CASE STUDY

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Non Newtonian Model for Two Phase Blood Flow in Hepatic Arterioles in Case of Dengue using Herschel – Bulkley Law

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Abstract

Herschel-Bulkley Non-Newtonian model has been studied taking blood flow in arterioles during dengue which have two phases. First one is a blood plasma and other one is RBC. Equation of continuity and equation of motion for two phase blood flow have been derived in tensorial form with respect to blood flow in hepatic circulatory sub-system. Model has been applied in biomechanical setup. A clinical data is taken in the case of dengue. Numerical methods are followed for finding the parameter. Relationship between hematocrit and blood pressure drop is also showed by graph.

Keywords: Blood pressure drop, Hematocrit, non Newtonian fluid, Power law model, Two phase blood.

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Introduction

Blood has anomalous viscous properties mainly due to suspension of cells in plasma. This anomaly produced due to 'low shear' and 'high shear'.[1] It has been revealed that blood flows in smaller blood vessels at lower shear rates. Here the finite yield stress is generated and blood behaves like a Non Newtonian fluid,[1,2] two fluid Newtonian model have been considered by Hynes[3] and Bugliarello and Sevilla[4] with both fluids as Newtonian fluid and different viscosities. When blood flow through larger diameter arteries at high shears rates, it behaves like a Newtonian fluid. With decreasing blood vessel diameter, apparent viscosity of blood decreases for diameter range 20. The apparent viscosity increases as the blood vessel diameter decreases and it shows a non-Newtonian character. This non-Newtonian character of blood gets to see in narrow arteries and veins where the cells induce the specific behavior. It has been reported by Tu and Deville[5] that the assumption of Newtonian behavior of blood is acceptable or high shear rate flow e.g in the case of larger arteries. When the shear stress is less than 100/sec blood shows non Newtonian character. Although Newtonian and several non-Newtonian model have been used to study the motion of blood. It is realized that Hershel-Bulkley model describes the behavior of blood very closely. H-B fluid are a class of non-Newtonian fluid that requires a finite stress known as yield stress, in order to deform. Therefore, these material behave like rigid solids when the local shear is below the yield stress. Once the yield stress is exceeded he material flows with non linear stress strain relationship either as a shear thickening fluid or shear thinning. H-B fluid is a non-Newtonian fluid model with yield stress which is generally used to model blood when it flow through narrow arteries (chaturani and Ponnalagar samy1985).

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Structure and Function of Hepatic Arterioles[7,8]

Blood is pumped from ventricles into large elastic arteries that branch repeatedly into smaller and smaller arteries until the branching results in microscopic arteries called arterioles. The arterioles play a key role in regulating blood flow into the tissue capillaries. The diameter of the arterioles is less compared to arteries. This diameter is adjusted to regulate the blood flow. The thickness of wall is also less compared to arteries. Arterioles further divide into capillaries which are the tiny blood vessel that facilitate of water and nutrients between blood and organ. Arterioles are considered as primary resistance vessels .It provide approximately 80% of the total resistance to blood flow through the body. Arterioles are made of three layers these are tunica externa, tunica media, tunica intima.

Dengue and liver[9]

This disease has been found to have profound effect on multiple organ system, the commonest being the liver . Starting from asymptomatic elevated transminase levels to acute liver failure, dengue has all the properties of hepatic illness. With dengue infection, high level of vermia is associated with involvement of deferent organs (liver, brain) in the sever from of the disease. The liver is the commonest organ to be involved in dengue.

Real Model

Blood is non Newtonian fluid containing plasma and cells. 45% of the cells in the blood, the rest is plasma.cells consist 98% of RBCs around 2% WBCs and platelets. which are ignorable with respect to RBC so one phase of the blood plasma and second phase of blood is RBCs. Boundary condition are as follows:

- Blood flow has a maximum velocity at r= 0 will be maximum and finite say
- At r=R (radius of transverse section of vessel)will be zero. It is called as no slip condition.

According to the Sherman I.W. and Sherman V.G. blood is mixed fluid.[10] Mainly there are two phases plasma and blood cells (enclosed with semi permeable membranes) whose density is greater than that plasma .Blood cells are constantly distributed into plasma. So blood can be considered as a homogeneous mixture of two phases.

