Modeling and Identification of the Non-linear Dynamics of a Piezoelectric Actuator-A Review

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ABSTRACT
There is currently no exact dynamic model which predicts hysteresis and creeps in a piezoelectric actuator under varying operating conditions (increasing frequency and amplitude of input, time of operation, temperature effects), and is stable against uncertainties. Thus, research needs to be carried out to predict the hysteresis and creep on the modeling and identification of the non-linear dynamics of a piezoelectric actuator. It would aid the implementation of a model-based control algorithm such as the precise positioning of a nano-positioning.

Keywords: Hysteresis, Modeling, Piezoelectric Actuator.

INTRODUCTION
A piezoelectric actuator is one of the most commonly used actuators to drive a nano-positioning application because of its quick response and high resolution.[1,2] However, presence of creep and hysteresis mainly affects the performance of a piezoelectric actuator driven nano-positioning application. To control the effect caused by hysteresis and creep, it is important to model these behaviors. This report provides a review of a few of the important modeling methodologies.

MODELING OF PIEZOELECTRIC ACTUATOR
A piezoelectric actuator suffers from hysteresis and creep, which affect its displacement precision and resolution. Therefore, the reduction of hysteresis and creep is vital if a piezoelectric actuator is used to drive any micro-and-positioning application. One way of minimizing hysteresis and / creep involves driving a piezoelectric actuator using a charge as input[3] or by insertion of a series capacitor[4] with the voltage-driven piezoelectric actuator. Another way involves accurate Modeling[5] of the non-linear dynamics of an actuator to predict hysteresis and creep from which a model-based feedback control[6,7] or an inverse control approach[8] can then be implemented to compensate for hysteresis and / creep.

Modeling of Hysteresis
It is known that a piezoelectric actuator exhibits hysteresis when it is driven using quasi-static or dynamic voltage input. Devasia[5] suggests that the amount of hysteresis exhibited by a piezoelectric actuator is not only dependent on the amplitude of the voltage input but depends on the frequency at which it is applied. Thus, the modeling of hysteresis in a piezoelectric actuator is not straightforward and, thus, numerous models have been derived to predict such complicated behavior. Purely phenomenological models such as the Preisach model[9,10] as well as the parametric models like the Lumped parameter model[12] and the first order Bouc-Wen model[11] have been derived to model hysteresis in a piezoelectric actuator. Also, various identification approaches.[13-15] Tong,Yeh[16] have been implemented to identify the model of a piezoelectric actuator to predict hysteresis. Presented next is a detailed review of the literature on the Modeling of hysteresis in a piezoelectric actuator.

Phenomenological Model
The most used model is by preisach model for the modeling of hysteresis in a piezoelectric actuator. Weiss and de Freundenreichin were the first to propose this model, in a conceptual form, to predict hysteresis in any ferroelectric material. The conceptual form was based on the fundamentals relating to the physical mechanism of magnetization, wherein the hysteresis is exhibited between...
the input and the output. Preisach provided the much-needed geometric interpretation to the conceptual form proposed by Weiss and de Freundenreich and implemented the derived model towards the modeling of hysteresis in magnetic materials. Although the model is well known as “preisach model,” it was Krasnoselskii and Pokrovskii who visualized the Preisach model as a continuous analog of a system of parallel connected elementary hysteresis “relay” operators (as shown in Figure 1). Based on this visualization, Krasnoselskii and Pokrovskii built a purely mathematical model; that could be used to predict hysteresis, between the input and the output, in any physical system.

![Figure 1: The block diagram representation of classical Preisach model](image)

In this figure $u(t)$ is the input, $\gamma_{\alpha\beta}$ is a relay operator, $\alpha$ and $\beta$ are the up/down switching values for the relay operator $\mu(\alpha, \beta)$ is the Preisach function and $x(t)$ is the output.

Mayergoz showed that the accurate modeling of hysteresis using the CPM was dependent on how well the hysteresis curve between the input and the output, was captured. To prove this, Mayergoz defined the wiping out and the congruency property and showed that accurate prediction of hysteresis required these properties to be satisfied. The wiping out property defined by Mayergoz implied that only the current input. The congruency property defined by Mayergoz implied that all minor hysteresis loops, formed due to the back and forth variations of the inputs between the same two fixed values, coincide with each other. Thus, above properties showed that the CPM proposed by Krasnoselskii and Pokrovskii could only be used to accurately model hysteresis when the bandwidth at which the input is applied was not varied. It would be fair to say that the CPM, in its original form, would not provide accurate prediction of hysteresis that is exhibited by a piezoelectric actuator used in dynamic applications.

