

# A Review on the Evolution of Vehicle Routing Problems

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## ABSTRACT

*This paper provides an indepth review on evolution of vehicle routing problems from savings matrix to time dependent vehicle routing problems. It is suggested that as to how VRPB framework of Goetschalckx and Jacobs-Blecha has offered better solution to routing problems that Clarke-Wright Savings matrix and with the introduction of time -window concept the result of heuristic has improved drastically. This is further extended by latest work on time dependent variable vehicle routing algorithm.*

**Keywords :** LHBH, GAP, Clarke-wright savings matrix, time dependent variable, vehicle routing problem.

## 1. INTRODUCTION

**T**HE vehicle routing problem (VRP) involves a set of delivery customers to be serviced by a set of vehicles housed at a depot or distribution center (DC), located in the same geographical region as the customers. The objective of the problem is to develop a set of vehicle routes such that all delivery points are serviced, the demands of the points assigned to each route do not violate the capacity of the vehicle that services the route, and the total distance traveled by all vehicles is minimized [1].

The importance of VRPB is related to the very large cost of physical distribution. The VRPB's significance can also be attributed to the continuing effort to reduce distribution costs by taking advantage of the unused capacity of an empty vehicle traveling back to the DC. The concept has helped Baroda Union to redesign their supply chain. In addition, government deregulation of interstate commerce restrictions in the Motor Carrier Act of 1980 has made it possible for backhauling to become a profitable venture for any company with a large fleet of vehicles. Commodities can now be backhauled not only for the owning company, but also for other companies who are willing to pay for the backhauls as though for common carriage. A company in Michigan increased its backhauling revenues from \$697,000 to almost \$2 million in just two distribution centers [2]. Other companies which are utilizing backhauling to generate revenues include Frito-Lay, K Mart, and Friendly Ice Cream [3]. Backhauling is truly emerging as an untapped resource for improved productivity in industry. This paper is organized as follows. A literature

review is given in Section 2, with some brief discussion of consideration for the addition of backhaul customers to the classical VRPB approaches. Also in section 2.0 some of our previously published background material is presented. Section 3 provides an explanation of the LHBH solution algorithm. The results of the computational study are detailed in Section 4, and in Section 5 conclusions are discussed.

## 2. LITERATURE REVIEW

Solution methodologies for the classical VRP include both exact and heuristic techniques. A comprehensive literature review can be found in Bodin et al., [4] and many other studies in the area of vehicle routing have been reported in the years since. Golden and Assad [5] provide an extensive review of the then-recent on vehicle routing. This section will describe how some of the methods for VRP could be adapted to VRPB, and report on current VRPB research.

### 2.1 Exact Procedures

The standard VRP can be thought of as a special case of VRPB, with the number of backhaul points equal to one (the distribution center). Since VRP is NP-complete (Lenstra and Rinnooy Kan, 1981), the VRP with backhauls is also NP-complete. The development of heuristic approaches is therefore a reasonable approach for practical applications. An exact procedure based on set covering was developed by Yano et al., [6] for a special case of the VRPB. Relaxing the special route conditions or increasing the number of backhaul points would make this exact procedure computationally intractable. Gelinas [7] also developed an exact procedure for the VRPB with

time windows which will be discussed in more detail.

## 2.2 Heuristic Procedures

The literature here proposes ways to solve the backhaul routing problem based on some well-known methods for the classical VRP. The solution methodologies are categorized according to a scheme suggested by Bodin et al. In addition to solution methodologies, work has been done to develop planning models for incorporating backhaul loads into an existing transportation system. Jordan and Burns [8, 9] examined the impact of backhauling on terminal locations and developed a method for determining which truckloads should be backhauled. Jordan extended this work to include systems with more than two terminals.

**2.2.1 Cluster-first/Route-second:** This strategy is illustrated by the sweep algorithm of Gillett and Miller [10]. The sweep approach can easily be extended to the VRPB by truncating the clusters when either linehaul or backhaul capacity is exceeded.

