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Propagation of Rayleigh Wave in Sandy Media with Imperfect Interface

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Abstract

In the present study, the propagation of Rayleigh wave in a sandy layer overlying a sandy semi-infinite media is investigated, with the interface considered imperfect. Expressions for displacement components are obtained. The dispersion frequency equation is derived using suitable boundary conditions. In particular cases, when interface is perfect and elastic media replace sandy media are also discussed. The effects of imperfectness and sandy parameter on the Rayleigh waves' phase velocity are investigated using MATLAB software. The theoretical results obtained may find useful applications in geophysics, civil engineering and soil mechanics.

Keywords: Rayleigh wave, Sandy layer, Imperfect interface, Dispersion equation, Phase velocity.

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Introduction

Theoretical studies regarding seismic wave propagation are crucial as they provide a vast amount of information about Earth's interior. Studies involving seismic wave propagation are important to seismologists and earthquake engineers since it contributes to determining the nature and cause of earthquakes and understanding the Earth's crust.

Materials of Earth may not always be isotropic and elastic. However, the Earth's crust consisting of sedimentary layers is not perfectly elastic but can be considered as sandy particles. A sandy layer consists of particles not retaining moisture or water vapours. Sand boils occurred due to the 7.1 Richter scale 1989 Loma Prieta earthquake causing the liquefaction of superficial sandy materials. So, role of a sandy layer in predicting seismic behavior is very important to seismologists. Various researchers studied seismic or Rayleigh wave propagation in sandy or elastic media. Rayleigh[1] and Bromwich[2] investigated Rayleigh waves propagation considering isotropic elastic solid and stratified media. The dynamics for dry sandy soil were explored by Weiskopf.[3] Kar *et al.*[4] studied Love wave propagation in sandy medium discussing the effects of irregularity. Abd-Alla[5] investigated Rayleigh wave propagation considering orthotropic material elastic half-space. Effects of the gravity field and initial stresses on Rayleigh wave propagation considering magnetoelastic half-space were discussed by Abd-Alla *et al.*[6]. Viswakarma and Gupta[7] investigated Rayleigh wave propagation in Earth's crustal layer for the sandy and elastic half-space cases obtaining the effects of inhomogeneity and rigid boundary. Pal *et al.*[8] explored Rayleigh wave propagation considering a sandy half-space and an anisotropic layer and derived the Rayleigh wave propagation dispersion frequency equation. Sahu *et al.*[9] examined Rayleigh wave propagation considering an orthotropic half-space with impacts of pre-stresses and self-weight and a liquid layer. Mandi et al. [10] investigated the propagation of Rayleigh waves in a geometry containing sandy media surrounded by couple stress media and orthotropic half-space.

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As Earth is a layered media, various interfacial conditions such as irregularities or imperfect interface have a significant effect on seismic wave propagation. Such studies provide rich information regarding Earth's seismic behavior. Various researchers studied the effects of these interfacial conditions. Hua *et al.*[11] studied effects of the imperfectness parameter on the propagation of Love wave considering a geometry consisting of layered graded composite structures. Vishwakarma and Xu[12] investigated dispersion of Rayleigh wave investigated considering sandy layer overlying an orthotropic mantle. The effects of irregular boundaries on upper plane has been discussed observing the initial stress and sandiness parameter effect. The dispersion equation for SH wave propagation in a layer of viscoelastic overlying couple stressed substrate with interface assumed to be imperfect was derived by Sharma and Kumar[13] and observed

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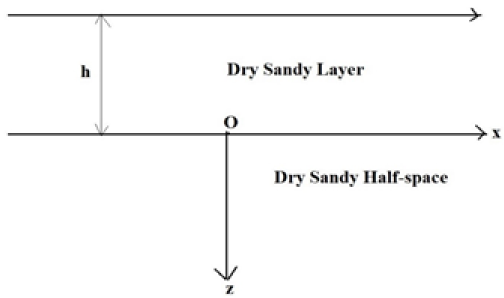

Figure 1: Geometry of the problem.

the effects of imperfectness, heterogeneity, friction and imperfectness parameter. Kumar *et al.*[14] examined shear wave propagation considering micropolar elastic half-space and piezoelectric layer under the effects of initial stresses with imperfect interface. Kumar and Madan[15] discussed effects of imperfectness and sandy parameter on Love wave propagation considering a layer consisting sand particles overlying an orthotropic semi-infinite media. Madan et al. [16] investigated propagation of Rayleigh wave considering orthotropic elastic medium under effects of pre-stresses. An explicit secular equation for perfect and sliding information has been derived.

