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Models of Super Dense Structures using a New Form of the Mass Function

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Abstract

In the present article, we perceive a new kind of mass function to solve Einstein's field equation and generate spherically symmetric models of ultra-dense stellar objects. By assuming a suitable form of mass function, we study the out-march of all physical parameters within the anisotropic fluid spheres. We find that the radial and transverse pressures, the density of matter, redshift, mass function are regular and well behaved inside the compact fluid spheres. Our model is stable under the action of hydrostatic, anisotropic, and gravitational forces and the causality condition is well satisfied inside the fluid spheres. The graphical representation of the adiabatic index reconfirms the stability of our model and the compactness parameter lies within the Buchdahl limit. The newly obtained solution is free from any singularity and satisfies all energy conditions, i.e., strong energy condition, weak energy condition, and null energy condition. We construct models of compact stars 4U 1820-30 and SAX J 1808-4-3658(SS2) and explore several physical features of the models.

Keywords: Anisotropy, Einstein's field equations, Mass function.

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Introduction

Einstein's gravity has produced very cogent and authentic outcomes about the compact stellar composition and their geometrical features. The field equations and their exact solutions have provided a solid framework for developing models of dense stellar objects. It's not a trivial task to find new exact solutions to field equations due to their highly complex and nonlinear character. Researchers have investigated many exact solutions by implementing different mathematical tactics and assumptions, keeping in view the physical compatibility of the solutions. Some investigations focus on the equation of state of the interior matter distribution, whereas some authors have investigated compact star models by making suitable assumptions on metric potentials, mostly based on embedded class one condition. An elegant and effective approach to finding out the new solutions of Einstein's field equations is to assume an appropriate mass function, and then trace out other essential ingredients to define the whole anatomy of the dense stars. Several interesting facts regarding compact stars have been explored by implementing the mass function idea. However, this approach has been rarely used due to the complexity of assuming the mass function's appropriate form. A particular type of mass function proposed by Matese and Whitman[1] has been used by several authors to investigate compact star models under different perspectives. Mak and Herko[2] used this mass function to construct anisotropic fluid configurations in the relativistic framework. Maharaj and Thirukkanesh[3] developed strange star models centered on the Matese and Whitman mass distribution. Schwarzschild[4] and Tolman[5] performed fundamental and pioneering research by working out initial solutions of Einstein's field equations, which revolutionized the investigations accruing to spherically symmetric stellar objects. Oppenheimer and Volkoff[6] developed simple neutron star models with the conception of Fermi gas, revealing many intriguing facts about dense astrophysical structures. In this direction, Buchdahl[7] made a significant contribution by establishing the upper mass to radius ratio limit for stable and spherically symmetric configurations. Initial investigations were mainly based on perfect fluid concepts, i.e., radial and transverse pressure possessed identical values. After that, advancements in theoretical physics and experimental techniques necessitated the presence of anisotropy in the interior of

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compact stellar objects. Intense magnetic field[8] Phase transition,[9] pion condensation[10] are some of the major factors causing anisotropy in the compact stars. Anisotropic structures differ in many features from their parental perfect fluid configurations. The redshift, stability, mass factor are some of the attributes in which the changes are inevitable due to the presence of anisotropy. Chan *et al.*[11] established that the local anisotropy has a great effect on the stability of stellar structures. Ruderman[12] investigated that pressure anisotropy plays a significant role when density in the compact fluid balls exceeds the nuclear density~ 10^{15} g/cm ^{3} as in X-ray pulsar, Her-X-1, X-ray buster 4U 1820-30, etc. Bowers and Liang[13] pointed out that in the presence of anisotropy the upper bound on the surface redshift exceeds the upper limit fixed for isotropic matter distribution. Herrera & Santosh[14] also investigated the characteristics of anisotropic stellar configurations. L. Herrera[15] investigated that energy inhomogeneities, dissipative forces, shear effects cause anisotropy in the relativistic compact stellar configurations. Herrera demonstrated that with the evolution of isotropic stellar configurations, the anisotropy develops inside the old compact stars. Bhar P, Ratanpal[16] presented the well-behaved models of Compact stars by assuming Matese & Whitman mass function and extensively analyzed the physical characteristics of the models. Bhar[17] constructed strange star models admitting Chaplygin Equation of state and carried out an extensive study of models. In the recent past, a lot of research work has been performed related to anisotropic compact star models.[18-21]

Bhar, Tamta and Pratibha[22,23] have elaborately analyzed the anisotropic stellar configurations in the relativistic framework of astrophysics. Feroze and Siddiqui employed different state linear and nonlinear forms to solve Einstein Field equations.[24] Recently some models of quark stars have been also analyzed by Malaver.[25,26] Takasia & Maharaj have explored compact stellar models using the quadratic state equation.[27] With these equations, models of astrophysical configurations have been proposed in the presence of electromagnetic field and anisotropy.[28] Some specific space-time manifolds like paraboloidal space-time and pseudo-spheroidal space-time have also been used to develop models of charged anisotropic compact stars.[29-31] A relative study of charged and uncharged anisotropic models has been done by Ratanpal and Bahar[32] in the framework of the general theory of relativity. Lobo has developed the dark energy star model by using Matese & Whitman's mass function.[33] Dayanandan *et al.*[34] also investigated compact stellar models based on Matese- Whitman mass function.

