

Effect of Artificial Viscosity on the Expansion of Dis-Continuities in a Rotating Interplanetary Medium with Material Pressure

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ABSTRACT

The solution of equations by seeking quasi-similar solution, in which the viscosity coefficient is taken to be at most a function of time but independent of space co-ordinates. an attempt is made to account for the material strength by including Newtonian- Viscosity term. In the present paper the characteristic method (Chester, Witham) is applied to obtain expressions of the density, the pressure, the particle velocity just behind the shock propagating in a rotating atmosphere. The effect of coriolis force is taken into account. Since the velocity effect has a tendency to smoothen out such discontinuities, the artificial viscosity coefficient suggested by Rithchmyer and Von Newmann is introduced. The problem is discussed for two different cases (i) for weak shocks and (ii) for strong shocks respectively.

Keywords: viscosity coefficient, artificial viscosity, shock velocity, non-ideal radiating gas

1. INTRODUCTION

The propagation of strong shock waves in space, due to a surface explosion or impact has been treated in many levels of approximation in one of these, an attempt is made to account for the material strength by including Newtonian- Viscosity term. Yuan [1] approximated the solution of equations by seeking quasi-similar solution, in which the viscosity coefficient is taken to be at most a function of time but independent of space co-ordinates. Pai [2] and Kumar [3] discussed the propagation of hydromagnetic cylindrical shock through a self-gravitating gas showing its velocity only for strong shocks. Singh and Mishra [4] obtained analytical relations for shock velocity and shock strength and the expressions for the pressure, the density and the particle velocity immediately behind the shock, assuming the fact that the initial density and azimuthal magnetic field are distribution variables. Vishwakarma [5] has obtained an exact analytic solution for self-similar flow behind

a magnetogasdynamic shock waves in radiative and self-gravitating gas. Nath[6] has discussed similarity solution for unsteady flow behind an exponential shock in a dusty gas. Vishwakarma et al. [7] have been discussed an analytical description of converging shock waves in a gas with variable density. Similarity solution for a cylindrical shock wave in a rotational axisymmetric dusty gas with heat conduction and radiation heat flux have also been studied by Vishwakarma and Nath [8]. Singh et al. [9] have been studied the evolution of weak discontinuities in a non-ideal radiating gas. Recently Singh et al. [10] have also been obtaining the evolution of weak shock waves in perfectly conducting gases. Liang and Chen [11] have discussed numerical study of spherical blast wave propagation and reaction. Srivastav and Yadav [12] have studied propagation of plane shock radiative shock wave in unstable homogeneous medium. the characteristic method (Chester [13], Witham [14]) is applied to obtain expressions of the density, the pressure, the particle velocity just behind

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the shock propagating in a rotating atmosphere. The effect of Coriolis force is taken into account. Since the velocity effect has a tendency to smoothen out such discontinuities, the artificial viscosity coefficient suggested by Richtmyer and von Neumann [15] is introduced. The problem is discussed for two different cases.

2. BASIC EQUATIONS, BOUNDARY CONDITIONS AND ANALYTICAL EXPRESSION FOR SHOCK VELOCITY

The equations governing the cylindrically symmetric flow of gas under the influence of its own Coriolis forces in the presence of transverse magnetic field, following Witham [14]

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial}{\partial r} (p + p_r + \bar{q}) + \frac{H}{\rho} \frac{\partial H}{\partial r} + \frac{H^2}{\rho r} - \frac{v^2}{r} = 0, \tag{1}$$

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} + \rho \frac{\partial u}{\partial r} + \rho \frac{u}{r} = 0, \tag{2}$$

$$\frac{\partial H}{\partial t} + u \frac{\partial H}{\partial r} + H \frac{\partial u}{\partial r} = 0, \tag{3}$$

$$\frac{\partial}{\partial t} (p + p_r) + u \frac{\partial}{\partial r} (p + p_r) \left\{ \frac{\gamma(p + p_r) + (\gamma - 1)\bar{q}}{\rho} \right\} \left(\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial r} \right) = 0, \tag{4}$$

$$\frac{\partial}{\partial t} (v_r) + u \frac{\partial}{\partial r} (v_r) = 0, \tag{5}$$

and $\bar{q} = \frac{1}{2} K^2 \rho r^2 \frac{\partial u}{\partial r} \left\{ \left| \frac{\partial u}{\partial r} \right| - \left(\frac{\partial u}{\partial r} \right) \right\}$ \tag{6}

where u, p, p_r, H, ρ represent the velocity, pressure, material pressure, magnetic field at distance r and time t and the density and ' \bar{q} ' is the artificial viscosity.