The effects of imperfect interfacial conditions for Rayleigh wave propagation in sandy media remain unexplored. So, an effort has been made to study Rayleigh wave propagation in a dry sandy layer overlying a dry sandy semi-infinite medium. The interface is assumed to be dislocation-like imperfect as various interfacial conditions significantly affect seismic wave propagation. Dispersion equation has been derived for the Rayleigh wave propagation using appropriate boundary conditions. Particular cases for perfect interface and for isotropic layer have also been discussed. MATLAB software has been used for plotting phase velocity against wave number graphically to demonstrate the significant effects of various parameters involved in the dispersion equation.

Geometry of the Problem

The model comprised of a dry sandy layer of 'h' thickness, overlying a dry sandy semi-infinite medium is considered. Cartesian coordinate system is used with Rayleigh wave assumed to be propagating along the x-direction, z-axis is in the increasing depth direction and origin O located at layer and semi-infinite medium interface. The interface is considered to be dislocation like the imperfect and keeping upper boundary of the layer rigid. The geometry for the considered problem is shown in Figure 1.

For 2-dimensional problem (-xz plane) discussion, we must have displacements components are independent of y *i.e.* $\frac{\partial}{\partial y} \approx 0$ and are zero in -y-direction.

Dynamics of Sandy Layer and Semi-infinite Medium

Dynamical equation of motion without external forces for Rayleigh surface wave propagation with displacement components u_1 and w_1 along x and z direction for a sandy layer is given as (Biot)[17]

$$\frac{\partial \tau_{xx}^1}{\partial x} + \frac{\partial \tau_{xz}^1}{\partial z} = \rho_1 \frac{\partial^2 u_1}{\partial t^2} \tag{1}$$

$$\frac{\partial \tau_{xx}^1}{\partial x} + \frac{\partial \tau_{xz}^1}{\partial z} = \rho_1 \frac{\partial^2 w_1}{\partial t^2} \tag{2}$$

where $\tau_{xx}^1, \tau_{xz}^1, \tau_{zz}^1$ denotes stress components and ρ_1 denotes density of material in sandy layer.

Using stress-displacement relations for the sandy layer,

$$\tau_{xx}^1 = \eta_1 \left\{(\lambda_1 + 2\mu_1)\frac{\partial u_1}{\partial x} + \lambda_1 \frac{\partial w_1}{\partial z}\right\} \tag{3}$$

$$\tau_{zx}^1 = \eta_1 \mu_1 \left(\frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x}\right)\left\{(\lambda_1 + 2\mu_1)\frac{\partial u_1}{\partial x} + \lambda_1 \frac{\partial w_1}{\partial z}\right\} \tag{4}$$

$$\tau_{zz}^1 = \eta_1 \left\{\lambda_1 \frac{\partial w_1}{\partial z} + (\lambda_1 + 2\mu_1)\frac{\partial u_1}{\partial x}\right\} \tag{5}$$

where λ_1 and μ_1 denote Lame's constant, η_1 denotes sandiness parameter.

Using equations (3), (4) and (5) in (1) and (3), we have

$$\eta_1(\lambda_1 + 2\mu_1)\frac{\partial^2 u_1}{\partial x^2} + \eta_1\mu_1\frac{\partial^2 u_1}{\partial z^2} + \eta_1(\lambda_1 + \mu_1)\frac{\partial^2 w_1}{\partial x \partial z} = \rho_1 \frac{\partial^2 u_1}{\partial t^2} \tag{6}$$

$$\eta_1 \frac{\partial^2 w_1}{\partial x^2} + \eta_1(\lambda_1 + 2\mu_1)\frac{\partial^2 w_1}{\partial z^2} + \eta_1(\lambda_1 + \mu_1)\frac{\partial^2 u_1}{\partial x \partial z} = \rho_1 \frac{\partial^2 w_1}{\partial t^2} \tag{7}$$