Motivated by the above-said investigations, we have investigated an anisotropic compact star model using a new form of the mass function. Models of compact stars 4U 1820-30 and SAX J 1808-4-3658(SS2) have been designed in close agreement with the known observational facts. The whole work is structured as follows: Section II includes basic Einstein's field equation. In section III, we have introduced the new kind of mass function. Moreover, section IV includes some additional features supporting the stability of the proposed model, and to solve the unknown parameters, and we have matched the interior space-time metric with an exterior metric in section V which is labeled as junction condition. In section VI, we have deduced the physical features of stellar models, strengthening the reliability of proposed models. At last, the conclusion of stellar models has been discussed in section VII.

Basic Einstein Field Equations

The spherically symmetric interior space-time geometry for the anisotropic static fluid sphere is defined as

$$ds^2 = e^{\lambda} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 - e^{\nu} dt^2 \quad (1)$$

Here ν and λ are the functions of radial coordinate r only. The energy momentum tensor for anisotropic compact star is considered as

$$T_j^i = (p_t + \rho c^2)v^i v_j - p_t \delta_j^i + (p_r - p_t)\chi_j \chi^i \quad (2)$$

Where all the symbols have their usual meaning, for the space-time metric given by equation (1) and energy momentum tensor given by equation (2) Einstein's field equations with are written as

$$8\pi p_r = e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right) - \frac{1}{r^2} \quad (3)$$

$$8\pi p_t = \frac{e^{-\lambda}}{2}\left[\left(\nu'' - \frac{\lambda'\nu'}{2} + \frac{\nu'^2}{2} + \frac{\nu' - \lambda'}{r}\right)\right] \quad (4)$$

$$8\pi\rho = e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} \quad (5)$$

The mass function for radius r is defined as

$$m(r) = 4\pi \int_0^r \omega^2 \rho(\omega) d\omega \quad (6)$$

Using equation (6) Einstein's field equations can be written as

$$e^{-\lambda} = 1 - \frac{2m}{r} \quad (7)$$

$$r(r - 2m)\nu' = 8\pi p_r r^3 + 2m \quad (8)$$

$$\frac{4}{r}(8\pi\Delta) = 8\pi(\rho + p_r)\nu' + 2(8\pi p_r') \quad (9)$$

Here Δ measures the anisotropy of the fluid sphere.

New Kind of Mass Function

We have assumed a new form of mass function as

$$m(r) = \frac{ar^3}{(2 + br + ar^2)} \quad (10)$$

Where a and b are taken as positive constants. On substituting eq.(10) into eq.(7), we have obtain metric potential as

$$e^{\lambda} = \frac{2 + br + ar^2}{2 + br - ar^2} \quad (11)$$

Expression for matter density can be written as

$$8\pi\rho = \frac{2a(6 + 2br + ar^2)}{(2 + br + ar^2)^2} \quad (12)$$

On the substitution of equation (10) into equation (8), we obtain

$$v' = (8\pi p_r)\frac{r(2+br+ar^2)}{(2+br-ar^2)} + \frac{2ar}{(2+br-ar^2)} \quad (13)$$

Equation (13) can be integrated by assuming the radial pressure as

$$8\pi p_r = bp_0\frac{(2-ar^2)}{(2+br+ar^2)^2} \quad (14)$$

Here we assume $p_0 > 0$. The above expression of radial pressure decreases monotonically with r and becomes zero at r= √2/a, representing the radius of the star. Substituting the above expression in equation (13) it can be integrated easily. The value of pressure at the centre is gives as $bp_0/2$. Plugging Eq. (14) into Eq. (13) we have obtained

$$v' = bp_0\frac{(2-ar^2)}{(2+br+ar^2)(2+br-ar^2)} + \frac{2ar}{(2+br-ar^2)} \quad (15)$$

