REVIEW ARTICLE

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# Review Paper on Green's Function- A Mathematical Approach for Investigation of Thermal Stresses in Steam Turbine Rotor

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## Abstract

In online systems, a numerical technique based on Green's function for estimating and analyzing thermal stresses subjected to transient conditions proved to be effective. The need for utility power plants to have great operating flexibility necessitates the use of online systems for controlling and monitoring damage to important components, such as steam turbine rotors. Different data and mathematical models are used in such systems to calculate thermal stresses and manage them continuously. Thermal fatigue is becoming more of a worry in power plant workers as the trend toward "two-shifting" operating methods grows. Thermal power plants are anticipated to be a form of energy mix for the near future when they're under pressure from society to produce in a more efficient and flexible manner.For transient thermal stress problems, Green's function approach offers a flexible approach for assessing benchmark elastic answers. Green's functions (deduced using finite element unit temperature phase answers) are frequently believed to be independent of temperature in order to ease integration. During startup, the turbine shafts of coal power plants are subjected to significant stresses caused by high pressures and temperatures. Thermal gradients approach the design threshold at the very same time that steam temperatures via components quickly transition from minimum to maximum temperature, reducing the rotor's useful lifetime. Thermal property assessments predicated on relatively stable heating are implicit procedures wherein the temperature-dependent properties and characteristics are determined by comparing the experimental results with heat-transfer concept.

**Keywords:** Steam turbine rotor, Green's function, Transient thermal stress, FEM.

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## Introduction

Thermal stresses in steam turbine components happen during quick startup, shutdown, or load variations. These stresses developed should not exceed the values which are specified earlier, which vary based on the material and current pressure and temperature values. The process conditions of a turbine rotor are harsh, with high temperatures and pressures. Long-term functionality under this kind of condition may cause low-cycle fatigue in the material of the turbine rotor, resulting in crack formation in the rotor. The primary goal of highly developed control techniques optimizing startup, shutdown, or variation of load is to minimize temperature variations during transient conditions. [1]. To stabilize these non-stationary processes economically, the maximum allowable stresses must be approached and sustained at the maximum temperature.Due to the obvious significance of turbine rotors in thermal power plants, the topic of turbine life estimation or monitoring piques the interest among researchers. The thermal power plant's complexities must be thoroughly understood to accomplish this goal. The turbine rotors were monitored in the earlier thermal stress control systems utilizing startup probables, that were thermo-mechanical models of steam turbine rotors. The transient thermal stresses were estimated using the difference in temperature between the test surface and the integral averaged temperature. Stress calculations were

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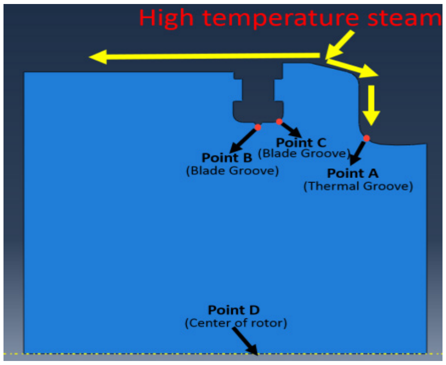


