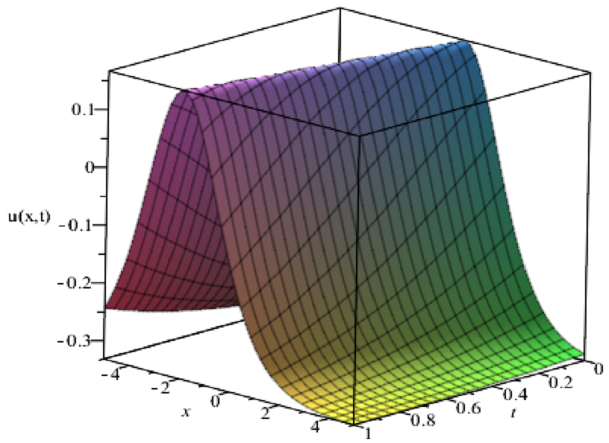

Figure1: The objective of this study is to determine the precise solution for the function $u(x,t)$.

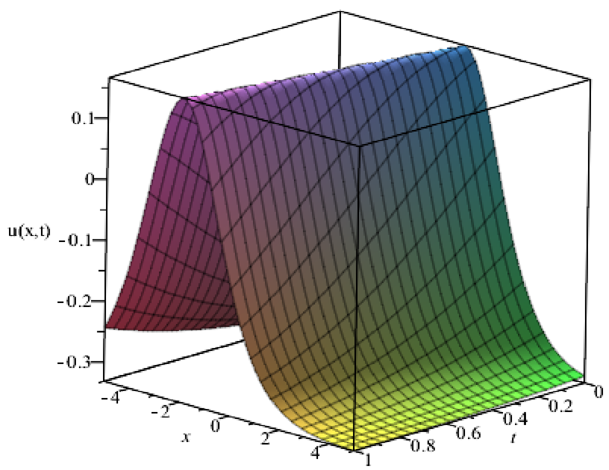

Figure 2: FVIM approximate $u(x,t)$ solution

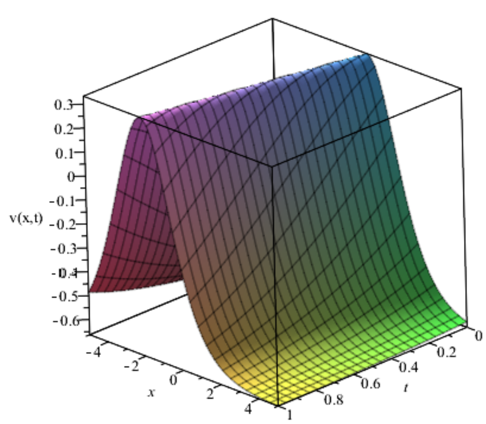

Figure 3: The objective of this study is to determine the precise solution for the function $v(x,t)$.

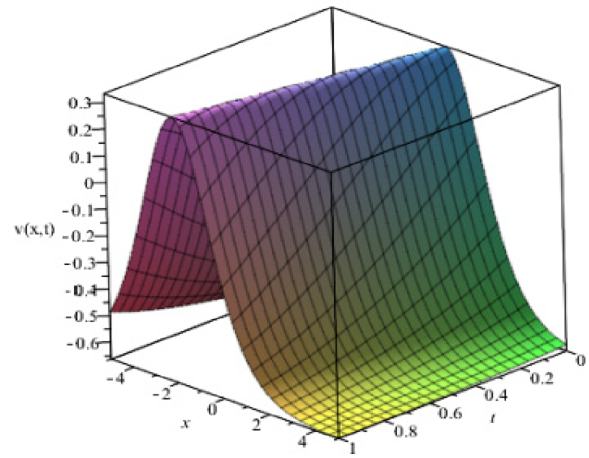

Figure 4: VIM approximate $v(x,t)$ solution

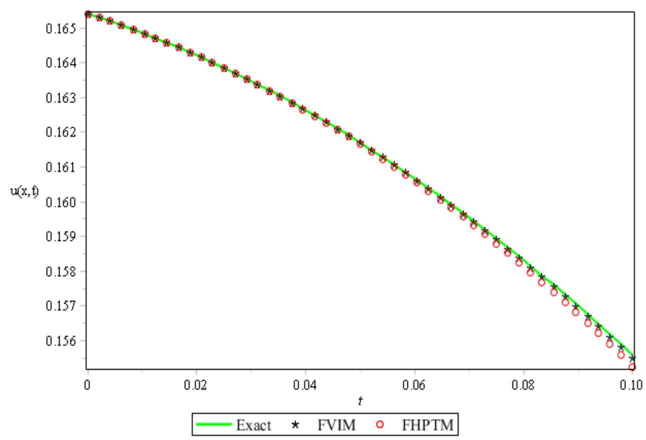

Figure 5: Exact solution $u(x,t)$ Vs. t behavior and assessment with approximation solution using FVIM and FHPTM at $\alpha = 1$