On integration of Eq. (15) we have deduce v as

$$v = \frac{bp_0 In\{r(ar+b)+2\}}{2a} + \frac{b^2p_0 In\left(\frac{2ar-\sqrt{b^2+8a}-b}{2ar+\sqrt{b^2+8a}-b}\right)}{2a\sqrt{b^2+8a}} - In\{r(ar-b)-2\} - b\frac{In\left(\frac{2ar-\sqrt{b^2+8a}-b}{2ar+\sqrt{b^2+8a}-b}\right)}{\sqrt{b^2+8a}} + C \quad (16)$$

Here C is the constant of integration. The anisotropic factor Δ is obtained from expression (9) as

$$8\pi\Delta = \frac{r}{4}\left\{\frac{2a(6+2br+ar^2)}{(2+br+ar^2)^2} + \frac{bp_0(2-ar^2)}{(2+br+ar^2)^2}\right\} - \left\{bp_0\frac{ar^2}{2(2+br+ar^2)^2}\right\} \left\{\frac{bp_0 r(2-ar^2)}{(2+br+ar^2)(2+br-ar^2)} + \frac{2ar}{(2+br-ar^2)}\right\} - \left\{bp_0 r\frac{(b+2ar)(2-ar^2)}{2(2+br+ar^2)^3}\right\} \quad (17)$$

The transverse pressure is given by

$$8\pi p_t = \frac{r}{4}\left\{\frac{2a(6+2br+ar^2)}{(2+br+ar^2)^2} + \frac{bp_0(2-ar^2)}{(2+br+ar^2)^2}\right\} - \left\{bp_0\frac{ar^2}{2(2+br+ar^2)^2}\right\} \left\{\frac{bp_0 r(2-ar^2)}{(2+br+ar^2)(2+br-ar^2)} + \frac{2ar}{(2+br-ar^2)}\right\} - \left\{bpr\frac{(b+2ar)(2-ar^2)}{2(2+br+ar^2)^3}\right\} \quad (18)$$

And the pressure and density gradients can be expressed as

$$8\pi\frac{dp_r}{dr} = \frac{-2abp_0 r}{(2+br+ar^2)^2} - \frac{2bp_0(b+2ar)(2-ar^2)}{(2+br+ar^2)^3} \quad (19)$$

$$8\pi\frac{d\rho}{dr} = \frac{2a(2ar+2b)}{(2+br+ar^2)^2} - \frac{4a(2ar+b)(6+2br+ar^2)}{(2+br+ar^2)^3} \quad (20)$$

$$8\pi\frac{d\Delta}{dr} = \frac{1}{\left\{\frac{2(ar^2-br-2)^2}{(ar^2+br+2)^4}\right\}} r \left[b\{(a^4b^2p^2 - 4a^5bp + 4a^6)r^8 - (a^3b^3p^2 + 5a^4b^2p - 14a^5b)r^7\} + \{(a^2b^4 - 12a^3b^2)p^2 + 2a^3b^3p + 8a^4b^2 + 48a^5\}r^6 + \{12a^2b^3p^2 + (30a^3b^2 - 3a^2b^4)p - 2a^3b^3 + 54a^4b)r^5\} + \{40a^2b^2p^2 + (80a^3b - 48a^2b^3)p + 64a^4\}r^4 - \{20ab^3p^2 + (6ab^4 + 172a^2b^2)p - 12a^3b^3 - 40a^3b\}r^3 - \{(4b^4 + 48ab^2)p^2 + (8ab^3 + 192a^2b)p - 96a^2b^2 - 64a^3\}r^2 + (72ab^2p + 240a^2b)r + 16b^2p^2 + 128abp + 192a^2 \right] \quad (21)$$

$$8\pi\frac{dp_t}{dr} = 8\pi\frac{d\Delta}{dr} + 8\pi\frac{dp_r}{dr} \quad (22)$$

Some Additional Features

Values of p_r, p_t and ρ at the centre are given as

$$p_r(r=0) = p_t(r=0) = \frac{bp_0}{16\pi} \quad (23)$$

$$\rho = \frac{3a}{8\pi} > 0 \; for \; a > 0 \quad (24)$$

The mass function m(r) can be obtained using Equation (11)

$$e^{-\lambda} = 1 - \frac{2m}{r} \quad (25)$$

$$m(r) = \frac{ar^3}{(2+br+ar^2)} \quad (26)$$

And the compactness factor is formulated with the help of equation (26) as

$$u(r) = \frac{m(r)}{r} = \frac{ar^2}{(2+br+ar^2)} \quad (27)$$

The surface redshift of the compact stellar structure is given by the formula

$$1 + z_s(r_b) = \{1 - 2u(r_b)\}^{1/2} \quad (28)$$

From above equation (28)

$$z(r_b) = \left(\frac{2+br_b-ar_b^2}{2+br_b+ar_b^2}\right) - 1 \quad (29)$$

The physical properties of the proposed relativistic model can be examined by observing the trends of metric potential, density, radial and transverse pressures, anisotropic factor, the mass function and compactness factor with respect to radial coordinate r. The metric potential e^{λ} has non-zero positive value at r=0 and is regular inside the boundary of stellar structures Figure 1. Density and pressures are monotonically decreasing with radial coordinate r, they decrease when we move towards surface from the centre Figures 2 and 3. So