**Figure 1:** Part of the steam turbine rotor.[1]

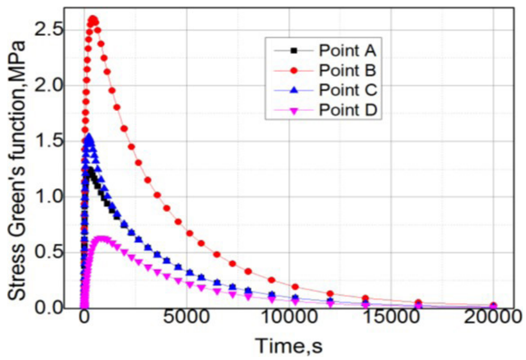


**Figure 2:** Green's function for stress of a high temperature rotor.[1]

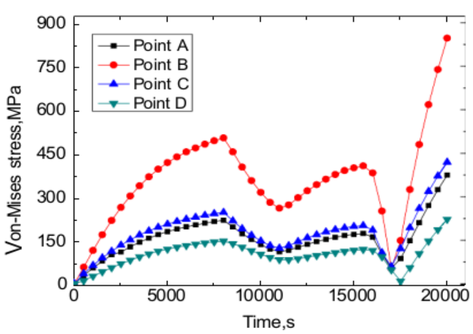


**Figure 3:** Von-Mises stress for Point A, B, C, D calculated by Green's function method.[1]

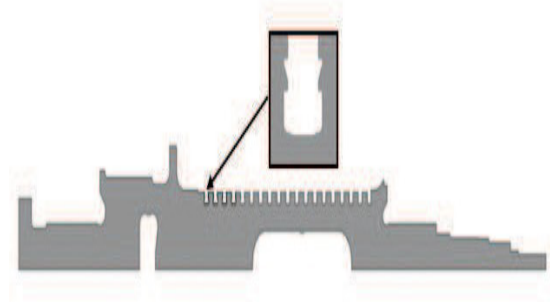

**Figure 4(a):** Rotor model with a zoomed blade groove.[2]

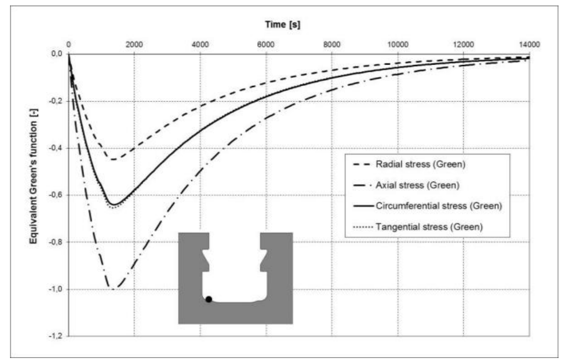


**Figure 4(b):** Rotor model with a zoomed blade groove.[2]

more accurate by using a mathematical model in place of the reported average temperature. A fresh approach to modeling thermal stress in turbine rotors using Duhamel's integral and Green's functions was proposed. For a long time, fatigue life monitoring has utilized Green's functions and Duhamel's integral for remnant life estimation of steam turbine rotors.

## Literature Review

Haitao wang *et al.* [1] This research used Green's Function methodology, steam temperature data, and FEM to determine the temperature and transient thermal stress in specific areas for a high-temperature turbine rotor. The outside surface and the axis of turbine rotor's thermal stress are both highlighted. Results showed that, whenever the constant coefficient of heat transfer was applied in the transient situation, the anticipated temperature and transient thermal stresses by the Green's function technique corresponded to those by the detailed Finite element method. The groove of the first blade or the thermal groove at the inlet are the places where the rotor at high temperature is weakest.As an undetectable sensor, Green's function can quickly and easily monitor and calculate r these specific regions of the rotor at high temperatures in each transient condition, allowing the rotor to be in service for an extended period of time without risk. It was recommended that Green's function approach based averaged coefficient of heat transfer coefficient should be employed in the individual stage for precise thermal stress control and monitoring.

M. Banaszkiewicz *et al.*[2] The method for controlling and calculating thermal stress in turbine rotors was provided in the paper. Themain components of the suggested system are presented in the study, and their application is covered from thermodynamics and heat transmission perspectives.The steam temperature at important points was calculated online using typical turbine readings as input signals utilizing thermodynamic principles that have been well-established in design calculations.The accuracy of the model predictions when compared to operational information from an actual thermal power plant operating at a warm startup was surprisingly high.The relationship of the variables coefficient of heat transfer and properties of the material on transient thermal stresses was examined numerically using the FEM on a cylindrical model. To

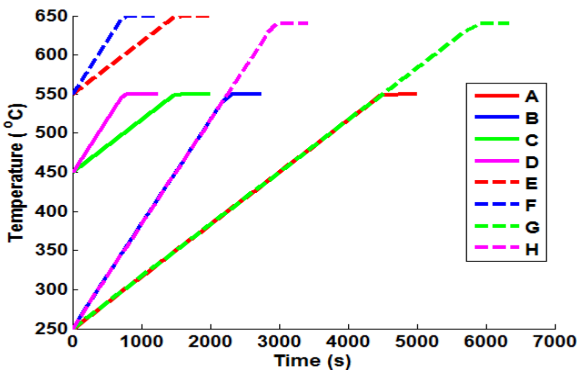


**Figure 5:** Rotor model with a zoomed blade groove.[3]


**Figure 6:** Predictions of thermal stress responses determine dusing the interpolation and weighted Green's functions methods for ramp up profile.[3]

consider this variability in the transient thermal stress, a model depending on Duhamel's integral, the concept of equivalent Green's function was introduced. By analyzing the transient thermal stresses at the groove of the rotor bladecomputed using finite element method and Duhamel's integral, it was demonstrated that this method produced correct findings for increasingly complex designs.