Upadhyay V. saw the factor affecting the flow of blood. The main reason for this is blood cells flow of blood is affected by the presence of blood cells. This effect is directly related to the volume of blood cells.[11] Choose a unit volume in blood in which cells volume is X where $X = \frac{H}{100}$ and H is hematocrit (volume % of blood cells). Hence the volume portion covered by plasma will be 1-X. If the mass ratio of blood cells to plasma is r then

$$r = \frac{X\rho_c}{(1-X)\rho_p} \quad (2.1)$$

Where P_c and P_p are densities of blood cells and plasma respectively. In general this mass ratio is not constant. In specific blood vessel ,it is constant throughout that vessel .The both phase the blood cells and plasma move with common velocity.

Formulation

Select the orthogonal curvilinear co-ordinate system briefly described as E 3 called as Euclidian space. According to Mishra the biophysical law hold good in any co ordinate system which is a compulsion for the truthfulness.

Equation of continuity for two phase.

$$\frac{\partial(X\rho_c)}{\partial t} + \left(X\rho_c v^i\right)_{,i} = 0 \quad (2.2)$$

$$\frac{\partial(1-X)\rho_p}{\partial t} + \left((1-X)\rho_p v^i\right)_{,i} = 0 \quad (2.3)$$

where V^i is common velocity of two phase blood cells and plasma, $(X\rho_c v^i)_{,i}$ is covariant derivative of $(X\rho_c v^i)$ with respect to X^i and $((1-X)\rho_p v^i)_{,i}$ is covariant derivative of $((1-X)\rho_p v^i)$ with respect to X^i.

If P_m be uniform density of mixture then

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \, [12]$$

where $\rho_m = X\rho_c + (1\text{-}X)\rho_p \quad (2.4)$

from equation (2.2)and (2.3) and using (2.4) we get

$$\frac{\partial \rho_m}{\partial t} + \left(\rho_m v^i\right)_{,i} = 0 \quad (2.5)$$

Equation of motion for two phase blood flow-

According to Ruch T.C. and H.D. The hydro dynamical pressure P between two phases of can be supposed to be uniform because the both phases are always in equilibrium state in blood (1973).[13] By the principle of conservation of momentum the equation of motion of two phase blood cells and plasma

$$X\rho_c \frac{\partial v^i}{\partial t} + \left(X\rho_c v^j\right) v^i_{,j} = -X_{P,j} g^{ij} + X\eta_c \left(g^{jk} v^i_{,k}\right)_{,j} \quad (2.6)$$

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \{(1-X)\rho_p v^j\} v^i_{,j} = -(1-X)_{P,j} g^{ij} + (1-X)\eta_p \left(g^{jk} v^i_{,k}\right)_{,j} \quad (2.7)$$

Christoffel's symbols of second kind for cylindrical co-ordinates

Now adding (2.6) and (2.7) and using (2.4) then equation of motion for blood flow will be

$$\rho_m \frac{\partial v^i}{\partial t} + \left(\rho_m v^j\right) v^i_{,j} = -P_{,j} g^{ij} + \eta_m \left(g^{jk} v^i_{,k}\right)_{,j} \quad (2.8)$$

Where $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity coefficient of blood as a mixture of two phases. Velocity of blood flow depends on viscosity of blood. As blood viscosity increases then velocity of blood flow decreases. Since arterioles are far away from the heart. So the pumping effect of the heart is very low in that vessel[14] and these vessels narrow down more rapidly in this situation blood cells line up on the axis to build up rouleaux. Hence a yield stress is very small even then the viscosity of blood is increases nearly ten times.[15]

The Herschel-Bulkley law hold good on two phase blood flow through the arterioles and whose constitutive equation as follows-

$T^{\circ} = \eta_m e^n + T_p \ (T' > T_p)$ and $e = 0 \ (T^{\circ} < T_p)$ where T_p is yield stress.

When strain rate $e = 0 \ (T^{\circ} < T_p)$ a core region is formed which flow just like a plug. Let radius of plug be r_p and the stress

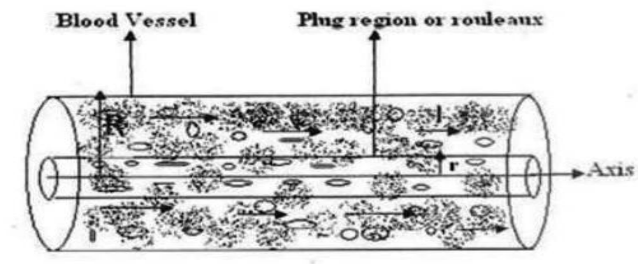

Figure 1: Herschel-Bulkley Blood Flow [Singh J.P,science and education publishing,2015]

acting on the surface of plug will be T_p
Equation of force acting on the plug

$$P\pi r_p^2 = T_p 2\pi r_p \text{ or, } r_p = 2\frac{T_p}{P} \qquad (2.9)$$

The constitutive equation for rest part of blood vessel is $T^{\circ} = \eta_m e^n + T_p$ or, $T^{\circ} - T_p = \eta_m e^n = T_e$ where T_e is effective stress whose generalized form will be $T^{ij} = -pg^{ij} + T_e^{ij}$ where $T_e^{ij} = \eta_m (e^{ij})^n$, where $e^{ij} = g^{jk} v_k^i$