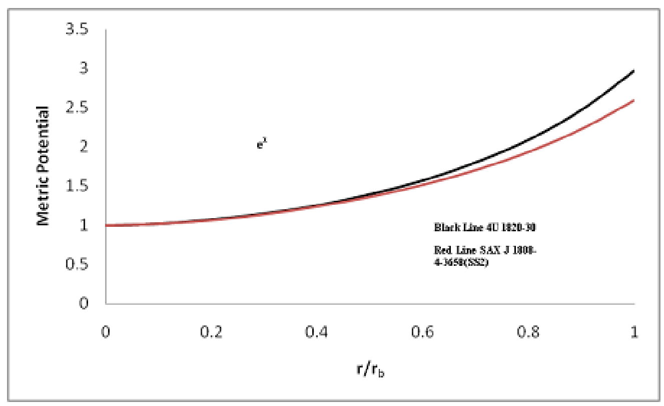

Figure 1: Variation of metric potentials with radial coordinate r

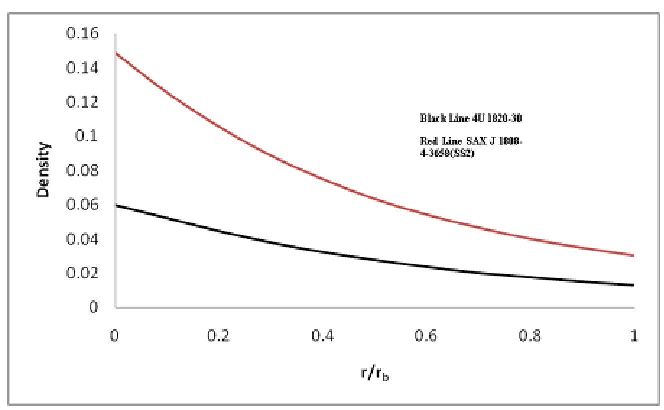

Figure 2: Variation of matter densities with radial coordinate r.

the newly obtained solution is well behaved and physically acceptable. The pressure to density ratios follow decreasing pattern when we move from centre toward surface Figure 4. The anisotropic factor is zero at the centre and increases towards the outward direction Figure 5. In order to satisfy the stability condition $-1 \le v_t^2 - v_r^2 \le 0$, we must have $d\Delta /dr>0$ as $d\rho /dr<0$. The density and pressure gradients decrease with radial coordinate r and have negative values inside the fluid sphere (6). The negativity of the gradients further validates the decreasing trends of pressure and density. As observed from graphical representation Figure 7, the mass function is monotonically increasing with radial coordinate r. Figure 8 shows the outmarch of compactness parameter with radial coordinate r inside the compact structure and manifests the increasing behavior in moving from centre to surface. The profile of surface redshift is shown in Figure 9. Figure shows that surface red shift is a monotonically increasing function of radial coordinate r. The behavior of the parameters mentioned above is consistent with the physically realizable structures. Substituting the values of constants, a and b, the surface red shift of the stars can be evaluated from equation

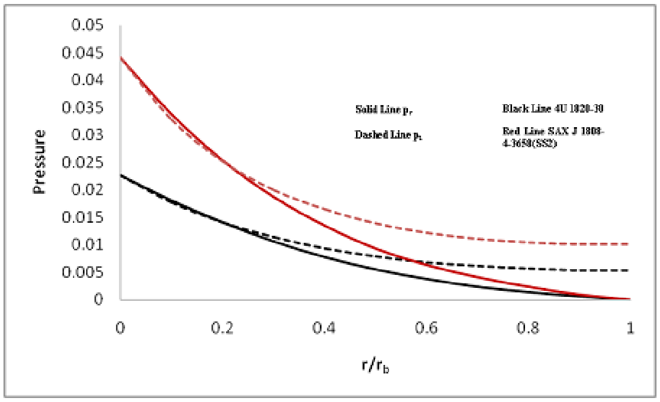

Figure 3: Variation of pressures with radial coordinate r.