James Rouse *et al.*[3]The current study provides a straightforward interpolation method and several reference unit solutions to estimate a material's temperature dependence.Using several approaches, thermal stress patterns for actual temperature cycles are anticipated and compared.In most cases, the suggested interpolation method outperforms the ideal Green's function or the earlier proposed weighting function method.The two techniques are often comparable for smaller temperature ranges. Nine unit temperature stages were used to Finite Element Analysis models and utilized in the current work's temperature-dependent approaches.In order to more accurately predict transient thermal stresses for bulk steam temperature increases in thick-walled parts, the present work has underlined the significance of adding temperature dependency in Green's function method. Peak stress variances between the true and projected databases always seem to be below 10 MPa, and determination coefficients are frequently higher than 0.96.

Mohammad Haghpanahi *et al.*[4] The FSW tool is viewed in the present article as a circular source of heat moving inside a finite rectangular section with a cooled surface and non-homogeneous initial and boundary non-uniform as well as non-homogeneous initial and boundary. A new analytical solution of transient in nature, relying on Green's function technique is formed to achieve the three-dimensional temperature distribution in the welding process.

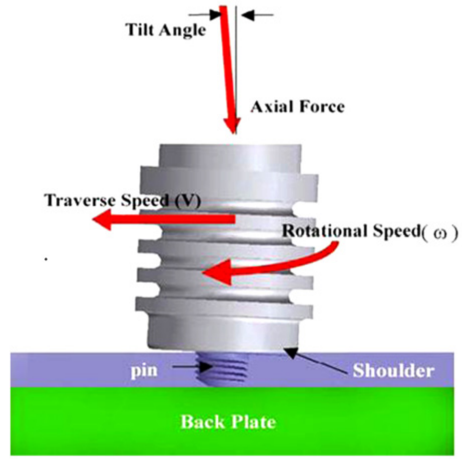


**Figure 7:** Schematic illustration of the FSW process.[4]

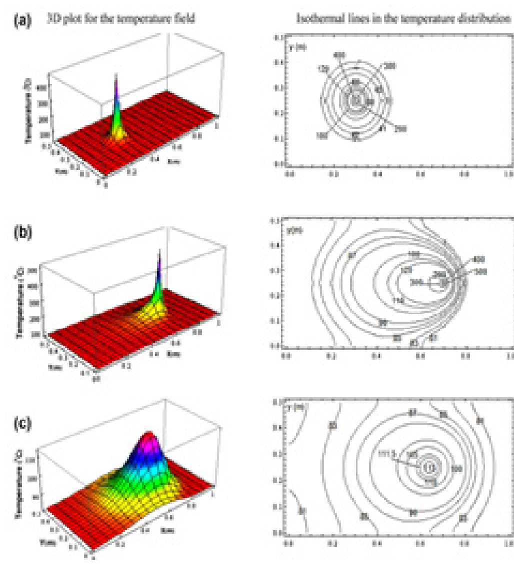


**Figure 8:** The temperature distribution on the top of the workpiece after (a) preheating stage (t = 30 s), (b) welding stage (t = 230 s) and (c) cooling stage (t = 290 s).[4]

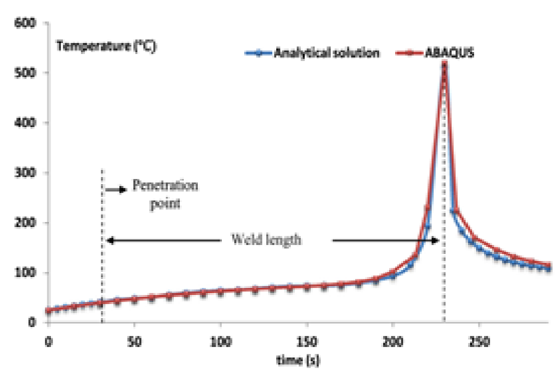


**Figure 9:** Thermal history for the point located in the end of the loading.[4]

By taking the temperature field produced by the pre-heating phase to be the non-uniform beginning state for the welding

