RESEARCH ARTICLE

SAMRIDDHI Volume 15, Issue 2, 2023 Print ISSN: 2229-7111 Online ISSN: 2454-5767

The Stability of Shear Flow of Viscous Electrically Conducting Fluid in the Presence of Velocity and Magnetic Shears

Vijayalakshmi A.R.

Department of Mathematics, Maharani's Science College for Women, Bangalore, Karnataka, India.

Abstract

The linear stability of the shear flow of an incompressible, viscous, electrically conducting fluid permeated by the sheared magnetic field is investigated. An unbounded two-layer model consisting of different viscosity and magnetic diffusivity fluids with different velocity shear and magnetic shears is examined for two-dimensional disturbances. An analytical study using the short wavelength approximation shows that the configuration is always unstable for different diffusivities and for different shears. When the magnetic field does not vanish on the interface, it may stabilize or destabilize the system depending on the values of certain parameters.

Keywords: Stability, Shear flow, Electrically conducting fluid, Unbounded two-layer model magnetic shear, Short wavelength approximation.

SAMRIDDHI : A Journal of Physical Sciences, Engineering and Technology (2023); DOI: 10.18090/samriddhi.v15i02.04

Introduction

When a fluid is driven away from thermal and mechanical equilibrium, it will undergo a sequence of instabilities, each leading to a change in spatial or temporal structure. The transition from laminar to turbulent flow begins with the instability of flow. The stability of shear flows in the presence of applied magnetic field is important in geophysics and astrophysics. This study is based on the previous investigations by Drazin & Reidl[3], Hooper & Boyd[4], Yih[6]. It was shown by Yih [6] that instability can occur when two co-flowing fluids have different viscosities. Hooper & Boyd [4] considered the stability analysis of two unbounded linear viscous shear flows with different shears, and showed that the configuration is unstable when the fluids are of different viscosities and shears. In the presence of a discontinuity in the electrical conductivity, a sheared magnetic field can give rise to a new instability. For two superimposed fluids of different electrical conductivities, it was shown by Sneyd[5], Davidson & Lindsay[2], and Bhattacharya & Gupta[1] configuration is unstable when there is continuous or discontinuous variation in electrical conductivity. In this study, we considered the combined effect of velocity and magnetic shears on the stability of the interface formed by two unbounded shear flows of different viscosity and electrical conductivities and have determined the criteria for the growth rate, using a regular perturbation analysis.

Mathematical Formulation

We consider the two co-flowing viscous unbounded electrically conducting shear flows separated by the interface at $y = 0$ in the presence of sheared magnetic field as shown in Figure 1. We assume both the fluids are incompressible. The governing equations in each fluid are

$$\frac{\partial \vec{q}}{\partial t'} + (\vec{q}.\nabla)\vec{q} = -\frac{1}{\rho}\nabla P + \frac{1}{\rho \mu_0}(\vec{B}.\nabla)\vec{B} + \nu \nabla^2 \vec{q} \quad (1)$$

$$\nabla.\vec{q} = 0 \quad (2)$$

$$\frac{\partial \vec{B}}{\partial t'} + (\vec{q}.\nabla)\vec{B} = (\vec{B}.\nabla)\vec{q} + \lambda \nabla^2 \vec{B} \quad (3)$$

$$\nabla.\vec{B} = 0 \quad (4)$$

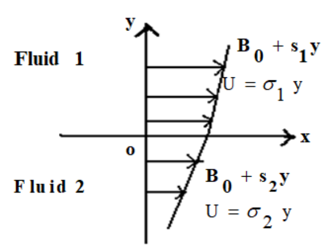

Figure 1: A sketch of the physical problem.

Corresponding Author: Vijayalakshmi A.R., Department of Mathematics, Maharani's Science College for Women, Bangalore, Karnataka, India, e-mail: drarv@rediffmail.com

How to cite this article: Vijayalakshmi A.R. (2023). The Stability of Shear Flow of Viscous Electrically Conducting Fluid in the Presence of Velocity and Magnetic Shears. *SAMRIDDHI : A Journal of Physical Sciences, Engineering and Technology,* 15(2), 206-209.

Source of support: Nil

Conflict of interest: None

where $P = p + \frac{\mu_m H^2}{2}$ is the total pressure. μ_0 is the magnetic permeability of the fluid. ρ, ν and λ are the density, kinematic viscosity and magnetic diffusivity of the fluid, $\vec{q}$ is the fluid velocity, $\vec{B}$ is the magnetic field.