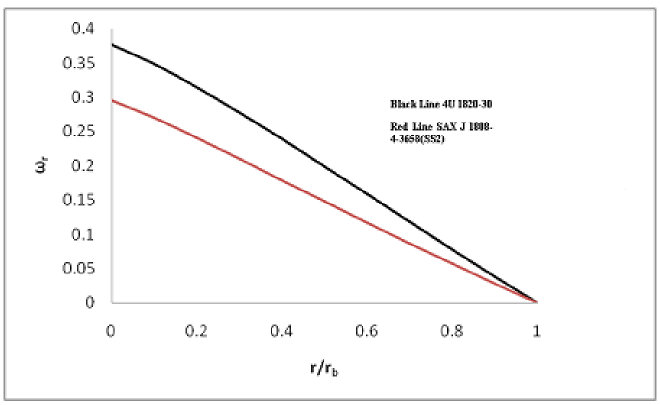

Figure 4: Variation of pressure to density ratios ($\omega r = p r / \rho$, $\omega t = pt / \rho$) with radial coordinate r.

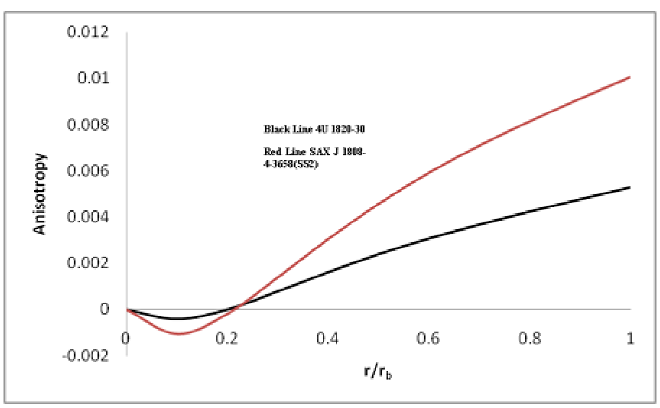

Figure 5: Variation of anisotropy with radial coordinate r.

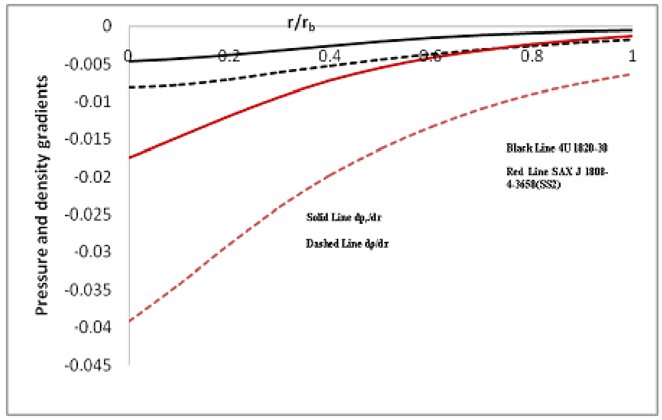

Figure 6: Variation of pressure and density gradients with radial coordinate r.

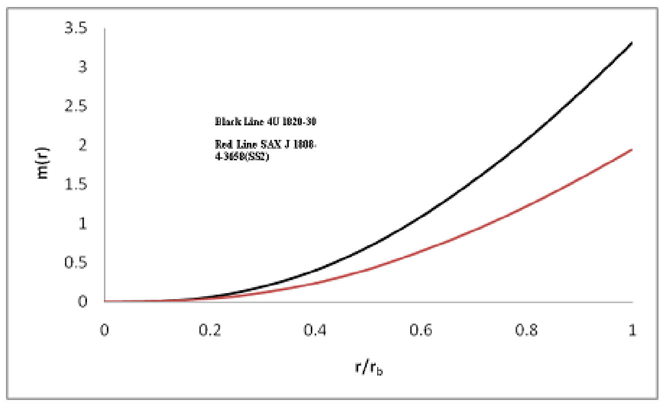

Figure 7: Variation of mass function with radial coordinate r.

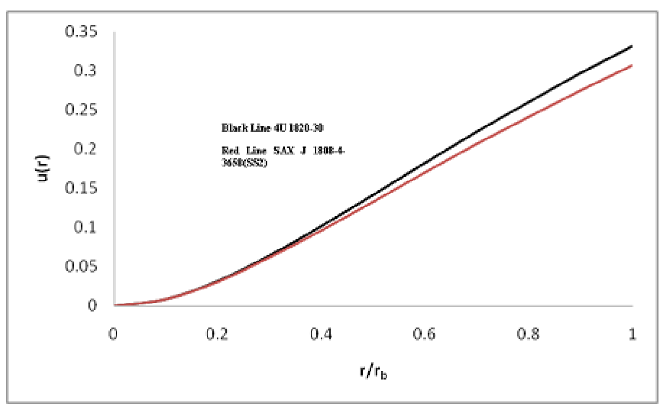

Figure 8: Variation of compactness parameter with radial coordinate r.

(29). We can have an idea about the range of redshift from equation (29) and Table 2, according to Bohmer & Herko[35] the redshift should be z ≤ 5.