Ge and Jouaneh built a numerical model by modifying the existing CPM to model the hysteresis behavior in a piezoelectric actuator. Although the error is maximum in the prediction of hysteresis was found to be approximately 2%, of the maximum displacement of the stack type actuator, it was seen to increase with a increase in the frequency of the voltage input. Continuing on their work in this area, Ge and Jouaneh developed a modified generalized Preisach model (MGPM) to model hysteresis exhibited by a piezoelectric stack type actuator. In comparison to the CPM, the MGPM was designed such that it only required the wiping out property to be satisfied for accurate modeling of hysteresis. The maximum error in the prediction of hysteresis was found to be approximately 2.3% of the displacement of the stack type actuators. However, the use of MGPM to model hysteresis exhibited by a piezoelectric actuator used in dynamic applications was not recommended by Ge and Jouaneh.

To access the accuracy of the CPM in the prediction of hysteresis in a piezoelectric stack type actuator, Hu and Mrad conducted a similar study. It was found that although the CPM is an accurate means to model hysteresis, its accuracy starts to deteriorate when the band of operating frequency gets wider, or, the load on the actuator increases. Mrad and Hu developed a model, an extension to the CPM, to accurately predict the hysteresis in a piezoelectric stack type actuator that was subject to varied operating frequencies. As mentioned before, the CPM proposed by Krasnoselskii and Pokrovskii is based on the concept of elementary relay operators with up and down switching values given by $\alpha$ and $\beta$. The model presented by Mrad and Hu modifies these operators by adding a frequency dependent parameter to their mathematical form. Numerical implementation and experimental verification of this model shows that a measured and predicted hysteresis behaves accurately.

Yu modified the geometric interpretation of the CPM. This was done to take in to account specific cases wherein the displacement of a piezoelectric actuator may not be not equal to zero even if the input is; especially the cases in which a piezoelectric actuator is pre-loaded. Also, Yu suggested a better numerical implementation of the CPM in order to eliminate any error that could creep in a while performing mathematical calculations. A good agreement was shown to exist between the predicted and the measured hysteresis.

### Parameteric Models

Wen proposed the use of a parametric model to predict hysteresis in a physical system. The basic ideology behind this parametric model was to define the hysteresis curve using a set of parameters. The Bouc-Wen model used six parameters to capture the hysteresis curve exhibited by a physical system. Vinogradov and Pivovarov verified that the Bouc-Wen model could be used to predict hysteresis in any non-linear system that was preceded by hysteresis and that the captured hysteresis curve was symmetric.

In this figure, $k$ is the spring constant for massless linear spring, $N$ is the input applied to the block, $x$ is output displacement, $F$ is force, and $\mu$ is Coulomb friction the block is subject to.

Wang presented an analogy that, mechanically, the hysteresis in any system is due to the Coulomb friction. Goldfarb and Celanovic, used this analogy as the basis to describe the static hysteresis in a piezoelectric actuator and
showed that when a single Elasto-slide element (as shown in Figure 2) was subject to an input, $N$, of sufficient amplitude, the relationship between this input and the generated output, $x/F$, exhibits a frequency-independent hysteretic behavior.

Connecting such Elasto-slide elements in parallel (as shown in Figure 3), Goldfarb and Celanovic developed a generalized Maxwell slip model to predict frequency-independent hysteresis in a piezoelectric actuator. However, this meant that the model proposed by Goldfarb and Celanovic could only be used to model hysteresis when the frequency remains the same accurately. Thus, it would be fair to say that the Maxwell slip model, in its original form, would not provide an accurate prediction of hysteresis that is exhibited by a piezoelectric actuator in dynamic applications. Moreover, the Maxwell slip model application could lead to an inaccurate prediction of hysteresis exhibited by a pre-loaded piezoelectric actuator when deriving this model.

Zhu and Zhou\cite{21} used the fractional order calculus (FOC) theory to establish a model for dynamic hysteresis non-linearities. Here in, a fictitious hysteresis force was introduced and mathematically described by a fractional order differential equation. The hysteresis force model (HFM) was characterized by non-linear phase shifts and non-linear modulations of amplitudes, both mainly depending on input frequencies and differential orders. Choosing proper model parameters has shown that the dynamic hysteresis effects could be well described.