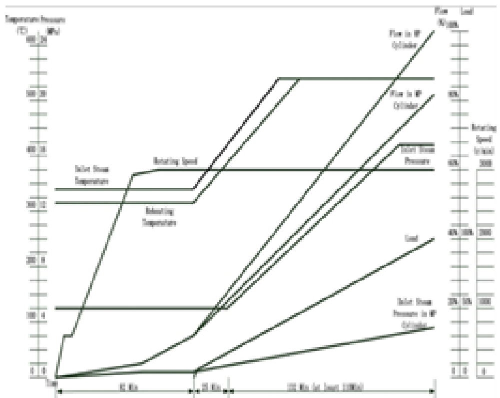

**Figure 10:** Cold start operation curve.[5]

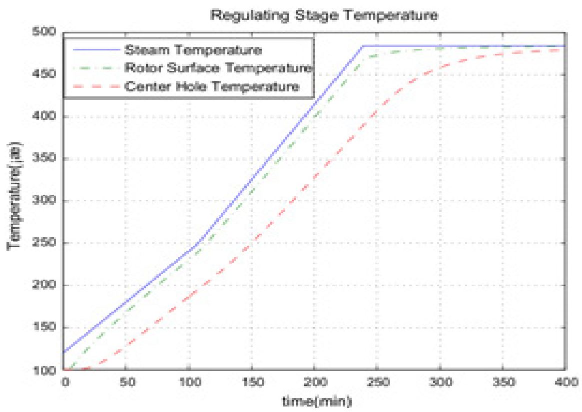


**Figure 11:** Temperatures evolution curve at the regulating stage.[5]

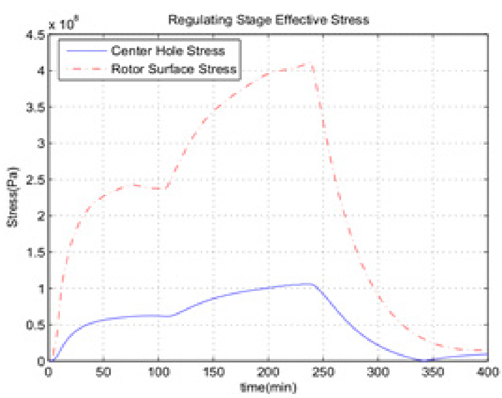


**Figure 12:** Temperatures evolution curve at the regulating stage.[5]

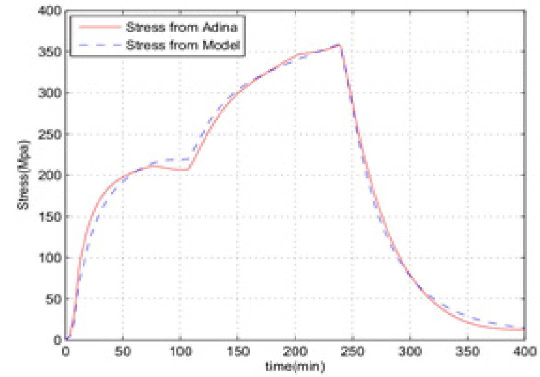


**Figure 13:** The comparison between thermal stress from ADINA and that from model.[5]

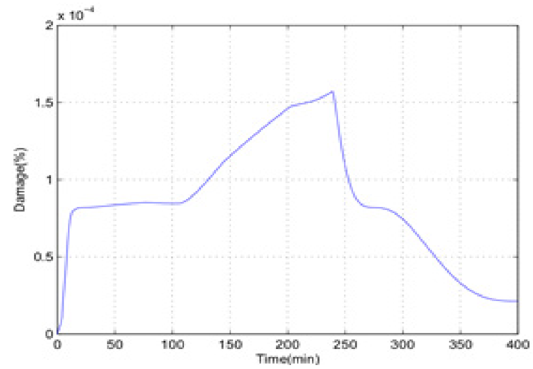


**Figure 14:** The rotor life damage curve.[5]

phase, the preheating/penetrating stage's impact is also considered. Similar calculations can be made for the cooling rate during the cooling phase. Additionally, the numerical and analytical findings are in strong agreement, which supports the accuracy of the established analytical solution, according to the modeling of the FSW operation using conventional FEM software. The current paper presents a Green's function-based solution to the transient temperature distribution in a finite plate induced by a moving circular source of heat with quasi-boundary and initial conditions, which enables temperature calculation at any point of the workpiece and at any moment during the FSW operation without the challenges associated with 3D numerical computation.The proposed analytical solution is a workable way to determine the temperature field surrounding the FSW tool, that, if compared with experimental data, leads to determining the actual heat input by the FSW tool. This is because the solution is straightforward and infinite series rapidly converge.The analytical solution that is being provided also sets the way for future investigation of thermomechanical states and the identification of residual stress distribution.