Junction Conditions

The values of constants a and b appearing in the solution can be obtained by smoothly joining the interior space time metric of anisotropic fluid distribution to the exterior Schwarzschild solution given by

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) + \left(1-\frac{2M}{r}\right)^{-1} dr^2 \quad (30)$$

The metric functions e^{ν} and e^{λ} must be continuous at the boundary of the compact stars. The radial pressure p_r should be vanishing at the boundary ($r = r_b$), accordingly we get

$$a = \frac{2}{r_b^2} \quad (31)$$

$$b = \frac{2}{M} - \frac{4}{r_b} \quad (32)$$

Constant of integration C from equation (16) can be evaluated as

$$C = \frac{In\left(1-\frac{2M}{r_b}\right) - \frac{bp_0 In\{r_b(ar_b+b)+2\}}{2a} - \frac{b^2 p_0 In\left(\frac{2ar_b - \sqrt{b^2+8a} - b}{2ar_b + \sqrt{b^2+8a} - b}\right)}{2a\sqrt{b^2+8a}}}{In\{r_b(ar_b - b) - 2\} + b\frac{In\left(\frac{2ar_b - \sqrt{b^2+8a} - b}{2ar_b + \sqrt{b^2+8a} - b}\right)}{\sqrt{b^2+8a}}} \quad (33)$$

Physical features of stellar model

TOV Equation and Hydro-static Equilibrium

A superdense structure is in hydrostatic equilibrium if all the three forces acting on the structure counterbalance each other. The stellar models we have studied here are influenced by three forces, i.e., the gravitational force, hydrostatic force and anisotropic force. The hydro-static equilibrium condition due to these three forces acting on the system is obtained by the generalized Tolman-Oppenheimer-Volkoff equation, which is given as

$$\frac{-M_G(r)(\rho+p)}{r} e^{\frac{\lambda-\nu}{2}} - \frac{dp_r}{dr} + \frac{2}{r}(p_t - p_r) = 0 \quad (34)$$

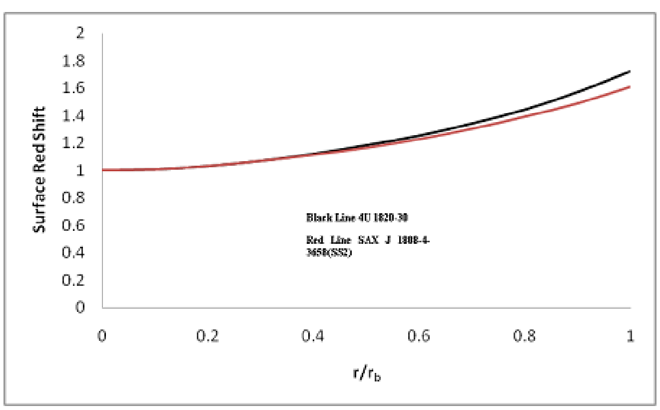

Figure 9: Variation of red shift with radial coordinate r.

Here $M_G = M_G(r)$ refers to effective gravitational mass inside the fluid sphere. The expression for gravitational mass is given by Tolman-Whitaker mass formula as

$$M(r) = \frac{1}{2} r^2 e^{\frac{\nu-\lambda}{2}} \nu' \quad (35)$$

Plugging the value of M G (r) from expression (35) into expression (34), we have the modified Tolman-Volkoff-Oppenheimer equation as

$$F_g + F_h + F_a = 0 \quad (36)$$

Where we have

$$F_g = -\frac{\nu'}{2}(\rho + p_r) \quad (37)$$

$$F_h = -\frac{dp_r}{dr} \quad (38)$$

$$F_a = \frac{2}{r}(p_t - p_r) \quad (39)$$

F_g, F_h and F_a represent the gravitational, hydro-static and anisotropic forces, respectively and the variation of these forces for the compact stars are shown in Figures 10 and 11. From these patterns, we can conclude that only Hydrostatic force shows some irregular behavior, whereas other forces follow a well-behaved trend inside the fluid spheres.

Energy Conditions

When all energy conditions are satisfied inside the star the solution is physically valid. Our model satisfies all inequalities

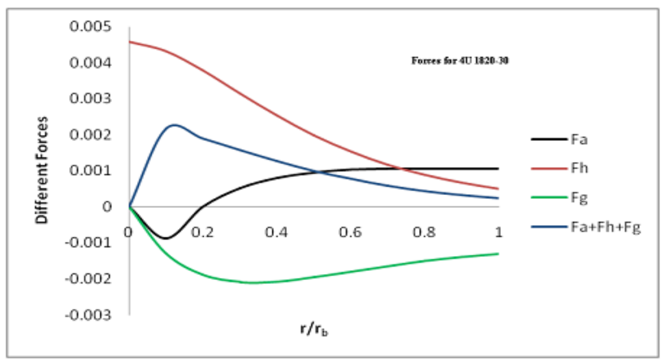

Figure 10: Variation of different forces with radial coordinate r.