**2.2.2 Route-first/Cluster-second :** Extension of this approach to VRPB can be accomplished by solving a Traveling Salesman Problem (TSP) for the delivery points, then solving a TSP for the pickup points. Each of the large tours can be broken up into individual delivery and pickup routes, which can then be patched together to form line haul-backhaul routes. Goetschalckx and Jacobs [11] investigated a similar approach based on space filling curves. This method is included in the experimental comparison in Section 4.

**2.2.3 Savings/Insertion :** This concept is a constructive approach whereby a configuration of points is changed to an alternative configuration which yields a 'savings' in terms of a particular objective. Perhaps the most widely known and used savings algorithm for the VRP was developed by Clarke and Wright [12], Deif and Bodin have proposed an extension of this algorithm for VRPB. This modified Clarke-Wright method will be described in detail. Golden et al., [13] and Casco et al., [14] report on an insertion procedure for VRPB where any VRP algorithm is used to initially sequence the delivery customers. Once the line haul customers are routed, the backhaul customers are inserted onto the delivery routes according to an insertion criterion based on a penalty for pickup before the end of the

delivery route. This is a relaxation of the line haul-backhaul sequencing constraints. This approach is most applicable to the cases where there are very few backhaul points.

**2.2.4 Improvement/Exchange :** Perhaps the best known method is the r-opt algorithm of Lin and Kernighan [15]. Other exchange procedures exchange customers between routes, instead of within routes. Such methods can easily be applied to a given solution for VRPB by taking into account the precedence relationship of deliveries before pickups whenever an exchange is considered.

**2.2.5 Optimization-Based Techniques :** Min et al., [16] develop a methodology for solving the VRPB when multiple depots are involved, denoted by MDVRPB. They use a decomposition approach, determining first the delivery/pickup clusters, then assigning those clusters to depots and routes, and finally the sequencing of the route itself. In determining the delivery/pickup clusters, they use a statistical clustering method to take advantage of the spatial nature of the problem. They claim that statistical clustering is computationally more efficient than mathematical programming clustering for large number of points. In the second step, delivery route, pickup route, and depot are assigned to each other by a three dimensional assignment formulation (3DAP). They solve only the linear relaxation for their example case. Finally, the line haul-backhaul routes are constructed by an optimal asymmetrical TSP algorithm, after all the distances from pickup points to delivery points have been set to infinity. Their method is computationally efficient because, for their example, the 3DAP can be solved with linear programming and the number of points on the line haul-backhaul routes is smaller than 19. Two acclaimed approaches for VRP were presented by Fisher and Jaikumar [17, 18] and by Cullen et al., [19] and Cullen [20]. The first approach is based on the Generalized Assignment Problem and the second on Set Partitioning. Desrochers et al., [21] present a set partitioning algorithm for VRP with time windows (VRPTW) which can be used to find optimal solutions to the problem. Gelinias [22] has extended this work for VRPB. There has apparently been no further published work to date on optimization-based

heuristic methods for VRPB.

### 2.3 Mathematical Model

Goetschalckx and Jacobs-Blecha [11] developed an integer programming formulation for the VRPB problem by extending the Fisher and Jaikumar [23] formulation to include pickup points. The model naturally decomposes into three sub problems. The first two sub problems correspond to the clustering decisions for the delivery customers and the pickup customers, which are independent Generalized Assignment Problems (GAP). The third sub problem consists of  $K$  independent, single route TSPs, each having one additional constraint, enforcing the completion of all deliveries before any pickups can be made. These precedence constraints impose a dependency relationship on all the model components. This relationship is also indicative of the importance of the routing links adjoining delivery to pickup in each route. They develop an efficient and effective heuristic solution algorithm for this problem based on space filling curves. Some of the properties of the VRPB are discussed in the next section, which leads to the algorithm specification in Section 3.0.