Equation of continuity - $\frac{1}{\sqrt{g(\sqrt{g v^i})_{,i}}} = 0 \quad (2.10)$

Equation of motion - $\rho_m \frac{\partial v^i}{\partial t} + \rho_m v^j v^i_{,j} = -T^{ij}_{e,j} \quad (2.11)$
Where all the symbols have their usual meaning

Solution

Let $X^1 = r, X^2 = \theta, X^3 = z$
$[g_{ij}]$ be matrix of metric tensor and $[g^{ij}]$ be matrix of conjugate matric tensor where

$$[g_{ij}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, [g^{ij}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Metric elements $g_{rr} = 1, g_{\theta\theta} = r^2, g_{zz} = 1$

Or $g_{11} = 1, g_{22} = r^2, g_{33} = 1$

$$\left\{ {1 \atop 22} \right\} = -r, \left\{ {2 \atop 21} \right\} = \left\{ {2 \atop 12} \right\} = \frac{1}{r}$$

Physical components

since $\sqrt{g_{11}} v^1 = v_r$ or, $v_r = v^1$

$\sqrt{g_{22}} v^2 = v_\theta$ or, $v_\theta = rv^2$

$\sqrt{g_{33}} v^3 = v_z or, v_z = v^3$

Again the physical component of $-P_{,j} g^{ij}$ are $-\sqrt{g_{ii}} P_{,j} g^{ij}$
Equation (2.10) and (2.11) are transformed into cylindrical form and solve by power law model we get

$$-\frac{dv}{dr} = \left(\frac{pr}{2\eta_m}\right)^{1/n} \qquad (3.12)$$

replace r from r $-r_p$ for non plug region

Table 1: Clinical data of dengue patient

Date	*Hemoglobin gm/dl*	*B.P mmhg*	*Hematocrit*
25/10/2021	12.3	110/80	36.9
26/10/2021	13.4	120/70	40.2
27/10/2021	13.3	120/80	39.9
29/10/2021	14	110/80	42
30/10/2021	14.3	120/80	42.9

$$\frac{dv}{dr} = -\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (r - r_p)^{\frac{1}{n}} \qquad (3.13)$$

Integrating equation (3.13)

$$v = -\left(\frac{p}{2\eta_m}\right)^{1/n} (r - r_p)^{\frac{1}{n}+1} + C \qquad (3.14)$$

Using no slip boundary condition in (3.14), v = 0 at r = R we get

$$C = \left(\frac{p}{2\eta_m}\right)^{1/n} (R - r_p)^{\frac{1}{n}+1} \qquad (3.15)$$

From (3.14) & (3.15)

$$v_p = \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \frac{n}{n+1} \left[(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1}\right] \quad (3.16)$$

This is velocity of blood flow in arterioles
Putting $r = r_p$ we get the velocity of plug flow as follows

$$v_p = \frac{n}{n+1}\left(\frac{P}{2\eta_m}\right)^{1/n} (R - r_p)^{\frac{1}{n}+1} \qquad (3.17)$$

Where value of r_p taken from equation of motion (2.7)

Result and Discussion

Hemoglobin v/s Blood pressure is taken from Shree Ji Hospital, Karwi (Chitrakoot)
Patient Name : Mr. Anand Mohan Shukla,
Age / Sex : 56 Years / Male
Clin: Dr. Shashank Agrawal

We will find the Pressure drop in arterioles by Table 1

$$\Delta P = \left(\frac{\frac{S+D}{2} + D}{3} - \frac{S+D}{2} \right)$$

The flow flux of two phased blood flow in arterioles

$$Q = \int_0^{r_p} 2\pi r v_p \, dr + \int_{r_p}^{R} 2\pi r v dr$$

$$Q = \int_0^{r_p} 2\pi r \frac{n}{n+1} \left(\frac{p}{2\eta_m}\right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1}$$

$$dr + \int_{r_p}^{R} 2\pi r \frac{n}{n+1} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \left[(R - r_p)^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1}\right] dr$$