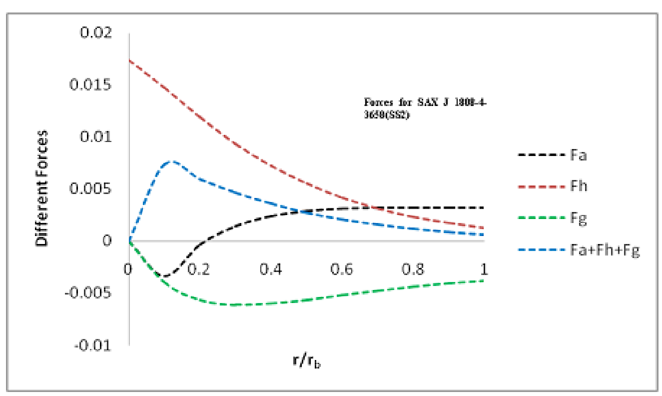

Figure 11: Variation of different forces with radial coordinate r.

Table I: The values of the parameters a and b used for two compact stars.

Compact stars	*M/ M$_\Theta$*	*Mass (km)*	*Radius (km)*	*a (km $^{-2}$)*	*a (km $^{-2}$)*
4U 1820-30	2.25	3.31875	10	0.02	0.202636535
SAXJ1808.4-3658(SS2)	1.323	1.951425	6.35	0.049600099	0.394970806

Table II: The values of the various parameters obtained for the compact stellar models.

Compact stars	*Central density (ρ_0 gms/cc)*	*Surface density (ρ_s gms/cc)*	*2M/r$_b$*	*Surface redshift(z_s)*
4U 1820-30	2.14×10^{15}	0.71×10^{15}	0.664	0.7245
SAX J 1808.4-3658(SS2)	4.91×10^{15}	1.64×10^{15}	0.615	0.611

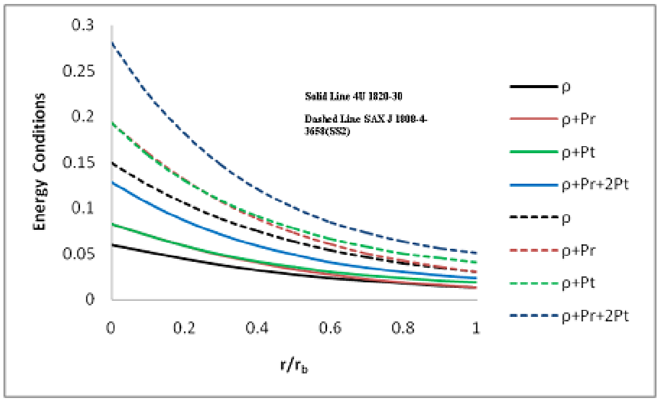

Figure 12: Variation of energy conditions with radial coordinate r.

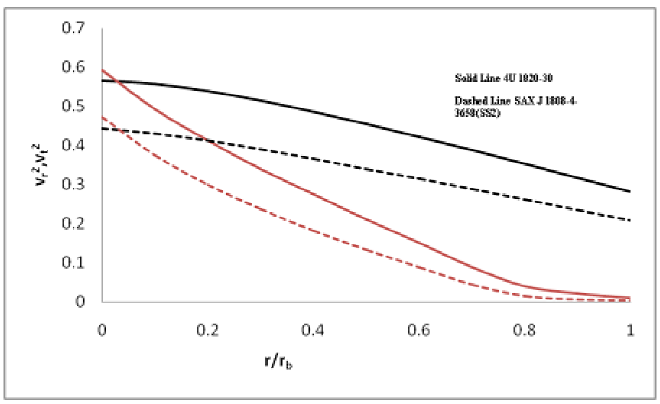

Figure 13: Variation of v_i^2 with radial coordinate r.

for null energy condition (NEC), strong energy condition (SEC) and weak energy condition (WEC) and dominant energy conditions (DEC), these inequalities are

$$\text{NEC}: \rho + p_r \geq 0; WEC: \rho + p_r \geq 0 \text{ and } \rho \geq 0;$$

$$SEC: \rho + p_r \geq 0: \rho + p_r + 2p_t \geq 0; DEC: \rho > |p_r|; \; \rho > |p_t| \quad (40)$$

In Figure 12, we have represented all energy conditions. From the trends, we can conclude that all inequalities pertaining to energy conditions hold good inside the stars.