### 2.4 Worst-case Bounds

Jacobs-Blecha (1987) showed that for Euclidean distances the VRPB routes will never be more expensive than executing separate delivery and pickup routes. The best savings occurs when the pickup and delivery customers are all collinear with the DC, and the pickup and delivery locations farthest from the DC are coincident. This example shows that the maximum savings achievable by backhauling is 50%, as opposed to sending a separate and independent truck for the backhauling. Jacobs-Blecha [24] also derived a worst case bound equal to 3 for the a simple heuristic for the VRPB by extending the results of Haimovich and Rinnooy Kan [25] for the classical VRP, whose bound equals 2.

## 3. LHBH: A GENERALIZED ASSIGNMENT HEURISTIC

The fundamental structure of a VRPB route consists of three parts. The first is a Hamiltonian path from the DC through all delivery points, ending at the delivery interface point. The second component is the interface link between delivery and pickup customers.

Third is a Hamiltonian path from the DC through all pickup points, terminating at the pickup interface point. The set of delivery customers on the delivery path comprises a sector of the plane anchored at the DC. A similar sector is defined by the set of pickup customers on the pickup path. Jacobs-Blecha (1987) showed that the best savings from backhauling can be attained by minimizing the angles of the delivery and pickup sectors as well as the angle between the delivery and pickup sectors. This property will be exploited in the initialization phase of the LHBH algorithm. The algorithm LHBH is based on the Generalized Assignment Problem, and is similar to the Fisher and Jaikumar GAP heuristic for VRP. However, this method differs most from Fisher and Jaikumar's approach in two respects. LHBH employs a fresh, new method for executing the process known as route seeding. In addition, the LHBH route seeds are extended into unique, high quality route primitives by exploiting the properties described in Sections 2.3 and 2.4.

The algorithm comprises three phases:

- Initialization
- Clustering and
- Sequencing.

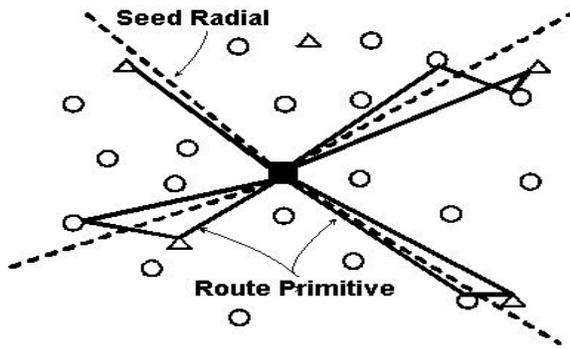
In the initialization phase, an initial solution is obtained and costs are estimated for solving the clustering problem. In phase (2), the costs from (1) are used to solve the Generalized Assignment Problem, which allocates the delivery and pickup customers to a set of minimal cost routes. Phase (3) is concerned with solving the TSP for each cluster formed in (2), taking into account the precedence relationship described in section 2.3. The following sections will explain each of these steps in further detail.

### 3.1 Clustering

In the clustering step of algorithm LHBH there are two tasks to be accomplished: (1) determine the cost of assigning a customer to a route, and (2) use the costs to make the route assignments by solving the Generalized Assignment Problem.

Once the seed radials have been established in phase two, a route primitive is generated by choosing points for the route that are near the seed radial. (In this case, an angle "distance" of 10 degrees or less was considered "near"). For each route, linehaul points

are sequenced by increasing distance to the distribution center and backhaul points by decreasing distance. Any point which is within 10 degrees of more than one seed radial is not placed in either primitive, and left unassigned. This assignment of points results in a polygonal route primitive from which the GAP costs can be determined. Such a set of route primitives is illustrated in Figure 2.