Since ahead the shock wave $u = 0 \Rightarrow \bar{q} = 0$, and $\rho_0 = \rho_c r^{-\omega}$ and $p_0 + p_{r_0} = H_c r^{-\bar{\omega}}$,

where $\omega = \frac{\bar{\omega}}{2}$,

$$\frac{\partial m}{\partial r} - 2\pi r \rho = 0, \text{ (for cylindrical shock wave)}$$

The magnetic-hydrodynamic condition can be

written in terms of single parameter $N = \frac{\rho_1}{\rho_0}$ are

$$\rho_1 = N \rho_0, \quad H_1 = N H_0, \quad u_1 = \left(1 - \frac{1}{N}\right) u$$

$$\bar{q} = q_0, \quad v = v_0$$

$$u^2 = \frac{2N}{(\gamma + 1) - (\gamma - 1)N} \left[a_0^2 + \frac{b_0^2}{2} \{(2 - \gamma)N + \gamma\} \right]$$

$$(p_1 + p_{r_1}) = (p_0 + p_{r_0}) + \frac{2\rho_0(N - 1)}{(\gamma + 1) - (\gamma - 1)N}$$

$$\left\{ a_0^2 + \frac{(\gamma - 1)}{4} b_0^2 (N - 1)^2 \right\},$$

where 0 and 1 respectively stand for the states just ahead and just behind the shock front, u is the shock velocity, a_0 is the sound speed given by

$$\left[\frac{\gamma(p_0 + p_{r_0})}{\rho_0} \right]^{1/2} \text{ and } b_0 \text{ is the Alfvén speed}$$

$$\left(\frac{\mu H_0^2}{\rho_0} \right)^{1/2}, \text{ where } \mu = 1 \text{ is magnetic permeability.}$$

2.1 Weak Shock

For a very weak shock we take the parameter as

$$\frac{\rho_1}{\rho_0} = N = 1 + \epsilon,$$

where ϵ is another parameter which is negligible in comparison with unity, now we consider two cases of weak and strong magnetic field.

Case -I: For weak magnetic field $\frac{b_0^2}{a_0^2} \ll 1$ under this condition the boundary condition reduces to as follows

$$\rho_1 = \rho_0 (1 + \epsilon), \quad q = q_0, \quad v = v_0, \quad H_1 = H_0 (1 + \epsilon),$$

$$u_1 = \left(1 - \frac{1}{N}\right) u = \epsilon q_0$$

$$= \left(1 - \frac{1}{1 + \epsilon}\right) u = \left(\frac{\epsilon}{1 + \epsilon}\right) u = \epsilon (1 + \epsilon)^{-1} u$$

$u_1 = \epsilon (1 - \epsilon) u = \epsilon u = \epsilon a_0$ (by neglecting the higher degree term of ϵ) now since

$$u^2 = \frac{2N}{(\gamma + 1) - (\gamma - 1)N} \left[a_0^2 + \frac{b_0^2}{2} \{(2 - \gamma)N + \gamma\} \right]$$

since, $\frac{b_0^2}{a_0^2} \ll 1$

$$u = \left(1 + \frac{\epsilon}{2}\right) \left\{1 + \frac{(\gamma + 1)}{4} \epsilon\right\} a_0$$

$$= \left(1 + \frac{\gamma + 1}{4} \epsilon\right) a_0 \quad (\text{by neglecting the higher order terms } \epsilon)$$

der terms ϵ)

$$\text{And } (p_1 + p_{r1}) = (p_0 + p_{r0}) + \frac{2\rho_0 \epsilon a_0^2}{\gamma + 1 - \gamma + 1 - \gamma \epsilon + \epsilon}$$

$$\text{where } a_0^2 = \frac{\gamma (p_0 + p_{r0})}{\rho_0}$$

$$= (p_0 + p_{r0}) \left[1 + \gamma \epsilon \left\{1 + \frac{\gamma - 1}{2} \epsilon + \dots\right\}\right]$$

$$= (p_0 + p_{r0}) [1 + \gamma \epsilon]$$

and $\bar{q} = q_0, \quad v = v_0,$ remain remain same as before.