Yong-Jian Sun *et al.*[5] In order to create an online damage assessment model of a turbine generator rotor, this research uses the output and input information from finite element analysis during the cold startup phase. The Nyquist plot examines the sensitivity of the thermal stress transfer function. The accuracy of this study in determining the low cycle fatigue effects is validated by comparison between the suggested model and the finite element analysis model. The current research introduces an engineering-relevant online damage monitoring system for steam turbine rotors.

In this study, a 300 MW steam turbine rotor's low cycle fatigue damage online monitoring model has been developed. Additionally, it investigates the sensitivity of the

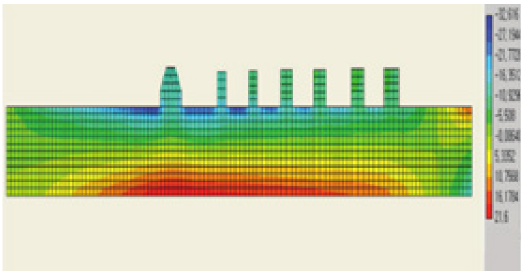

**Figure 15(a):** Thermal stress distribution of HP turbine rotor after 40 minutes power generation.[6]

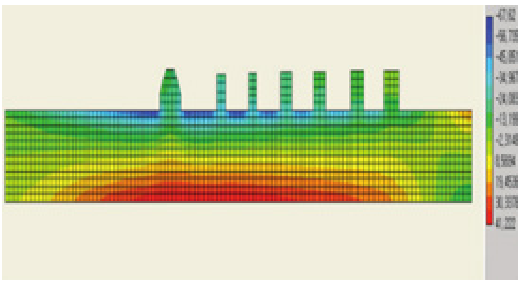

**Figure 15(b):** Thermal stress distribution of HP turbine rotor after 60 minutes power generation.[6]

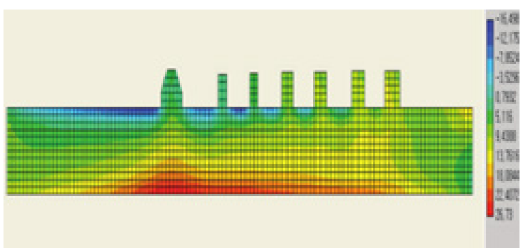

**Figure 15(c):** Thermal stress distribution of HP turbine rotor after 140 minutes power generation.[6]

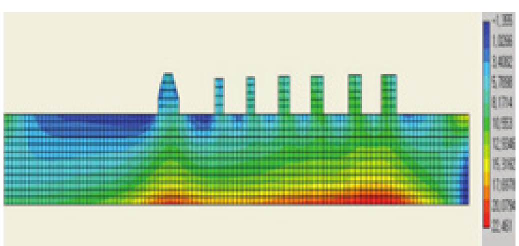

**Figure 15(d):** Thermal stress distribution of HP turbine rotor after 200 minutes power generation.[6]

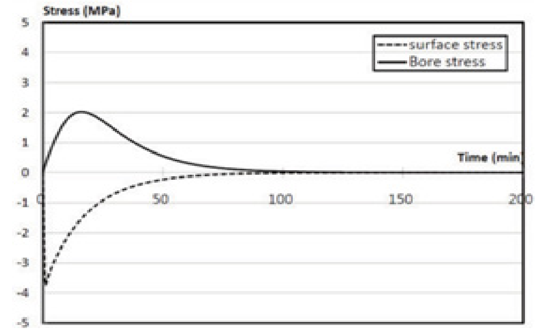


**Figure 16:** Green's function.[6]

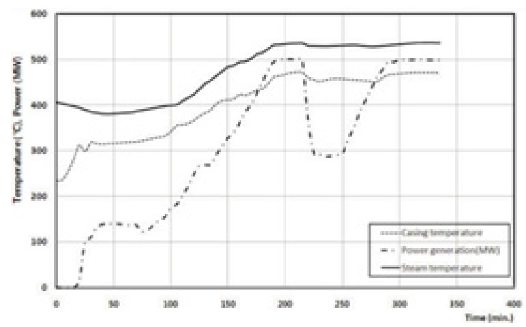


**Figure 17:** Cold startup condition.[6]

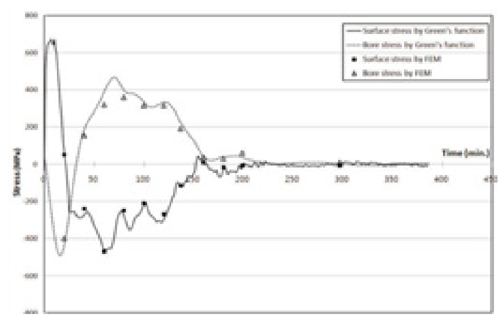

**Figure 18:** Comparison between Green's function method and FEM with start up schedule.[6]

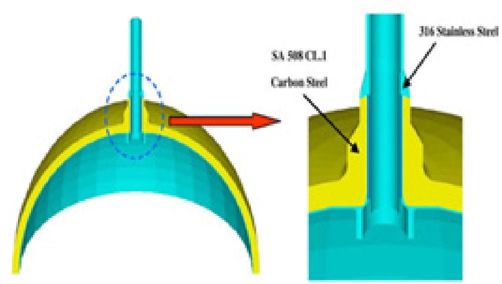


**Figure 19:** Configuration of pressurizer spray nozzle.[7]

thermal stress transfer function and performs a quantitative study of the FE model of the turbine rotor.The correctness and validity of the FEA model are demonstrated by a comparison between it and the current model when computing LCF damage during cold startup.Thus, our work offers a fresh perspective on online LCF damage monitoring in turbine rotors.

Geewook Song *et al.*[6] This paper's objective is to conduct fatigue studies that take into account the outcomes of thermal stress prediction during startup and shutdown. With the help of Green's Function, a quick and easy simulation of turbine startup has been created.A transfer of stress and temperature Critical areas of the rotor receive the application of Green's function. Green's function is the thermal stress caused by a unit change in steam temperature. This thermal stress model can be used at the startup cycle. As a result, it is