**Fig. 1:** Route Primitives for VRPB (Adapted from Goetschalckx and Blecha)

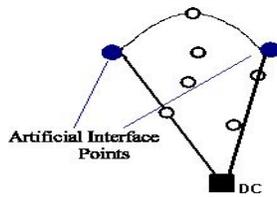
Once the route primitives are found, the Euclidean distance metric is used in the remainder of the clustering phase. For problems where customers are randomly located in the region around the DC, determining routes that fit the model of minimal sector angles is not likely to happen. In such cases, Euclidean distance is a better estimator of nearness than the ring-radial metric. In cases where customer locations are naturally clustered, the ring-radial metric would be a better choice. Since this study focuses on randomly located customers, in the following discussion the Euclidean metric is applied.

In LHBH, Martello and Toth's [26] savings regret heuristic is implemented for solving the GAP. This method assigns points to routes in a sequential manner based on a computation of the regret to be experienced by waiting until later to make the assignment. Possible improvements to the resulting routes are sought with the application of a Lagrangean heuristic which will be described below. The cost of assigning each remaining point  $i$  to route  $k$  is estimated as the minimum insertion cost of point  $i$  into the links of the primitive for route  $k$ . Since the savings regret heuristic is sequential in its assignments, each time a point is assigned to a route, the primitive for that route grows and the insertion costs for points yet to be assigned to that route may change. Thus, the cost

estimates for assigning the remaining points to the growing route primitives change dynamically. It is a simple computational update to keep the costs current as the GAP is solved. This dynamic implementation of the savings regret method has been implemented in LHBH. The GAPs for VRPB are solved in two stages. First, the dynamic sequential step builds a set of customer-route assignments based on the savings regret heuristic. Then, an improvement step is implemented by applying a Lagrangean heuristic developed by Jacobs [27]. This Lagrangean heuristic takes an initial set of dual multipliers, solves the Lagrangean to obtain a lower bound, adjusts the solution to feasibility to obtain an upper bound, adjusts the multipliers, and iterates. The procedure stops when either the difference between the upper and lower bounds is sufficiently close, or when a maximum number of iterations have been executed.

### 3.2 Route Sequencing

The sequencing problem consists of a TSP for each cluster with a side constraint restricting the tour to only one link from delivery to pickup. A practical solution method is to apply any construction heuristic followed by 2-opt and 3-opt. Golden et al., [28] have shown this to be on average within 2% of optimality for the TSP which occurs in the classical VRP. Goetschalckx and Jacobs have confirmed this for the VRPB and have shown this to be the best tradeoff between tour length and computation time. The best sequence of points for a cluster is highly dependent on the link selected as the interface between delivery and pickup. The heuristic for approximating the best sequence begins by determining a pair of artificial interface points. The cluster of delivery points geometrically defines a pie-shaped sector of the plane, anchored at the DC, (see Figure 4). The two "corner" points of this sector are candidates for the artificial delivery interface point. Similarly, there are two candidates for the artificial pickup interface point. The closest pair of candidate points, one from each sector, are selected as the initial interface points. Note that these points are artificial in the true sense of the word. It is likely that neither of them are actually located at a customer site. This selection of interface points creates an artificial interface link, joining the two sectors.



**Fig. 2:** Artificial Sequencing Interface points  
(Goetschalckx and Blecha)

Using the artificial interface link as the base, the cheapest insertion procedure is performed to create an initial feasible tour for the route. At this point, the artificial interface points are discarded and the actual interface points for pickup and delivery point are designated as the current interface points. To allow the two sides of the route to interact with each other (as the mathematical model indicates the case), two-opt and three-opt are performed in a special way. The current pickup interface point is "added" to the delivery route and forced to remain as the final point to be visited by setting its distance to the DC = -1. The two-opt and three-opt procedure is then performed on this set of points. Since the current delivery interface point is not restricted from being changed in the delivery route, the delivery interface point is then updated. This procedure is now performed for the pickup route, adding the current delivery interface point to the set, as before. The procedure then repeats until there is no further reduction in the cost of the route. This procedure converges quickly to an interface link which determines a good sequence for the overall VRPB route.