Case - II: For strong magnetic field $\frac{b_0^2}{a_0^2} \gg 1$, then the boundary conditions are reduces to

$$\rho_1 = \rho_0 (1 + \epsilon), \quad H_1 = H_0 (1 + \epsilon),$$

$$\bar{q} = q_0 \quad \text{and} \quad v = v_0,$$

$$u^2 = \frac{2N}{(\gamma + 1) - (\gamma - 1)N}$$

$$\text{Since } \left[a_0^2 + \frac{b_0^2}{2} \{(2 - \gamma)N + \gamma\} \right]$$

$$= \left[1 + \frac{1}{2} \left(2 - \frac{\gamma}{2}\right) \epsilon\right] \left[1 + \frac{1}{4} (\gamma - 1) \epsilon\right] b_0$$

$$u = b_0 \left[1 + \frac{3}{4} \epsilon\right], \quad (\text{by neglecting the higher order terms } \epsilon)$$

$$\text{since } u_1 = \epsilon (1 - \epsilon) u \quad u_1 = \epsilon u \quad u_1 = \epsilon b_0,$$

$$p_1 + p_{r1} = (p_0 + p_{r0}) + \frac{2\rho_0 (1 + \epsilon - 1)}{(\gamma + 1) - (\gamma - 1)(1 + \epsilon)} b_0^2$$

$$\text{and } \left(\frac{\gamma - 1}{4}\right) (1 + \epsilon - 1)^2,$$

$$= (p_0 + p_{r0}) + b_0^2 \rho_0 \epsilon^3 \left[1 + \frac{\gamma - 1}{2} \epsilon\right],$$

2.2 Strong Shock

In the limiting case of a strong shock $\frac{\rho_1}{\rho_0}$ is large,

and in the presence of the magnetic case this can be brought about in two ways

Case-I : The purely non-magnetic way when

$$N = \frac{\gamma + 1}{\gamma - 1} \text{ is small.}$$

Case-II : When $b_0^2 \gg a_0^2$ or when $\mu H_0^2 \gg \rho_0$, that is when the ambient magnetic pressure is large compared with the ambient fluid pressure in terms of N . The boundary conditions are now becomes as follow

$$\rho_1 = N \rho_0, H_1 = N H_0, u_1 = \left(1 - \frac{1}{N}\right)u$$

$$(p_1 + p_{r1}) = (p_0 + p_{r0}) +$$

Since,
$$\frac{2 \rho_0 (N-1)}{(\gamma+1) - (\gamma-1)N} \left[a_0^2 + \frac{b_0^2}{4} (\gamma-1)(N-1)^2 \right],$$

where
$$a_0^2 = \frac{\gamma(p_0 + p_{r0})}{\rho_0},$$

hence
$$\left(\frac{p_1 + p_{r1}}{p_0 + p_{r0}}\right) = Y(N) + X(N) \frac{u^2}{a_0^2},$$

where
$$Y(N) = \frac{(\gamma+1)N - (\gamma-1)}{(\gamma+1) - (\gamma-1)N}$$
 and

$$X(N) = \frac{\gamma(\gamma-1)}{2N} \frac{(N-1)^3}{\{(2-\gamma)N - \gamma\}}$$

3. CHARACTERISTIC EQUATION AND SOLUTION OF EQUATIONS

For diverging shock the characteristic form of system of equation (1) to (5) that is form in which equation contains derivatives only in (r,t) is

$$\begin{aligned} \frac{dp}{dt} + \frac{dp_r}{dt} + \rho c \frac{du}{dt} + H \frac{dH}{dt} - \frac{H^2(u-c)}{(u+c)} \frac{dr}{r} + \\ \frac{\rho c^2 u}{(u+c)} \frac{dr}{r} - \frac{\rho c v^2}{(u+c)} \frac{dr}{r} + \frac{\partial \bar{q}}{\partial r} \frac{dr}{(u+c)} \\ + \frac{\bar{q}(\gamma-1)u}{(u+c)} \frac{dr}{r} = 0 \end{aligned} \tag{7}$$