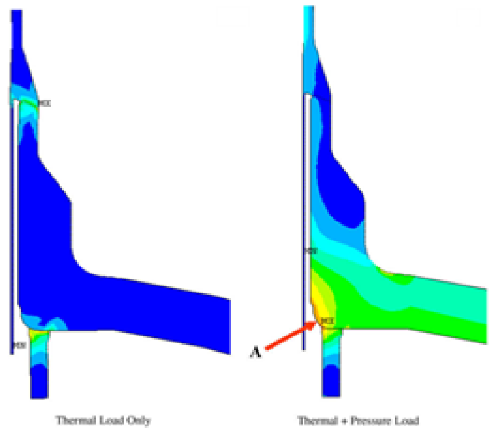


**Figure 20:** Stress distributions and selected monitoring point.[7]

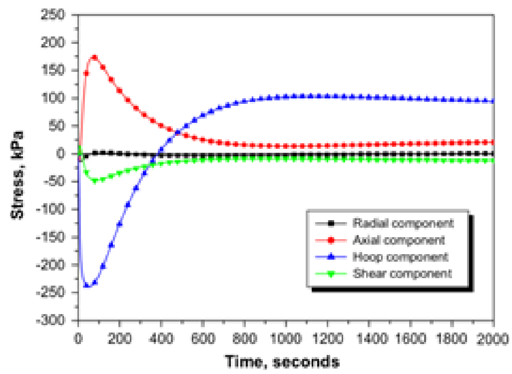


**Figure 21(a):** Calculated Green's functions for pressurizer spray nozzle.[7]

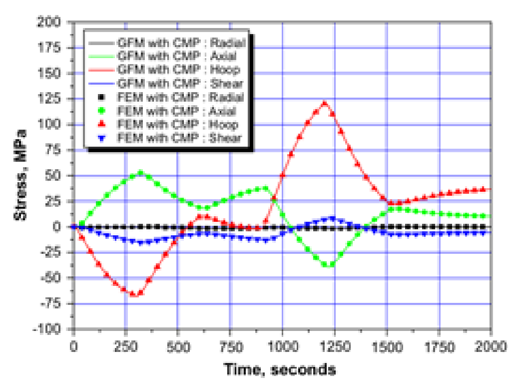


**Figure 21(b):** Comparison between GFM and FEM with CMP.[7]

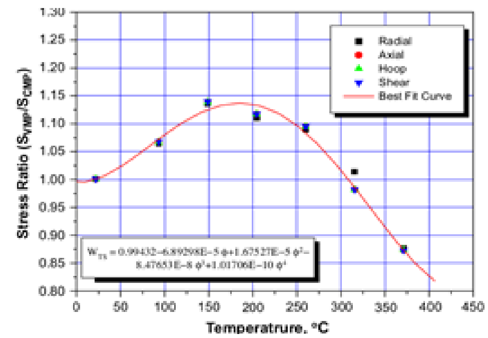


**Figure 21(c):** Weight function for steady state Green's functions.[7]

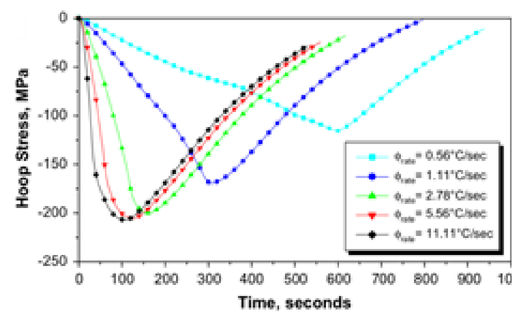


**Figure 22:** Hoop stress vs. time for assumed transient temperature rates.[7]

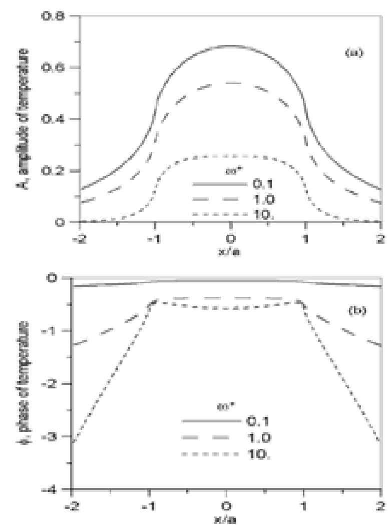


**Figure 23:** Amplitude and phase of the temperature on the heated surface of a slab with Ba = 1 and W / a = 1 for three heating frequencies.[8]

envisaged that the suggested scheme using Green's function and performance histories can be used for evaluating the turbine rotors' remaining useful lives.