In the unperturbed state, $\vec{q} = (U(y) = \sigma y', 0)$, $\vec{B} = (B_0 + s y', 0)$. $P = P_0$, where σ, s, B_0, P_0 are constants, σ and s are respectively the shear intensity of the mean flow and the magnetic shear intensity. We now consider a two-dimensional perturbation given by $\vec{q} = (\sigma y' + u', v')$, $P = P_0 + P'$ $\vec{B} = (B_0 + s y' + b_x', b_y')$. In the unperturbed state, the balance of normal stress and continuity of the tangential component of the magnetic field required

$$(B_0)_1 = (B_0)_2, (P_0)_1 = (P_0)_2. \qquad (5)$$

Whereas the continuity of the tangential component of the electric field and velocity field require

$$\lambda_1 s_1 = \lambda_2 s_2, \ \mu_1 \sigma_1 = \mu_2 \qquad (6)$$

where s_1, s_2 are magnetic shears, σ_1, σ_2 are velocity shears, ν_1, ν_2 are viscous diffusivities and λ_1, λ_2 are magnetic diffusivities in fluid 1 and 2, respectively.

Let $v' = \eta'(x', t')$ be the equation of the interface between the two fluids in the perturbed state. The kinematic boundary condition at the interface, after linearizing and assuming normal modes as before, gives

$$v' = -i\alpha' c' \eta' \text{ at } y' = 0. \qquad (7)$$

We now impose the requirement of continuity of the tangential and normal components of velocity, shear and normal stress, tangential and normal components of the magnetic field and tangential component of the electrical field at the perturbed interface. We linearize and assume normal modes each of the dependent variables with (x, t') - dependence in the form $\exp[i\alpha'(x' - c't')]$, We now introduce a stream function $\psi(y')$ and $\phi(y')$ for the magnetic field such that

$$u' = \frac{d\psi}{dy'},\ v' = -\frac{d\psi}{dx'},\ b_x' = \frac{d\phi}{dy'},\ b_y' = -\frac{d\phi}{dx'} \qquad (8)$$

The linearized equations by eliminating pressure reduce to

$$i\alpha'(\sigma y' - c')\left(\frac{d^2}{dy'^2} - \alpha'^2\right) = \left(\frac{d^2}{dy'^2} - \alpha'^2\right)^2 \psi + \frac{i\alpha'}{\rho\mu_0}(B_0 + s y')\left(\frac{d^2}{dy'^2} - \alpha'^2\right)\phi \qquad (9)$$

$$i\alpha'(\sigma y' - c')\phi = i\alpha'(B_0 + s y')\psi + \lambda\left(\frac{d^2}{dy'^2} - \alpha'^2\right)\phi \qquad (10)$$

The requirement that the perturbations vanish as $y' \to \pm\infty$, together, constitute the eigenvalue problem governing linear stability.

The length scale L and time scale T are defined as $L = \left(\frac{\lambda_2^2 \mu_0 \rho_2}{s_2^2}\right)^{\frac{1}{4}}$, $T = \frac{(\mu_0 \rho_2)^{\frac{1}{2}}}{s_2}$ and non-dimensional variables are defined as $\left(X, Y, \frac{1}{\alpha}\right) = \left(\frac{s_2^2}{\lambda_2^2 \mu_0 \rho_2}\right)^{1/4}\left(x', y', \frac{1}{\alpha'}\right)$

$$(\varphi_1, \varphi_2) = \left(\frac{1}{\mu_0 \rho_2}\right)^{1/2}(\phi_1', \phi_2'),\ (\psi_1, \psi_2) = \left(\frac{1}{\mu_0 \rho_2}\right)^{1/2}(\psi_1', \psi_2').$$

$$C = \left(\frac{\mu_0 \rho_2}{\lambda_2^2 s_2^2}\right)^{1/4} c. \qquad (11)$$

The rescaled coordinates and a rescaled phase speed are defined by

$$(x, y) = \alpha(X, Y),\ C_1 = \alpha C \qquad (12)$$

Substituting the above into equations (8) and (9) and writing the equations separately for the two fluids, we have

$$\left(\frac{d^2}{dy^2} - 1\right)^2 \psi_1 = -\frac{i}{\alpha^2}\frac{m}{rP_2}\left\{r(\alpha M + \chi y)\varphi_1 + (C_1 - N_1 Q y)\left(\frac{d^2}{dy^2} - 1\right)\psi_1\right. \qquad (13)$$

$$\left(\frac{d^2}{dy^2} - 1\right)^2 \psi_2 = -\frac{i}{\alpha^2}\frac{m}{rP_2}\left\{r(\alpha M + y)\varphi_2 + (C_1 - N_2 Q y)\left(\frac{d^2}{dy^2} - 1\right)\psi_2\right. \qquad (14)$$

$$\left(\frac{d^2}{dy^2} - \phi\right)\varphi_1 = -\frac{i}{\alpha^2}\chi\{(\alpha M + \chi y)\ \ + (C_1 - N_1 Q y)\ \varphi_1\} \qquad (15)$$