### 3.3 Summary

Algorithm LHBH can now be specified as follows:

- Initialization. Find initial seed radials by solving the location-allocation problem, utilizing the ring-radial distance metric.
- Clustering. Find polygonal route primitives from the current seed radials. Assign the delivery and pickup points to the routes using a savings regret heuristic, dynamically re-estimating the GAP costs as the points are assigned. Attempt to improve the route clusters by applying a Lagrangean heuristic.
- Routing. Heuristically solve the special TSP problem for each route by cheapest insertion, using an iterative technique to search for a good interface link. Apply two-opt and three-opt to the resulting routes.

- Iteration. Using the ring-radial metric, locate a new set of seed radials from the current route clusters.
- Convergence. If the seed radials are unchanged, or the maximum number of iterations has been executed, then stop. Otherwise, return to step 2.

## 4. TESTING AND EVALUATION

To evaluate the efficiency and effectiveness of the LHBH heuristic, the algorithm is compared computationally with three other algorithms for solving the VRPB, providing results for both solution costs and execution times (Goetschalckx and Blecha).

### 4.1 Clarke-Wright Savings Heuristic for VRPB

Clarke and Wright (1964) developed an algorithm for the vehicle routing problem based on the computation of a savings for combining two customers into the same route. Initially, each customer is considered to be on a separate route. The savings  $S_v$  for combining points  $i$  and  $j$  into a single route for a symmetric distance matrix is then computed as:

$$S_v = d_{oi} + d_{oj} - d_{ij}$$

where  $d_{ab}$  is the distance from point  $a$  to point  $b$  and point  $o$  is the DC.  $S_v$  is computed for every pair of points  $i$  and  $j$  and arranged in non-increasing order.

Deif and Bodin [29] proposed an extension of this algorithm in an effort to produce good solutions to the VRPB. Their procedure is based on two modifications to the original Clarke-Wright algorithm. The first modification adds the constraint that only one link from delivery to pickup (or vice versa) is allowed on any route. Second, the definition of savings is modified to include a penalty to reduce the size of savings for a changeover from delivery to pickup. The modified savings computation is:

$$S_v = d_{oi} + d_{oj} - d_{ij} - \alpha \hat{S}$$

where  $\hat{S}$  is an estimate of the maximum value of savings and  $\alpha$  is a penalty multiplier. If  $i$  and  $j$  are either delivery points, or both pickup points, then is 0. The maximum savings is estimated by as an efficiency measure, since the savings list grows as a quadratic function of the number of customers. The penalty reduces the size of the savings for a changeover from delivery to pickup.

## 4.2 Solving VRPB with Sequential VRPs

An alternative approach for solving the VRPB is to treat the problem as two separate VRP's, forming routes for the delivery points independently of the pickup points, and vice versa. A solution to VRPB can then be formed by simply patching the two sets of routes into a single set of routes by matching the delivery and pickup routes.

To solve the two separate routing problems, the Generalized Assignment heuristic method, introduced by Fisher and Jaikumar [30] was selected, and will be referred to as ROVER. The ROVER algorithm is a benchmark that includes an exact GAP solution, developed by Fisher, Jaikumar, and Van Wassenhove [31]. Once the VRPs are solved, the VRPB solution is completed by pairing the delivery routes need with the pickup routes. This can be accomplished by solving a simple assignment problem with an appropriate definition of the cost of assigning a pickup route to a delivery route. Two approaches were developed.