**Modeling of Creep**

Fett and Thun\cite{22} showed that the response obtained is non-linear displacement when a piezoelectric actuator is driven using a static input signal. From their investigation, Salapaka\cite{6} showed that creep leads to a loss in precision of a piezoelectric actuator, when positioning is required over extended periods. Therefore, it would be fair to precisely say that it is important to model the creep behavior to position a piezoelectric actuator accurately, as noted by Jung and Gweon\cite{23} (2000) "in comparison to hysteresis in a piezoelectric actuator, creep has been investigated less frequently and, therefore, very few approaches have been designed to model it".

In this figure $k$ is the spring constant, $c$ is the damping coefficient, $x$ is the output displacement and $u_x$ is the voltage input.

Malvern\cite{25} is possibly the first to modeled the creep behavior in a piezoelectric actuator. He modeled the displacement response of a piezoelectric actuator, which exhibits the creep behavior, as a series connection of $n$ pairs of parallel-connected spring and damper (as shown in Figure 4). An experimental investigation conducted by Croft, which incorporated a system identification approach, showed that three pairs of spring and damper were sufficient to accurately model the creep behavior exhibited by a piezoelectric tube scanner, a type of piezoelectric actuator.

Basedow and Cocks\cite{26} suggested that when an electric field is applied to a piezoelectric actuator, the actuator’s instantaneous displacement is followed by a slow relaxation that reduces over time. Carrying forward the work done by Basedow and Cocks,\cite{26} Vieira\cite{27} found that the instantaneous displacement follows a constant value; the instantaneous constant depends only on the applied static electric field. The slow relaxation approximately follows a logarithmic behavior. It can, thus, be modeled as a product of a relaxation
parameter, which depends on a combination of the applied static electric field and the operating temperature and a time-dependent logarithmic function. Combining above all, Vieira came up with a logarithmic model wherein the total displacement of a piezoelectric actuator was a combination of the instantaneous displacement and the slow relaxation. Jung and Gweon, Salapaka and Ru have all based their model, of creep in a piezoelectric actuator, on the work done by Vieira.

Identification of Hysteresis

Devasia suggests that the amount of hysteresis exhibited by a piezoelectric actuator depends not only on the amplitude but also on the frequency at which the voltage input is applied. Moreover, as Song and Li mentioned, the tedious calibration process is necessary when using the phenomenological model; the same would apply to the parametric model. Thus, it will be fair to say that implementing such models in real-time will not be easy. This calls for an identification approach, which assists in predicting the frequency-dependent hysteresis and real-time implementation. Various types of identification techniques are implemented for the prediction of hysteresis. A review of few important techniques is presented below.

In this figure, $u(t)$ is the voltage input while $x(t)$ is the displacement output.

![Figure 6: Static hybrid model](image)

In this figure, $u(t)$ is the voltage input, $v(t)$ is the derivative of the input, and $x(t)$ is the displacement output.

![Figure 7: Dynamic hybrid model](image)

Song and Li implemented a neural network (NN) algorithm to identify the CPM to predict hysteresis using a sine wave of frequency 2 Hertz and 32 Hertz. First, a static hybrid model (as shown in Figure 6) was implemented to predict frequency-independent hysteresis. Then a dynamic hybrid model (shown in Figure 7) was implemented to predict frequency-dependent hysteresis. When using the static model, the maximum error in the hysteresis prediction was found to be less than 5% compared to the measured hysteresis.

The error was further reduced to less than 2.5% when the dynamic hybrid model was used. In addition, the introduction of the NN algorithm enabled model identification using simple displacement measurement data of a piezoelectric actuator, thereby eliminating the tedious calibration process associated with the CPM. However, the prediction error seemed to increase with an increase in the frequency from 2 Hertz to 32 Hertz.

Banning implemented a system identification approach to predict hysteresis in a high precision mechanical translation system driven using a piezoelectric actuator. The reason behind realizing a state-space model was implementing a state-space control approach in the future. A good agreement; a prediction error of less than 2%, was shown to exist between the predicted and the measured hysteresis when the actuator was driven using a periodic sine wave input of frequency 2 Hertz.