At $y' = 0$, it follows that $\psi_1 = \psi_2$ and $\phi_1 = \phi_2$

$$\left(\frac{d^2}{dy^2} - \phi\right)\varphi_2 = -\frac{i}{\alpha^2}\{(\alpha M + y)\ \ _2 + (C_1 - N_2 Q y)\ \varphi_2\} \qquad (16)$$

Here

$$m = \frac{\mu_2}{\mu_1} = \frac{\sigma_1}{\sigma_2},\ r = \frac{\rho_2}{\rho_1},\ \chi = \frac{\lambda_2}{\lambda_1} = \frac{s_1}{s_2},\ P_2 = \frac{\gamma_2}{\gamma_1},\ N_2 = \frac{\sigma_2}{s_2},$$

$$M = \frac{B_0}{(\mu_0 \rho_2 \lambda_2^2 s_2^2)^{1/2}},\ Q = (\mu_0 \rho_2)^{1/2}\ S = \frac{T}{\rho_2 \lambda_2}\left(\frac{\mu_0 \rho_2}{\lambda_2^2 s_2^2}\right)^{1/4} \qquad (17)$$

Where M and S are the magnetic and surface tension parameters. Further $\psi_1 \to 0$, $\varphi_1 \to 0$ as $y \to \infty$;

$$\psi_2 \to 0,\ \varphi_2 \to 0 \text{ as } y \to \infty. \qquad (18)$$

A Regular Perturbation Analysis for Short Wavelength

From the equations (13) – (16) and the boundary conditions it is evident that $1/\alpha^2$ can be used as an expansion parameter for carrying out a regular perturbation analysis for disturbances of short wavelength. Accordingly, we assume the expansions.

$$\psi_1(y) = \sum_{n=0}^{\infty}\frac{a_n(y)}{\alpha^{2n}}e^{-y},\ \psi_2(y) = \sum_{n=0}^{\infty}\frac{b_n(y)}{\alpha^{2n}}e^{y},\ \varphi_1(y) = \sum_{n=0}^{\infty}\frac{g_n(y)}{\alpha^{2n}}e^{-y},$$

$$\varphi_2(y) = \sum_{n=0}^{\infty}\frac{d_n(y)}{\alpha^{2n}}e^{y},\ C_1 = \sum_{n=0}^{\infty}\frac{c_n}{\alpha^{2n}} \qquad (19)$$

We substitute from (19) into (13)-(16) and (17), to obtain the zeroth order solutions of the problem

$$a_0(y)=0,\ b_0(y)=0,\ g_0(y)=K_0,\ d_0(y)=K_0, \quad (20)$$

where K_0 is a non-zero constant. We find

We find that $C_0=0$ (21)

The first-order perturbation solutions are

$$a_1(y)=c_1+c_2\,y-\frac{i\,m^2}{P_2}(\alpha M+\chi y)\,K_0 \quad (22)$$

$$b_1(y)=c_3+c_4\,y-\frac{i\,m^2}{P_2}(\alpha M+y)\,K_0, \quad (23)$$

$$g_1(y)=c_5+c_6\,y-i\div\left(C_1-N_1Q\,y\right)K_0, \quad (24)$$

$$d_1(y)=c_7+c_8\,y+i\left(C_1-N_1Q\,y\right)K_0 \quad (25)$$

Using the boundary conditions, we determine the eigenvalue C1 , given by

$$C_1=\frac{i}{4P_2}\left(\frac{m}{1+m}\right)\left(2\alpha M(\chi-1)+(\chi-1)^2-2S\alpha^3+2P_2\left(\frac{1-m}{1+m}\right)\left(1-\frac{m^2}{r}\right)-2P_2S\alpha^3\right) \quad (26)$$

It can be seen that at $O\left(\alpha^{-2}\right)$ the configuration will be stable or unstable depending on whether

$$\left(2\alpha M(\chi-1)+(\chi-1)^2-2S\alpha^3-2P_2S\alpha^3+2P_2\left(\frac{1-m}{1+m}\right)\left(1-\frac{m^2}{r}\right)\right)<\text{ or }>0 \quad (27)$$

This shows that if $S=0, M=0, \nu=0$ configuration is always unstable provided $\chi\neq 1$, when the magnetic field does not

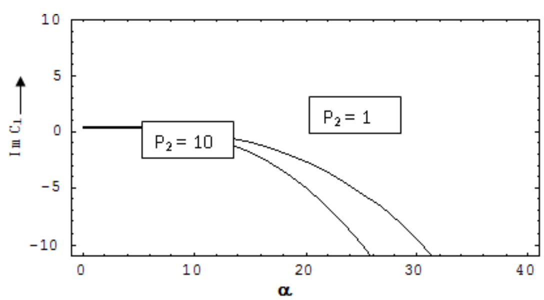

Figure 2: Growth rate Im (C_1) vs. wavenumber α for different values of magnetic diffusivity ratios χ with r = 1, m = 2, S = 0 , M = 0 and $P_2 = 0.025$.