Now, assume solution of equations (6) and (7) be

$$u_1 = (Ae^{-kpz} + Be^{kpz})\, e^{ik(x-ct)} \tag{8}$$

$$w_1 = (Ce^{-kpz} + De^{kpz})\, e^{ik(x-ct)} \tag{9}$$

Using values of u_1 and w_1 in equations (6) and (7) and separating coefficients of e^{-kpz} and e^{kpz}, we have

$$[\rho_1 c^2 - \eta_1(\lambda_1 + 2\mu_1) + \eta_1\mu_1 p^2]A_1 - i\eta_1(\lambda_1 + \mu_1)pC_1 = 0 \tag{10}$$

$$[\rho_1 c^2 - \eta_1(\lambda_1 + 2\mu_1) + \eta_1\mu_1 p^2]B_1 + i\eta_1(\lambda_1 + \mu_1)pD_1 = 0 \tag{11}$$

$$[\rho_1 c^2 - \eta_1\mu_1 + (\lambda_1 + 2\mu_1)\eta_1 p^2]C_1 - i\eta_1(\lambda_1 + \mu_1)pA_1 = 0 \tag{12}$$

$$[\rho_1 c^2 - \eta_1\mu_1 + (\lambda_1 + 2\mu_1)\eta_1 p^2]D_1 + i\eta_1(\lambda_1 + \mu_1)pB_1 = 0 \tag{13}$$

Writing equations (10)-(13) in determinant form in order to eliminate A_1,B_1,C_1 and D_1, we must have:

$$\begin{vmatrix} \rho_1 c^2 - \eta_1(\lambda_1 + 2\mu_1) + \eta_1\mu_1 p^2 & 0 & -i\eta_1(\lambda_1 + \mu_1)p & 0 \\ 0 & \rho_1 c^2 - \eta_1(\lambda_1 + 2\mu_1) + \eta_1\mu_1 p^2 & 0 & i\eta_1(\lambda_1 + \mu_1)p \\ -i\eta_1(\lambda_1 + \mu_1)p & 0 & \rho_1 c^2 - \eta_1\mu_1 + (\lambda_1 + 2\mu_1)\eta_1 p^2 & 0 \\ 0 & i\eta_1(\lambda_1 + \mu_1)p & 0 & \rho_1 c^2 - \eta_1\mu_1 + (\lambda_1 + 2\mu_1)\eta_1 p^2 \end{vmatrix} = 0$$

On expanding the determinant, we obtain a biquadratic equation in p, given as

$$\eta_1^2 \frac{\beta_1^2}{\alpha_1^2} p^4 + \left\{\eta_1^2\left(1 - \frac{\beta_1^2}{\alpha_1^2}\right)^2 + \eta_1\left(\frac{c^2}{\alpha_1^2} - \eta_1\right) + \eta_1 \frac{\beta_1^4}{\alpha_1^4}\left(\frac{c^2}{\beta_1^2} - \eta_1\right)\right\}p^2 + \frac{\beta_1^2}{\alpha_1^2}\left(\frac{c^2}{\alpha_1^2} - \eta_1\right)\left(\frac{c^2}{\beta_1^2} - \eta_1\right) = 0 \tag{14}$$

where, $\alpha_1^2 = \frac{(\lambda_1 + 2\mu_1)}{\rho_1}$ and $\beta_1^2 = \frac{\mu_1}{\rho_1}$.

Assume $\pm p_1$ and $\pm p_2$ be solution of equation (14), then displacement expression in equations (8) and (9) can be written as:

$$u_1 = (A_1 e^{-kp_1 z} + A_2 e^{-kp_2 z} + B_1 e^{kp_1 z} + B_2 e^{kp_2 z})\, e^{ik(x-ct)} \tag{15}$$

$$w_1 = (n_1 A_1 e^{-kp_1 z} + n_2 A_2 e^{-kp_2 z} - n_1 B_1 e^{kp_1 z} - n_2 B_2 e^{kp_2 z})\, e^{ik(x-ct)} \tag{16}$$

$$n_j = \frac{\frac{c^2}{\alpha_1^2} - \eta_1 + \eta_1 \frac{\beta_1^2}{\alpha_1^2} p_j^2}{i\,\eta_1\left(1 - \frac{\beta_1^2}{\alpha_1^2}\right)p_j} \ (j = 1,2), \text{ with } p_1 = \sqrt{1 - \frac{c^2}{\beta_1^2}} \ and \ p_2 = \sqrt{1 - \frac{c^2}{\alpha_1^2}} \tag{17}$$