is a characteristic form of equations (1) to (5) [in positive direction],

where
$$c^2 = a^2 + b^2 = \frac{\gamma(p + p_r)}{\rho} + \frac{H^2}{\rho}, \mu = 1$$

for final step we substitute the shock condition (4) or (5) or (6) in equation (7), first order differential equation in ϵ (r) or u^2 is obtained which determines

the shock, for weak shocks in the presence of a weak transverse magnetic field, on substituting the shock condition (4) in (7), neglecting the higher-order terms of ϵ , since ϵ is very less than unity.

where, $H_1 = H_0(1+\epsilon) \Rightarrow dH_1 = dH_0(1+\epsilon) + H_0 d\epsilon,$

$$u_1 = \epsilon a_0 \Rightarrow du_1 = \epsilon da_0 + a_0 d\epsilon$$

$$H_1 dH_1 = H_0(1+\epsilon)[dH_0(1+\epsilon) + H_0 d\epsilon]$$

$$\Rightarrow H_0(1+\epsilon)^2 dH_0 + H_0^2(1+\epsilon) d\epsilon$$

$$= (H_0 + 2\epsilon H_0) dH_0 + H_0^2 d\epsilon$$

(by neglecting the higher terms of ϵ and product $\epsilon d\epsilon$)

$$H_1 dH_1 = H_0 dH_0 + 2\epsilon H_0 dH_0 + H_0^2 d\epsilon$$

since, $v_1 = v_0 \Rightarrow dv_1 = dv_0$

$$\bar{q} = q_0 \Rightarrow d\bar{q} = dq_0,$$

now we know that

$$c^2 = a^2 + b^2 = \frac{\gamma(p + p_r)}{\rho} + \frac{H^2}{\rho}$$

for weak magnetic field equation (7), becomes as

$$2 + \left[\frac{H_0^2}{\gamma(p + p_{r0})} \right] d\epsilon + \left[\frac{dp_0 + dp_{r0}}{(p_0 + p_{r0})} + \frac{dH_0^2}{(p_0 + p_{r0})} + \frac{dr}{r} + \frac{da_0}{a_0} - \frac{1}{\gamma(p_0 + p_{r0})} \frac{\partial \bar{q}}{\partial r} dr + \frac{\bar{q}(\gamma-1)}{\gamma(p_0 + p_{r0})} \right] \epsilon$$

$$= - \frac{1}{q_0^2} \left[\frac{dp_0}{\rho_0} + \frac{dp_{r0}}{\rho_0} + \frac{1}{2} \frac{dH_0^2}{\rho_0} + \frac{H_0^2}{\rho_0} \frac{dr}{r} - v^2 \frac{dr}{r} - \frac{1}{\rho_0} \frac{\partial \bar{q}}{\partial r} dr \right], \tag{8}$$

since in view of hydrostatic equilibrium

$$v^2 \frac{dr}{r} = \frac{dp_0}{\rho_0} + \frac{dp_{r0}}{\rho_0} + \frac{1}{2} \frac{dH_0^2}{\rho_0} + \frac{H_0^2}{\rho_0} \frac{dr}{r} - \frac{1}{\rho_0} \frac{\partial \bar{q}}{\partial r} dr \quad (9)$$

hence equation (8) becomes

$$\left[2 + \frac{H_0^2}{\gamma(p_0 + p_{r0})} \right] d\epsilon + \left[\frac{dp_0 + dp_{r0}}{(p_0 + p_{r0})} + \frac{dH_0^2}{\gamma(p_0 + p_{r0})} + \frac{dr}{r} + \frac{da_0}{a_0} - \frac{1}{\gamma(p_0 + p_{r0})} \frac{\partial \bar{q}}{\partial r} dr + \frac{\bar{q}(\gamma - 1)}{(p_0 + p_{r0})} \frac{dr}{r} \right] \epsilon = 0,$$