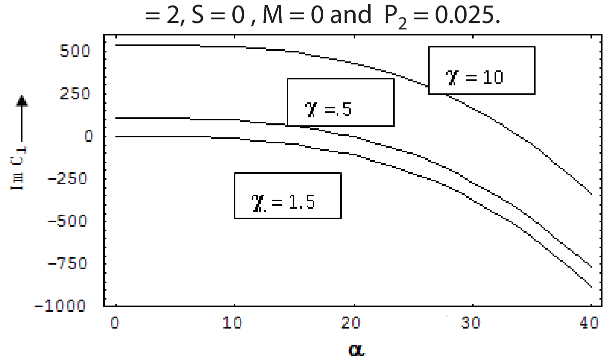

Figure 3: Growth rate Im (C_1) vs. wavenumber α for different magnetic Prandtl numbers P_2 with r = 1, m = 2, χ = 2, S = 0.001 and M = 0

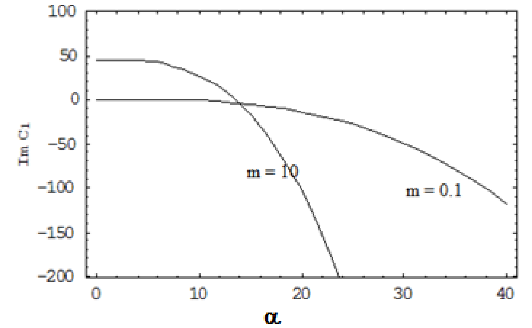

Figure 4: Growth rate Im (C_1) vs. wavenumber α for different viscosity ratios m with r = 1, χ = 2, S = 0.001 and M = 0 and $P_2 = 0.025$.

vanish. If $M\neq 0$ it may have a stabilizing or destabilizing effect depending on the sign of M. The dimensional growth rate, correct to first order in $1/\alpha^2$ is given by

$$\alpha C=\left(\frac{s_2^2}{\mu_0\,\rho_2}\right)^{1/2}\frac{1}{\alpha^2}\frac{i}{4P_2}\left(\frac{m}{1+m}\right)\left(2\alpha\,M(\chi-1)+(\chi-1)^2-2S\alpha^3+2P_2\left(\frac{1-m}{1+m}\right)\left(1-\frac{m^2}{r}\right)-2P_2S\alpha^3\right) \quad (28)$$

From equation (28) it can be seen that the growth rate vanishes when $s_2=0$. The condition for stability or instability given by (26) holds, provided the magnetic shears s_1 and s_2 are positive.

Results and Discussion

It is shown that an unbounded configuration of viscous electrically conducting parallel flows permeated by a sheared magnetic field is always unstable for short-wavelength disturbances (in the absence of surface tension and viscosity) if the magnetic field vanishes at the interface and the magnetic diffusivities of the two fluids are different.

The graphical representations of growth rate Im(C_1) against the values of α for various values of the other parameters are shown in Figures 2-4. Figure 2 shows that the maximum growth rate occurs for shorter wavelengths as χ it increases. Figure 3 shows the growth rates for different magnetic Prandtl numbers P_2 with r = 1, m = 2, χ = 2, S = 0.001 and M = 0 . Figure 4 shows the growth rates for different viscosity ratios m for m = 0.1 and m = 10 with r = 1, χ = 2, S = 0.001, M = 0 and $P_2 = 0.025$. We find the maximum growth rate for $\alpha = O(1)$ and is in agreement with equation (25) for short wavelengths. Figures 3 and 4 show that the largest growth rate shifts to longer wavelengths with an increase in P_2 or a decrease in m.

References

[1] Bhattacharya and Gupta, Instability due to a discontinuity in magnetic diffusivity in the presence of magnetic shear, J. Fluid Mech. 509(2004) 125-144.

[2] Davidson,P. A. and Lindsay, R.. Stability of interfacial waves in aaluminium reduction cells, J. aluminium reduction cells, J. Fluid Mech. 362(1998) 273-295.
[3] Drazin, P.G. and Reid W. H., Hydrodynamic Stability. Cambridge University Press., 1981.
[4] Hooper A. P. and Boyd, W.G.C., Shear-flow instability at the interface between two viscous fluids, J. Fluid Mech. 128(1983) 507-528.
[5] Sneyd, A.D, Stability of fluid layers carrying a normal electric current, J. Fluid Mech. 156(1985), 223-236.
[6] Yih C. S., Instability due to viscosity stratification J. Fluid Mech. 27(1967), 337-352.