In a similar manner displacement expression for sandy semi-infinite medium can be written as:

$$u_2 = (E_1 e^{-kq_1 z} + E_2 e^{-kq_2 z} + F_1 e^{kq_1 z} + F_2 e^{kq_2 z})\, e^{ik(x-ct)} \tag{18}$$

$$w_2 = (n_1' E_1 e^{-kq_1 z} + n_1' E_2 e^{-kq_2 z} - n_1' F_1 e^{kq_1 z} - n_2' F_2 e^{kq_2 z})\, e^{ik(x-ct)} \tag{19}$$

$$n'_j = \frac{\frac{c^2}{\alpha_2^2} - \eta_2 + \eta_2 \frac{\beta_2^2}{\alpha_2^2} q_j^2}{i\,\eta_2\left(1 - \frac{\beta_2^2}{\alpha_2^2}\right) q_j} \ (j = 1,2), \text{ with } q_1 = \sqrt{1 - \frac{c^2}{\beta_2^2}} \ and \ q_2 = \sqrt{1 - \frac{c^2}{\alpha_2^2}} \quad (20)$$

Also, we have displacement vanishing as the depth increases, i.e. $u_2, w_2 \to 0$ as $z \to \infty$. So, F_1 and F_2 must be zero. Then from equations (18) and (19):

$$u_2 = \left(E_1 e^{-kq_1 z} + E_2 e^{-kq_2 z}\right) e^{ik(x-ct)} \quad (21)$$

$$w_2 = \left(n'_1 E_1 e^{-kq_1 z} + n'_2 E_2 e^{-kq_2 z}\right) e^{ik(x-ct)} \quad (22)$$

Equations (21) and (22) represents expressions for displacement components for the semi-infinite medium.

Boundary Conditions and Dispersion Equation

To examine Rayleigh wave propagation in a dry sandy layer overlying a sandy semi-infinite medium with dislocation like imperfection present at the interface, the boundary conditions involving vanishing of stresses, continuity of displacements and stresses may be presented as:

- Displacement vanishes at rigid upper boundary plane. i.e. $u_1 = 0$ and $w_1 = 0$ at $z = -h$.
- Displacement components for semi-infinite medium and layer are proportional at the imperfect interface, i.e., $u_2 = \eta_T u_1$ and $w_2 = \eta_N w_1$ at $z = 0$ where η_T and η_N denotes imperfectness parameters along tangential and normal direction.
- Stresses are continuous at the imperfect interface, i.e., $\tau_{zx}^1 = \tau_{zx}^2$ and $\tau_{zz}^1 = \tau_{zz}^2$ at $z = 0$.

Using these boundary conditions and expressions for displacements and stresses for sandy layer and half-space, we obtain the following set of six homogeneous equations in terms of A_1, A_2, B_1, B_2, E_1 *and* E_2

$$A_1 e^{kp_1 h} + A_2 e^{kp_2 h} + B_1 e^{-kp_1 h} + B_2 e^{kp_2 h} = 0 \quad (23)$$

$$A_1 n_1 e^{kp_1 h} + A_2 n_2 e^{kp_2 h} - B_1 n_1 e^{-kp_1 h} - B_2 n_2 e^{kp_2 h} = 0 \quad (24)$$

$$A_1 \eta_T + A_2 \eta_T + B_1 \eta_T + B_2 \eta_T - E_1 - E_2 = 0 \quad (25)$$

$$A_1 \eta_N n_1 + A_2 n_2 \eta_N - B_1 n_1 \eta_N - B_2 n_2 \eta_N - n'_1 E_1 - n'_2 E_2 = 0 \quad (26)$$

$$A_1\eta_1\mu_1(-kp_1 + ikn_1) + A_2\eta_1\mu_1(-kp_2 + ikn_2) + B_1\eta_1\mu_1(kp_1 - ikn_1) + B_2\eta_1\mu_1(kp_2 - ikn_2) + E_1\,\eta_2\mu_2(kq_1 - ikn'_1) + E_2\eta_2\mu_2(kq_2 - ikn'_2) = 0 \quad (27)$$