Using (3.16) & (3.17) then

Table 2: Blood pressure drop vs hematocrit

Date	*Hematocrit(H)*	*B.P Drop(ΔP)*
25/10/2021	36.9	0.311047
26/10/2021	40.2	0.337627
27/10/2021	39.9	0.335213
29/10/2021	42	0.352111
30/10/2021	42.9	0.359353

$$Q=\frac{2\pi n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}(R-r_p)^{\frac{1}{n}+1}\left[\frac{r^2}{2}\right]_0^{r_p}+\frac{2\pi n}{n+1}\left(\frac{P}{2\eta_m}\right)^{1/n}\left[\frac{r^2}{2}(R-r_p)^{\frac{1}{n}+1}-\frac{r(r-r_p)^{1/n+2}}{\frac{1}{n}+2}+\frac{r(r-r_p)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)}\right]_{rp}^{R}$$

$$Q=\frac{\pi n}{n+1}\left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}}(R)^{\frac{1}{n}+3}\left[\frac{r_p^2}{R^2}\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}+1}+\left(1+\frac{r_p}{R}\right)\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}+2}+\frac{2\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)}-\frac{2\left(1-\frac{r_p}{R}\right)^{\frac{1}{n}+2}}{\left(\frac{1}{n}+2\right)}\right] \quad (3.18)$$

Now let $R = 1 \; and \; r_p = 1/3$

$$Q=\frac{2\pi}{27}\left(\frac{P}{3\eta_m}\right)^{1/n}\left[\frac{26n^3+33\,n^2+9n}{6n^3+11\,n^2+6n+1}\right]$$

$$\frac{27Q}{2\pi}=\left(\frac{P}{3\eta_m}\right)^{1/n}\left[\frac{26n^3+33\,n^2+9n}{6n^3+11\,n^2+6n+1}\right]$$

Now $Q = 1000 \; ml/min = 0.016661 \; lit/sec$

According to Gustafson, Daniel R.(1980)

$$\eta_p = 0.0015 \; pascal \; sec$$

$$\eta_m = 0.035 \; pascal \; sec$$

By equation

$$\eta_m = 0.0008524 \; H + 0.0015$$

After putting the value of $\pi, \eta_m, \Delta P$

$$0.071627 = \left(\frac{5332.9}{3 \times 0.035}\right)^{\frac{1}{n}}\left(\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right)$$

Solve by Numerical method

$$n = \; -2.526234$$

Terminal hepatic arterioles length = 50 micrometer[15]

$$z_f - z_i = 0.00005 \; meter$$

Let $B = \left(\frac{26n^3+33n^2+9n}{6n^3+11n^2+6n+1}\right)$

Further by equation pressure drop $\Delta P = 3\eta_m\left(\frac{27Q}{2\pi B}\right)^n \Delta Z$

$$\Delta P = 3(0.0008524\,H + 0.0015)\left(\frac{27 \times 0.016661}{2 \times 3.14 \times 5.684741}\right)^{-2.526234} \times 0.00005$$

$$\Delta P = 0.00804649 \; H + 0.014158932$$

After putting value of H in above equation we will form the table of blood pressure drop versus Hematocrit.

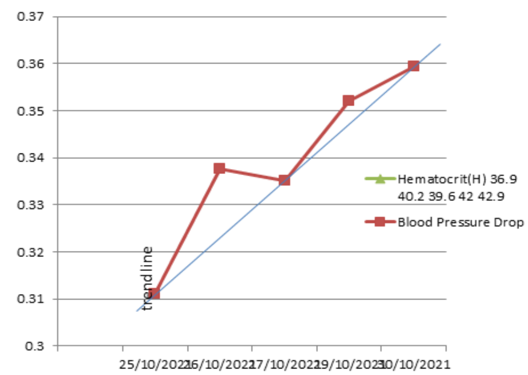

Figure 2: Graph of Hematocrit v/s Blood Pressure Drop [Pandey B.,2022]

Further we will analyse our hematocrit and blood pressure drop by grapg which will be drawn with the help of Table 2

Conclusion

After calculation we found a linear relationship between hematocrit and blood pressure drop $\Delta P = 0.00804649 \; H + 0.014158932$ that implies both quatity are connected each other. By graphical analysis of above graph, hematocrit is increasing with increasing date. Due to increasing sense of graph we will not give the high dose of drug to dengue patient. In this graph straight line shows that hematocrit is directly proportional to blood pressure drop.

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