The first approach is a simple one which is computationally inexpensive. For each pair of routes, only points adjacent to the depot on the separate routes are considered as candidates for interface points. This means that four pairs of delivery and pickup points are considered, and the one yielding the most savings in travel distance determines the candidate interface link and the assignment cost for that pair of routes. The cost matrix for the assignment problem is found by computing this savings for all pairs of routes from the VRP solutions. The VRPB algorithm incorporating this simple matching solution will be denoted by RVR. The first method of computing the assignment costs clearly may not yield the best pairing, nor the best interface links for the resulting set of paired routes. To find the best set of interface links for two given sets of delivery and pickup routes, it is necessary to consider every delivery point with every pickup point as a candidate interface, for every pair of routes. For each of these candidates, a TSP solution must then be computed, and the total cost of the resulting routes compared. The TSP problems were solved by cheapest insertion followed by 2-opt and 3-opt. This time-consuming approach will be referred to as RVRBST. A greedy nearest-neighbor heuristic based on travel time between customers was proposed, as well as a branch and cut algorithm to

solve TDVRP without time windows. Computational results for one vehicle and five customers were reported. The modifications to the savings, insertion, and local improvement algorithms to better deal with TDVRP. In randomly generated instances, they reported computation time reductions as a percentage of "unmodified" savings, insertion, and local improvement algorithms. An important property for time dependent problems is the First In - First Out (FIFO) property. A model with a FIFO property guarantees that if a vehicle leaves customer  $i$  to go to customer  $j$  at any time  $t$ , any identical vehicle with the same destination leaving customer  $i$  at a time  $t + \epsilon$ , where  $\epsilon > 0$ , will always arrive later. This is an intuitive and desirable property though it is not present in all models. Earlier formulations and solutions methods, Malandraki and Daskin (1989, 1992), Hill and Benton (1992), and Malandraki and Dial (1996), do not guarantee the FIFO property as reported by Ichoua et al. (2003). Later research efforts have modeled travel time variability using "constant speed" time periods which guarantees the FIFO property, as shown by Ichoua et al. (2003). Ichoua et al. (2003) proposed a tabu search solution method, based on the work of Taillard et al. (1997), in order to solve time dependent vehicle routing problems with soft time windows. Ichoua et al. showed that ignoring time dependency, i.e. using VRP models with constant speed, can lead to poor solutions. Ichoua et al. tested their method using the Solomon problem set, soft time windows, three time periods, and three types of time dependent arcs. The objective was to minimize the sum of total travel time plus penalties associated with time window violations. Fleischmann et al. (2004) utilized route construction methods already proposed in the literature, savings and insertion, to solve uncapacitated time dependent VRP with and without time windows. Fleischmann et al. tested their algorithms in instances created from Berlin travel time data. Jung and Haghani proposed a genetic algorithm to solve time dependent problems. Using randomly generated test problems, the performance of the genetic algorithm was evaluated by comparing its results with exact solutions (up to 9 customers and 15 time periods) and a lower bound (up to 25 customers and 10 time periods). Ichoua et al. (2003) used the widely known Solomon problems for the

VRP with time windows. However, capacity constraints were not considered, optimal fleet size was given, and no details were provided regarding how links were associated with "categories" that represent differences in the urban network (i.e. main arteries, local streets, etc.). Donati et al. also used Solomon instances, however, the results cannot be compared with previous results by Ichoua et al. because a different time speed function was used and capacity constraints were considered. Comparisons are also problematical because objective functions and routing constraints for time dependent problems are often dissimilar, unlike VRPTW research where the objective function is hierarchical and usually considers fleet size (primary objective), distance (secondary objective), and total route duration. Ichoua et al. study the TDVRP with soft time windows and consider as the objective function total duration plus lateness and assume that the optimal fleet size is given a priority (Dubey et.al. [32]).

## 5. CONCLUSIONS AND FURTHER RESEARCH

The computational results indicate that the proposed RD heuristics can solve soft and hard time window time-dependent vehicle routing problems in relatively small computation times. The analysis and experimental results of the computational complexity indicate that average computational time increases proportionally to the square of the number of customers. The solution quality of the new algorithm appears to be comparable to other approaches that can be used to solve constant speed and soft time windows problems with time dependent speeds. However, the proposed RD approach seems to have an advantage in TDVRP with hard time windows; problems that cannot be readily tackled by local search heuristics and have not yet been studied in the literature.

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