$$A_1\eta_1\{\lambda_1 ik - (\lambda_1 + 2\mu_1)n_1 kp_1\} + A_2\eta_1\{\lambda_1 ik - (\lambda_1 + 2\mu_1)n_2 kp_2\} + B_1\eta_1\{\lambda_1 ik - (\lambda_1 + 2\mu_1)n_1 kp_1\} + B_2\eta_1\{\lambda_1 ik - (\lambda_1 + 2\mu_1)n_2 kp_2\} - E_1\eta_2\{\lambda_2 ik - (\lambda_2 + 2\mu_2)n'_1 kq_1\} - E_2\eta_2\{\lambda_2 ik - (\lambda_2 + 2\mu_2)n'_2 kq_2\} = 0 \quad (28)$$

For non-trivial solution of this homogeneous system from (23) - (28), we must have,

$a_{ij} = 0$, i, j = 1 to 6. (29)

where a_{ij} are coefficients of A_1, A_2, B_1, B_2, E_1 and E_2 in the system of six equations represented by equations (23) - (28).

The real part of equation (29) represents the dispersion frequency equation for Rayleigh wave propagating in a dry sandy layer overlying a sandy semi-infinite medium with imperfect interface.

Special Cases

Case I. When $\eta_T = \eta_N = 1$, then equation (29) becomes frequency equation for propagation of Rayleigh wave in sandy layer overlaying a sandy semi-infinite medium.

Case II. When $\eta_1 = \eta_2 = 1$, then equation (29) becomes frequency equation for propagation of Rayleigh wave in an isotropic elastic layer overlaying an isotropic semi-infinite medium with in- terface assumed to be imperfect.

Case III. When $\eta_1 = \eta_2 = \eta_T = \eta_N = 1$, then equation (13) becomes frequency equation for Rayleigh wave propagation in an isotropic elastic layer overlying an isotropic semi-infinite medium with perfect interface.

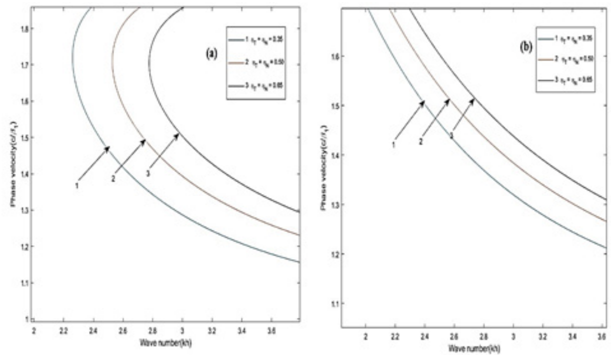

Figure 2: Imperfectness parameter effect, using variations of phase velocity (c/β1) against wave number (kh) for (a) sandy media case, (b) for isotropic elastic case, $\eta_1 = \eta_2 = 1$.

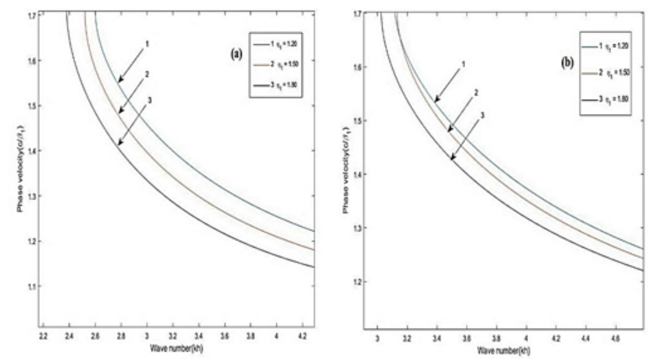

Figure 3: Sandy parameter (η_1) effect, using phase velocity (c/β1) variations against wave number(kh) for (a) imperfect interface, (b) perfect interface.