Stability Condition

Causality Condition

The radial and transverse components of velocity of sound for compact stellar structure are given by

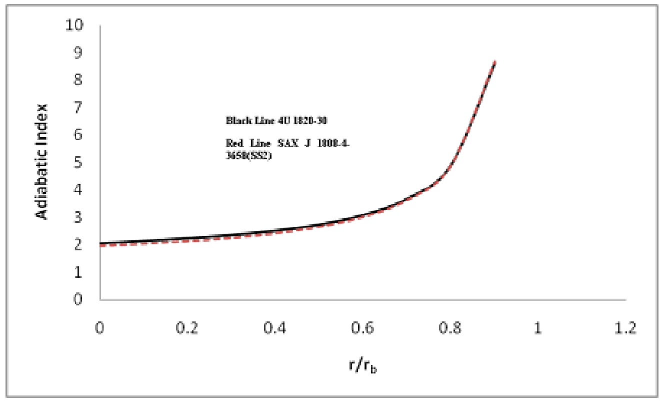

Figure 14: Variation of adiabatic index with radial coordinate r.

$$v = \frac{dp_r}{d\rho} = \frac{dp_r/dr}{d\rho/dr}\,;\, v = \frac{dp_t}{d\rho} = \frac{dp_t/dr}{d\rho/dr} \quad (41)$$

If the velocity of sound inside the structure is less than the velocity of light, any astrophysical system is said to be in stable state. In astronomical units the velocity of light is taken as unity. Both the squares of radial and transverse velocities are less than 1 and have positive values within the stellar configuration Figure 13. Using Herrera's cracking method,[36] we can determine the stability of anisotropic compact stars against the radial perturbations. We must have -1 ≤ vt2 - vr2 ≤ 0 in the region inside the compact stars for potentially stable structures. Figure 13 indicates that our models correspond to potentially stable structures.

Adiabatic Index

The relativistic adiabatic index Γ is defined as[37]

$$\Gamma = \frac{\rho + p_r}{p_r}\frac{dp_r}{d\rho} \quad (42)$$

According to Newtonian approximation, if Γ >4/3 then the Newtonian sphere is stable. Γ =4/3 indicates the condition for equilibrium and stability.[38] The profile of adiabatic index with radial coordinate r is shown in Figure (14). The graphical pattern of adiabatic index exhibits the well-defined trends inside the compact stellar configurations

Conclusion

In this composition we employ a new form of mass function to develop a well-behaved model of anisotropic compact stars 4U 1820-30 and SAX J 1808.4-3658(SS2). We have examined all physical properties of models such as metric potential, density, pressure, mass function, surface red-shift, gravitational redshift, and the stability of the stellar configurations. The physical parameters of our model, such as (e^{λ}, ρ, p_r, p_t, p_r / ρ, p_t / ρ) are free from central singularities and follow all necessary physical constraints. The metric potential increases and density, pressure, pressure to density ratios decrease monotonically outwards as shown in figs. 1 to 4. Anisotropy factor Δ, and adiabatic index Γ increase when we move outwards from centre to surface, which shows our models are physically realizable Figure (5,13). When we follow the radial and tangential pressure pattern, we conclude that the radissal pressure vanishes at the star's surface but the tangential pressure does not vanish. So there exists anisotropy in the interior of our stellar configuration. The anisotropic factor Δ, having minimum value at the centre and maximum at the surface Figure (5). The mass function and compactness parameter are plotted graphically in figs. 7 and 8, respectively. The surface red shift z s increases when we move towards the surface from centre shown in Figure (9). By anatomizing the graphical representations, we can say that our model is physically viable. All energy conditions are satisfied, which is necessary for the physical existence of our configurations. The null energy condition (NEC), the weak energy condition (WEC), and strong energy condition (SEC) have been shown in Figure (12). The inclination of pressure and density gradients are negative, which signifies that the pressure and density decrease radially outwards Figure 6. The hydrostatic equilibrium condition representing the counterbalancing of the different forces acting on the fluid sphere is shown in Figures. 10 and 11, approve the hydrostatic equilibrium of proposed models of compact stars. However, the hydrostatic force shows some unusual behaviour value of adiabatic index is more than 4/3 throughout the stellar structure, which justifies the stability of our model Figure 14. The patterns of squares of radial and transverse velocities have been shown in Figure (13), both v_r^2 and v_t^2 are less than 1. For any stable configuration v_t^2 - v_r^2 must have values between -1 and 0 ($-1 \leq v_t^2 - v_r^2 \leq 0$).

We have systematically examined our models for two compact stars and all the parameters of these stars have been depicted in Tables 1 and 2. All characteristics of this model are well-matched with the observed data for 4U 1820-30 and SAX J 1808-4-3658(SS2).

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