$$\frac{d\epsilon}{\epsilon} = -\frac{1}{2} \left(1 - \frac{H_0^2}{2\gamma(p_0 + p_{r0})} \right) \frac{dp_0 + dp_{r0}}{(p_0 + p_{r0})} + \frac{dH_0^2}{\gamma(p_0 + p_{r0})} + \frac{dr}{r} + \frac{da_0}{a_0} - \frac{1}{\gamma(p_0 + p_{r0})} \frac{\partial \bar{q}}{\partial r} dr + \frac{\bar{q}(\gamma - 1)}{\gamma(p_0 + p_{r0})} \frac{dr}{r} \quad (10)$$

where $\gamma(p_0 + p_{r0}) = \rho_0 a_0^2$

Since equation of mass for cylindrical shock wave is

$$\frac{\partial m}{\partial r} - 2\pi \gamma \rho = 0,$$

where, $\rho_0 = \rho_c r^{-\omega}$, $H_0 = H_c r^{-\bar{\omega}}$, $\mu = 0$, $\bar{q} = 0$

ahead the shock wave $\bar{\omega} = \frac{\omega}{2}$,

at $r = R$, $\rho = \rho_0$

$$\frac{\partial m}{\partial r} = 2\pi R \rho_0 = 2\pi R \rho_c R^{-\omega},$$

on integration, we get

$$m = 2\pi \rho_c \frac{R^{2-\omega}}{2-\omega} = \frac{2\pi \rho_c r^{2-\omega}}{2-\omega}$$

Since equation of motion at a head the shock, become as

$$(p + p_r) = - \left[\frac{v^2 \rho_c R^{-\omega}}{\omega} + \frac{(\omega - 1) H_c^2 R^{-2\omega}}{2\bar{\omega}} \right] \quad (11)$$

if, $\bar{\omega} = \frac{\omega}{2}$,

$$p + p_r = - \left[\frac{\rho_c v^2 R^{-\omega}}{\omega} + \frac{\left(\frac{\omega}{2} - 1\right) H_c^2 R^{-\omega}}{\omega} \right] = \frac{\rho_c v^2 R^{-\omega}}{\omega} + \frac{\left(1 - \frac{\omega}{2}\right) H_c^2 R^{-\omega}}{\omega}$$

$$= \left[-\frac{\rho_c v^2}{\omega} + \frac{H_c^2}{2\omega} \right] R^{-\omega},$$

if we take $K_1 = -\frac{\rho_c v^2}{\omega} + \frac{H_c^2}{2\omega}$,

$$p + p_r = K_1 R^{-\omega}, \quad (12)$$

and a_0 is given by $a_0^2 = \frac{\gamma(p_0 + p_{r0})}{\rho_0}$

$$a_0 = \sqrt{\frac{\gamma(p_0 + p_{r0})}{\rho_0}} = \sqrt{\frac{\gamma K_1 \bar{R}^\omega}{\rho_c \bar{R}^\omega}} = \sqrt{\frac{\gamma K_1}{\rho_c}} = K_2,$$

$$a_0 = K_2 \text{ which is constant.} \quad (13)$$

This implies that as pressure varies, in similar way density varies positively and finiteness of the equilibrium pressure as defined by equation (12) requires that the constant ω should obey the inequality

$$1 < \omega < 2, \quad (14)$$

$$\frac{\partial \epsilon}{\epsilon} = \frac{1}{4} [2(\omega - 1) + \beta^2(\omega + 1)]$$

$$\frac{dr}{r} + \frac{1}{4} \frac{(2 - \beta^2)}{\gamma k_1} [r^\omega \partial \bar{q} - r^{-\omega-1} \bar{q}(\gamma - 1) dr], \quad (15)$$

now by integration, we get,

$$\epsilon = k r^{\frac{1}{4} [2(\omega-1) + \beta^2(\omega-1)]} e^{\frac{1}{4} \frac{(2-\beta^2)\bar{q}r^\omega}{\gamma k_1 \omega} [\omega-\gamma+1]} \quad (16)$$