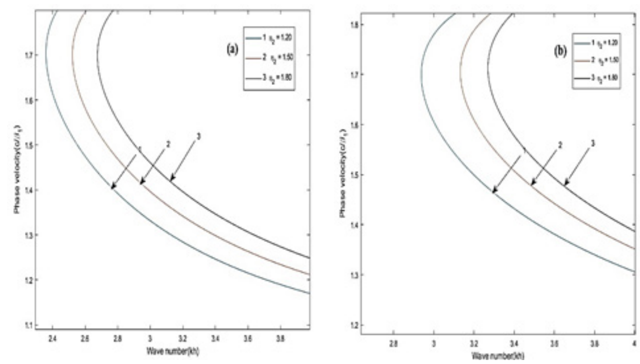

Figure 4: Effects of sandy parameter (η_2) using phase velocity (c/β1) variations against wave number (kh) for (a) imperfect, (b) perfect interface.

Numerical Computations and Discussion

Propagation of Rayleigh wave considering a geometry comprised of a sandy layer and sandy semi-infinite medium has been examined with the interface assumed to be dislocation like imperfect. The derived frequency equation, involving various parameters like imperfectness and sandiness, has been derived for the considered model. To examine the effects of various parameters, following data has been considered:

For the dry sandy layer (Gubbins [18])

$\rho_1 = 2300000\ Kg/m^3, \mu_1 = 6\ Gpa, \lambda_1 = 9$ Gpa.

For the dry sandy half-space (Pal *et al.* [8]):

$\rho_2 = 7800\ Kg/m^3, \mu_2 = 5.66\ Gpa, \lambda_2 = 2.46$ Gpa.

Graphs are plotted to show the effects of imperfectness, sandiness parameter for layer and half-space on phase velocity variation against wave number using real part of equation (29) with help of MATLAB software.

Figure 2 shows the effects of the imperfectness parameter (η_T, η_N). Variation of phase velocity against wave number to observe the effects of imperfectness parameter is shown for the two cases, i) for the sandy layer and sandy semi-infinite medium and ii) when layer and half-space becomes isotropic, i.e. $\eta_1 = \eta_2 = 1$. Three different values of the imperfectness parameter (0.35, 0.50 and 0.65) are used for plotting with values of η_1 and η_2 taken as 1.5 for Figure 2(a). It can be depicted that phase velocity increased with the increment in the imperfectness factor, for both cases.

Figure 3 shows the effects of sandiness factor (η_1) for the layer. Variation is shown for the imperfect and perfect interface, respectively. Phase velocity is plotted against wavenumber using three values of η_1 = 1.20, 1.50 and 1.80 with value of η_T and η_N = 0.5 for Figure 3(a). Sandiness parameter for the layer decreased the phase velocity of the Rayleigh wave for both the imperfect and perfect interface but with a slightly different behavior.

Figure 4 shows the effects of the sandiness parameter (η_1) for the half-space. Variation is shown for the imperfect and perfect interface, respectively. Plotting has been done using three different values of η_2 = 1.20, 1.50 and 1.80 with value of η_T and η_N = 0.5 for Figure 4(a). It has been found that sandiness parameter for the semi-infinite medium enhanced the Rayleigh wave phase velocity.

Conclusion

A mathematical analysis of Rayleigh wave propagation in dry sandy layer overlying dry sandy half-space has been studied with the interface assumed to be dislocation like imperfect. The dispersion frequency equation has been obtained by applying appropriate boundary conditions for the geometry. Graphs have been plotted to show the impacts of imperfectness and sandiness factor ($\eta_1 = \eta_2$) on Rayleigh waves phase velocity. The conclusions for the present study may be summarized as:

- Imperfectness parameter enhances the phase velocity of Rayleigh wave for the dry sandy layer and isotropic elastic layer case.
- Sandiness parameter for the layer affected inversely the phase velocity. As sandiness parameter increases, phase velocity decreases both for imperfect and perfect interface case.
- Sandiness parameter for the semi-infinite medium also significantly effected the phase velocity by enhancing the phase velocity with enhancement in sandiness parameter for both imperfect and perfect interface.

These theoretical studies regarding seismic wave propagation considering layered media have various applications in geophysics, civil engineering and understanding the effects and causes of earthquakes. Rayleigh wave causes more damage during earthquakes. The present results could be used to connect modeling results to field applications.