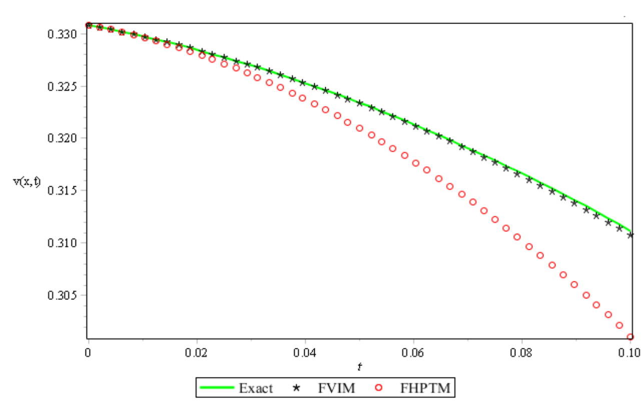

Figure 6: Exact solution $v(x,t)$ Vs. t behavior and assessment with approximation solution using FVIM and FHPTM at $\alpha = 1$

Table 1: Assessment of approximate solutions by proposed and existing methods with exact results

x	t	$\lvert u_{EXACT}\rvert$	$\lvert u_{EXACT}\rvert$	$\lvert u_{FVIM}\rvert$	$\lvert u_{FHPTM}\rvert$
-4	0.1	0.2905283	0.2905247	0.2905310	0.2905611
-2	0.15	0.0721218	0.0721655	0.0720566	0.0728324
0	0.2	0.1471883	0.1471556	0.1455867	0.1474670
2	0.25	0.1931262	0.1931627	0.1927110	0.1891865
4	0.3	0.3136286	0.3132390	0.3138466	0.3126970

Table 2: Assessment of approximate solutions by proposed and existing methods with exact results

x	t	$\lvert v_{EXACT}\rvert$	$\lvert v_{EXACT}\rvert$	$\lvert v_{FVIM}\rvert$	$\lvert v_{FHPTM}\rvert$
-4	0.1	0.581057	0.581019	0.581165	0.583319
-2	0.15	0.144244	0.144086	0.145086	0.127368
0	0.2	0.294376	0.294118	0.293326	0.240469
2	0.25	0.386252	0.386180	0.380978	0.407184
4	0.3	0.627256	0.627256	0.627256	0.636955

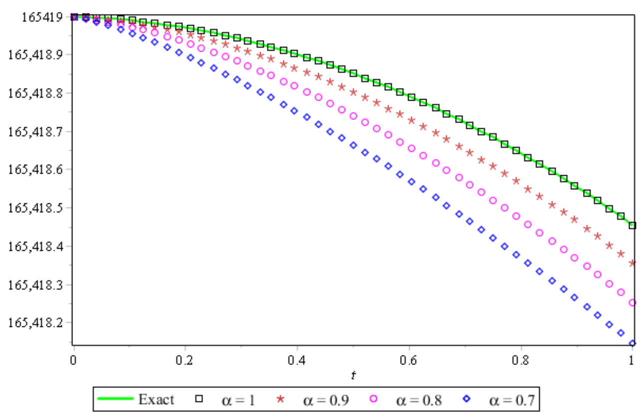

Figure 7: FVIM similarity of the approximate solution $u(x,t)$ for various values of α.

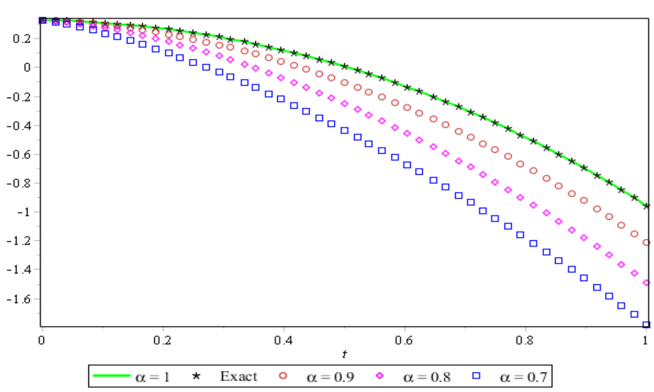

Figure 8: FVIM comparison of the approximate solution $v(x,t)$ for various values of α.

$t = 0.1, 0.15, 0.2, 0.25,$ and 0.3 using the suggested FVIM approach with the precise result. Numerical solutions show that our proposed FVIM solves the generalized time-fractional Ito problem better than other approaches, even with a lower order approximation solution. Still, higher-order approximate solutions improve accuracy.

Conclusion

This article effectively solves generalized time-fractional Ito system using FVIM. Some descriptive paradigms shows that, FVIM is uncomplicated to build and a dominant numerical method for approximate solutions. Convergence study shows that FVIM's numerical solution is bounded positive and converse. FVIM is utilized thoroughly without linearization, Adomain polynomials, perturbation, or further restrictions. FVIM's generalized time-fractional Ito system results are closer to exact solutions than hybrid approaches.

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