where k is the constant of integration

since $u = \left(1 + \frac{\gamma+1}{4}\epsilon\right)a_0$, and $a_0 = k_2$,

$$\frac{u}{k_2} = 1 + \frac{\gamma+1}{4} k r^{\frac{1}{4} [2(\omega-1)]} e^{\frac{1}{4} \frac{(2-\beta^2)r^\omega}{\gamma k_1 \omega} [\omega-\gamma+1]\bar{q}} \quad (17)$$

$$\frac{u}{a_0} = 1 + \frac{\gamma+1}{4} k r^{\frac{1}{4} [2(\omega-1) + \beta^2(\omega+1)]} e^{\frac{1}{4} \frac{(2-\beta^2)r^\omega}{\gamma k_1 \omega} [\omega-\gamma+1]\bar{q}} \quad (18)$$

Strong magnetic field:- For strong magnetic field, we use shock conduction (5) in equation (7) and hence obtain

$$\frac{d\epsilon}{\epsilon} = -\frac{1}{2} \left[1 - \frac{\gamma(p_0 + p_{r0})}{2H_0^2} \right] \left[\frac{\gamma d(p_0 + p_{r0})}{H_0^2} + 2 \frac{dH_0}{H_0} + \frac{db_0}{b_0} + \frac{dr}{r} - \frac{\partial \bar{q}}{\partial r} \frac{dr}{H_0^2} + \frac{(\gamma-1)\bar{q}}{H_0^2} \frac{dr}{r} \right] \quad (19)$$

pressure and density both are varying in similar manner; this effect gives us b_0 as constant hence

$$\frac{db_0}{b_0} = 0,$$

by substituting the values of $d(p_0 + p_{r0})$, dH_0 and $\frac{db_0}{b_0}$ in equation (19) and by integration, we get

$$\epsilon = \bar{K} r^{\frac{1}{4} \left[\frac{3\omega-1}{\beta^2} + 2(1-\omega) \right]} e^{-\frac{1}{2} \left[1 + \frac{1}{2\beta^2} \right] \bar{q} r^\omega \frac{(\gamma-1)}{\omega} k_1} \quad (20)$$

where \bar{K} is the constant of integration.

Since $u = \left(1 + \frac{3}{4}\epsilon\right)b_0$,

substituting the value of ϵ , we have

$$\frac{u}{k_2} = 1 + \frac{3}{4} \bar{r}^{\frac{1}{4} \left[\frac{3\omega-1}{\beta^2} + 2(1-\omega) \right]} e^{\frac{1}{2} \left[1 - \frac{1}{2\beta^2} \right] \frac{(\gamma-1)}{\omega} \bar{q} r^\omega} \quad (21)$$

$$\frac{u}{a_0} = 1 + \frac{3}{4} \bar{K} r^{\frac{1}{4} \left[\frac{3\omega-1}{\beta^2} + 2(1-\omega) \right]} e^{\frac{1}{2} \left[1 - \frac{1}{2\beta^2} \right] \frac{(\gamma-1)}{\omega} \bar{q} r^\omega} \quad (22)$$

4. RESULT AND DISCUSSION

Finally the expression for the pressure and particle velocity just behind the shock for above can be expressed as

$$p + p_r = K_1 r^{-\omega} + K K_1 r^{\frac{1}{4} [-2(\omega+1) + \beta^2(\omega+1)]} e^{\frac{1}{4} [(2-\beta^2)(\gamma-1)\bar{q}r^\omega / \gamma k_1 \omega]}, \quad (23)$$

$$\rho = \rho_c \bar{r}^\omega + K \rho_c r^{\frac{1}{4} [-2(\omega+1) + \beta^2(\omega+1)]} e^{\frac{1}{4} [(2-\beta^2)(\gamma-1)\bar{q}r^\omega / \gamma k_1 \omega]}, \quad (24)$$

$$u = \epsilon a_0 + K K_2 r^{\frac{1}{4} [-2(\omega-1) + \beta^2(\omega+1)]} e^{\frac{1}{4} [(2-\beta^2)(\gamma-1)\bar{q}r^\omega / \gamma k_1 \omega]}, \quad (25)$$