References

[1] Rayleigh, L. (1885). On waves propagated along the plane surface of an elastic solid. *Proceedings of the London Mathematical Society*, *1*, 4–11.https://doi.org/10.1112/plms/s1-17.1.4

[2] Bromwich, T. J. (1898). On the influence of gravity on elastic waves, and, in particular on the vibrations of an elastic globe. *Proceedings of the London Mathematical Society, 1*, 98–165. https://doi.org/10.1112/plms/s1-30.1.98

[3] Weiskopf, W.H. (1945). Stresses in soils under foundation. *Journal of Franklin Institute, 239*I, 445-465. https://doi.org/10.1016/0016-0032(45)90189-X

[4] Kar, B. K., Pal, A.K., & Kalyani, V.K. (1986). Propagation of Love waves in an irregular dry sandy layer. *Acta Geophysica Polonica, 34*, 157–170.

[5] Abd-Alla, A. (1999). Propagation of Rayleigh waves in an elastic half-space of orthotropic material. *Applied Mathematics and Computation, 99*, 61–69. https://doi.org/10.1016/S0096-3003(97)10170-9

[6] Abd-Alla, A., Hammad, H., & Abo-Dahab S. (2004). Rayleigh waves in a magnetoelastic half-space of orthotropic material under influence of initial stress and gravity field. *Applied Mathematics and Computation, 154(2)*, 583–597. DOI: 10.1016/S0096-3003(03)00767-7

[7] Vishwakarma, S. K., & Gupta, S. (2014). Rayleigh wave propagation: a case wise study in a layer over a half space under the effect of rigid boundary. *Archives of Civil and Mechanical Engineering, 14(1),* 181–189. http://dx.doi.org/10.1016/j.apm.2016.05.020

[8] Pal, P.C., Kumar, S., & Bose, S. (2015). Propagation of Rayleigh waves in anisotropic layer overlying a semi-infinite sandy medium. *Ain Shams Engineering Journal, 6*, 621-627. https://doi.org/10.1016/j.asej.2014.11.003

[9] Sahu, S. A., Chaudhary S., Saroj, P. K., & Chattopadhyay, A. (2004). Rayleigh waves in liquid layer resting over an initially stressed orthotropic half-space under self-weight. *Arabian Journal of Geosciences, 10(5).* DOI:10.1007/s12517-017-2924-1

[10] Mandi, A., Kundu, S., Pati, P., & Pal, P.C. (2020). An analytical study on the Rayleigh wave generation in a stratified structure. *Applied Mathematics and Mechanics (English Edition).* https://doi.org/10.1007/s10483-020-2625-9

[11] Hua, L., Jia-ling, Y., & Kai-Xin, L. (2006). Love waves in layered graded composite structures with imperfectly bonded interface. *Chin J Aeronaut, 20 (3),* 210-214. https://doi.org/10.1016/S1000-9361(07)60034-X

[12] Vishwakarma, S.K., & Runzhang, X. (2016). Rayleigh wave dispersion in an irregular sandy Earth's crust over orthotropic mantle. *Applied Mathematical Modelling,* 1-13. https://doi.org/10.1016/j.apm.2016.05.020

[13] Sharma, V., & Kumar, S. (2017). Dispersion of SH waves in a viscoelastic layer imperfectly bonded with a couple stress substrate. *Journal of Theoretical and Applied Mechanics, 55 (2),* 535-546. DOI: https://doi.org/10.15632/jtam-pl.55.2.535

[14] Kumar, R., Singh, K., & Pathania, D. S. (2019). Shear waves propagation in an initially stressed piezoelectric layer imperfectly bonded over a micropolar elastic half space. *Structural Engineering and Mechanics, 69(2),*121-129. https://doi.org/10.12989/sem.2019.69.2.121

[15] Kumar, N., & Madan, D.K. (2021). Propagation of Love waves in dry sandy medium laying over orthotropic semi-infinite medium with imperfect interface. *International Journal of Grid and Distributed Computing, 14(1),* 2057-2064.

[16] Madan, D. K., Rani, A., & Punia, M. (2021). A note on the effect of rigidity and initial stress on the propagation of Rayleigh waves in prestressed orthotropic elastic layered medium. *Proceedings of the Indian National Science Academy, 87*, 487-498. https://doi.org/10.1007/s43538-021-00044-3

[17] Biot, M.A. (1965). Mechanics of Incremental Deformations. *John Wiley and Sons, New York*.

[18] Gubbins, D. (1990). Seismology and Plate Tectonics. *Cambridge University Press, London*.