Green's function method is used in conjunction with the turbine rotor thermal stress study. Under conditions of transitory thermal loading, Green's function approach enables the unspecific stress analysis at specified locations, for example the grooves of the turbine rotor.The thermal stresses calculated using the Green's function approach and the detailed FEM are equivalent in the transient state.The findings of the transient thermal stress modeling using the Green's function method are used to determine the fatigue damage. This technique is helpful for evaluating the fatigue life of turbine rotors. Therefore, under actual operating settings of power plants, the fatigue damage of rotor shaft can be measured by applying the Green's function approach.

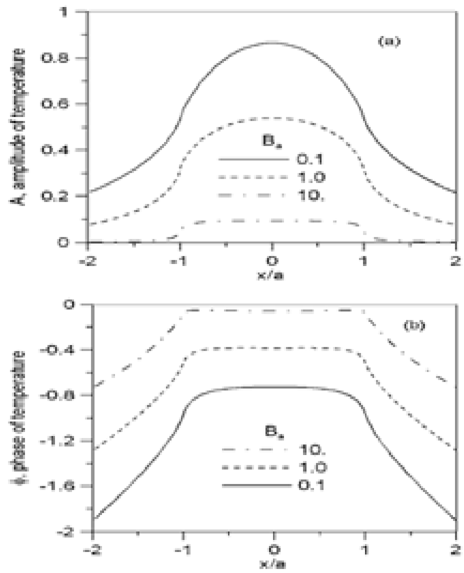


**Figure 24:** Amplitude and phase of the temperature on the heated surface of a slab with heating frequency and W / a = 1 for three Biot values.[8]