Parali implemented a two-step approach to identify hysteresis in a piezoelectric actuator. In the first step (refer to Figure 8), a rough hysteresis curve between the voltage input and the displacement output of a piezoelectric actuator, was generated using an analytic, geometric model-driven using voltage, $u(t)$. In the second step, an ANN was used to approximate the hysteresis curve. One of the inputs to the ANN was the voltage, $u(t)$, while the other input was the...
rough hysteresis curve output from the geometric model. In simulations, the approach worked successfully. On similar lines, Qin and Jia\(^{[12]}\) have proposed an inverse modeling approach using the ANN. However, to recommend such an approach for the accurate identification of hysteresis in a piezoelectric actuator, it would be important to validate this approach against experimentally generated hysteresis curves and tabulate the error in the prediction of hysteresis. Application of the Maxwell slip model\(^{[12]}\) could lead to inaccurate prediction of hysteresis exhibited by a pre-loaded piezoelectric actuator. Yeh\(^{[31]}\) implemented a linear programming approach to predict the parameters of the Maxwell slip model\(^{[12]}\) such that hysteresis in a pre-loaded piezoelectric actuator could be accurately determined. Although the error in the prediction of hysteresis was only 1.25\%, the approach was only implemented at a single operating frequency.

### Modeling of Vibration

Driving flexible micro-motion application in the zone of resonance could lead to vibration or chattering. This could lead to significant instability, thereby restricting the operating bandwidth of such applications. Therefore, it is important to model such micro-motion applications to predict the effect due to vibration.

Croft\(^{[24]}\) and Leang\(^{[8]}\) have shown that a system identification approach, which identifies a transfer function based on the measured frequency response between the input and the output, is best suited for modeling vibration in a piezoelectric tube scanner. Yong\(^{[34]}\) has shown that an approach similar to the one presented by Croft\(^{[24]}\) can be successfully implemented to model vibrations in a piezoelectric actuator-driven micro-and nano-positioning application. In addition, Yong noted that this approach’s success depends on the accuracy with which the actual response between the input and the output is measured.

### GAPS: Technologies for Compensation of Hysteresis, Creep, and Vibration

A thorough review of the literature relating to the modeling of a piezoelectric actuator is provided in the preceding sections. In addition, a review of the literature relating to modeling of vibrations in a micro-and nano-positioning application is also provided.

A detailed review of the various modeling techniques used to predict hysteresis creep, and vibration shows that although every technique provides certain advantages, it suffers from certain disadvantages. In addition, the performance of some of these techniques, because they have not been tested experimentally over a wider spectrum of operating conditions, is questionable. Overall, there are specific gaps to be filled if precise positioning of a piezoelectric actuator-driven micro-and nano-positioning application over a wide band of operating frequencies is to be achieved.

Although various models have been derived for the prediction of hysteresis, they are all static in nature i.e., they are designed to predict hysteresis that is not dependent on the frequency of the voltage input. However, Devasia suggested that the amount of hysteresis exhibited by a piezoelectric actuator depends not only on the amplitude of voltage input but also on the applied frequency. In addition, Song and Li note that implementing these static models in real-time is quite complex. Only a few models are derived for the prediction of creep\(^{[25,27]}\) in a piezoelectric actuator. However, these models are not robust to a change in the operating conditions, such as temperature, of a piezoelectric actuator. While Croft notes that the model proposed by Malvern can predict creep with reasonable accuracy, Devasia notes that Vieira’s model is only approximate.

Various signal processing-based identification approaches have been developed to identify a model that accurately predicts hysteresis\(^{[13,30,31]}\) in a piezoelectric actuator. The basic idea behind implementing these approaches is to update a static model using a measured displacement response of a piezoelectric actuator. Although these approaches have kept the error in identifying hysteresis between 1–2\%, their stability against multi-amplitude and or multi-frequency signal has not been tested. In addition, these identification approaches have only been tested at low frequencies of up to 40 Hertz. To the stability against uncertainties, these approaches would need to be tested at higher frequencies.

To minimize the effect of vibration and provide a high-frequency piezoelectric actuator to a frequency up to 250 Hertz, Croft and Leang have implemented a feed-forward control scheme. However, in both cases, the control scheme has been implemented to minimize vibration in the actuator. In some instances, like this thesis, the resonance frequency of a micro-and nano-positioning application is much lower than that of a piezoelectric actuator used to drive the application. Thus, the operating bandwidth is constrained by the resonance frequency of the application and not by the actuator. Other than Yong\(^{[34]}\) not many have successfully implemented the feed-forward control scheme to minimize vibration in a micro-and nano-positioning application.

### References