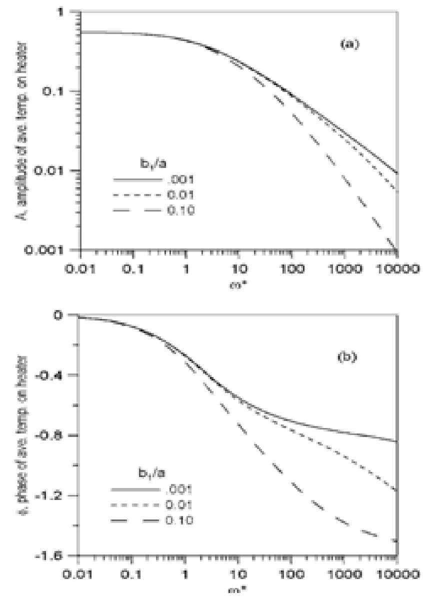


**Figure 26:** Amplitude and phase of the spatial average temperature on the heater as a function of frequency at B a = 1 for various thicknesses of films on a large substrate.[8]

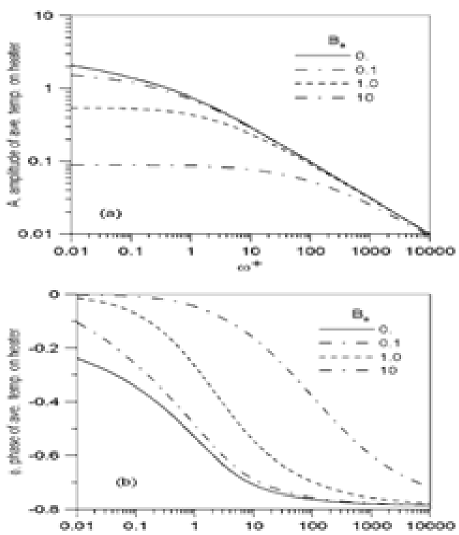


**Figure 25:** Amplitude and phase of the spatial average temperature on the heater for a thick substrate with no surface film as a function of frequency for three convection values.[8]

Gyeong-Hoi Koo *et al.*[7] In this research, a numerical weight function approach[7] is used to provide a way to take temperature dependent material attributes into account while applying the Green's function method. Using thorough finite element studies for a pressuriser sprayer with a variety of a prior thermal transient load situations, this is proved. For a pressurizer sprayer with various posited thermal transient load situations, thorough finite element calculations are used to verify this. The stress distribution ranges for a fatigue test are influenced by the temperature dependence of the material properties, it is established. Therefore,when utilising Green's function approach for the actual operating conditions at a nuclear power plant, it is vital to consider this effect to monitor fatigue damage.

In this research, a numerical weight function approach[7] is used to provide a way to take temperature-dependent material attributes into account while applying Green's function method. The findings show that the highest peak stresses can be greatly influenced by the thermal properties of the material.

Kevin . D. Cole *et al.*[8] The theory of heat transmission for a number of two-dimensional geometries relevant to steady-periodic thermal property approaches is discussed in this study. Rectangles, blocks (two dimensional), and quasi bodies are all treated methodically using the Green's functions approach.Convection and boundaries with a slim, highly conductive film are two boundary conditions that are addressed. Through the use of various but related geometries, the range of solutions offered here offers a chance for numerical results to be verified.Alternative Green's function expressions that can be used to create different series expressions for the same specific temperature solution provide a second chance for verification. Both methods of verification for the steady-periodic behavior to a heated strip are shown using numerical examples.

There are five different boundary conditions addressed. In order to measure the thermal properties of two geometries with bodies heated across a small area, numerical findings are presented. Results were reported throughout sixty years of frequency under various settings due to the method's computational efficiency.

## Conclusion

In this paper transient thermal Analysis of turbine rotor using FEM and Green's function from a variety of aspects, such as references, features, analytical treatment and technologies are reviewed. For any changes in fluid temperature causing the heating-up and cooling-down of an element, Green's function and Duhamel's method enable quick computation of temperature and thermal stresses at monitored areas at specific critical locations like the thermal slot and the first blade slot of the turbine rotor. In the transient condition, when the steady coefficient of heat transfer was applied, the projected temperature and thermal stresses using the Green's function technique match those by the detailed FEM.The proposed research work focuses on estimation of transient thermal stresses arising in a steam turbine rotor using Finite Element Analysis and validation the results with Green's function method. when utilising a Green's function approach for the actual operating conditions at a nuclear power plant, it is vital to take this effect into account to monitor fatigue damage.

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