$$p + p_r = K_1 \bar{r}^\omega + \bar{K} K_1 r^{\frac{1}{4} \left[\frac{3\omega-1}{\beta^2} + 2(1-3\omega) \right]} e^{\frac{1}{2} \left[1 - \frac{1}{2\beta^2} \right] (\gamma-1)\bar{q}r^\omega / \gamma \omega k_1 \beta^2}, \quad (26)$$

$$\rho = \rho_c \bar{r}^\omega + \rho_c \bar{K} r^{\frac{1}{4} \left[\frac{3\omega-1}{\beta^2} + 2(1-3\omega) \right]} e^{\frac{1}{2} \left[1 - \frac{1}{2\beta^2} \right] (\gamma-1)\bar{q}r^\omega / \gamma \omega k_1 \beta^2}, \quad (27)$$

$$u = \bar{K} K_2 r^{\frac{1}{4} [2(\omega-1) + \beta^2(\omega+1)]} e^{\frac{1}{2} \left[1 - \frac{1}{2\beta^2} \right] (\gamma-1)\bar{q}r^\omega / \gamma \omega k_1 \beta^2}, \quad (28)$$

The expression (23) and (28) represent the propagation of weak diverging cylindrical shock wave through a rotating gas with effect of artificial viscosity in the presence of transverse weak and strong magnetic field respectively.

In above relations first term represents the solution obtained by the Witham, showing the effect of density distribution. The second term arises obviously on account of the coriolis force and the magnetic field. The velocity of magnetogasdynamic shock wave is determined by first term only when magnetic field is strong, for the large values of r , the first term is very small in comparison to the second term.

Table : 1
Weak Shock, Weak Magnetic Field

| N = 0.5, $\bar{q} = 3.2, \omega = 0.5$ | | | | | |
|--|------------------|------------------|-----------------|------------------|------------------|
| $\beta^2 = 1.2$ | | | $\beta^2 = 2.4$ | | |
| r | u/K ₂ | u/a ₀ | r | u/K ₂ | u/a ₀ |
| 0.1 | 10357.24 | 10357.24 | 0.1 | 12450.92 | 12450.92 |
| 0.2 | 13249.83 | 15463.14 | 0.2 | 16776.68 | 14227.91 |
| 0.3 | 17361.19 | 21802.29 | 0.3 | 23850.72 | 22776.37 |
| 0.4 | 21782.37 | 38153.14 | 0.4 | 30185.50 | 31980.03 |
| 0.5 | 35970.96 | 44093.76 | 0.5 | 39896.19 | 34530.60 |

From Table : 1 gives the variation of shock velocity and shock strength with propagation distance for different values of parameters for weak shock with weak magnetic field. The nature of shock velocity of shock strength can be read through tables, from table we observed that shock velocity and shock strength are increasing.

Table : 2 Weak Shock, Strong Magnetic Field

| N = 7.5, $\bar{q} = 3.2, \omega = 0.5$ | | | | | |
|--|------------------|------------------|-----------------|------------------|------------------|
| $\beta^2 = 1.2$ | | | $\beta^2 = 2.4$ | | |
| r | u/K ₂ | u/a ₀ | r | u/K ₂ | u/a ₀ |
| 0.1 | 11789.49 | 11789.49 | 0.1 | 11688.80 | 11688.80 |
| 0.2 | 13805.33 | 24026.60 | 0.2 | 13324.01 | 15693.13 |
| 0.3 | 14539.93 | 30798.32 | 0.3 | 17160.47 | 18112.92 |
| 0.4 | 15104.31 | 34770.16 | 0.4 | 18932.53 | 20932.19 |
| 0.5 | 18509.91 | 48565.21 | 0.5 | 19817.60 | 22761.45 |

Table : 2, gives the variation of shock velocity and shock strength with propagation distance for different values of parameters for weak shock with strong magnetic field from table we observed that shock velocity and shock strength are increasing.

Table : 4
Strong Shock, Strong Magnetic Field

| N = 7.5, $\bar{q} = 3.2, \omega = 0.5$ | | | | | |
|--|------------------|------------------|-----------------|------------------|------------------|
| $\beta^2 = 1.2$ | | | $\beta^2 = 2.4$ | | |
| r | u/K ₂ | u/a ₀ | r | u/K ₂ | u/a ₀ |
| 0.1 | 3684.33 | 3684.33 | 0.1 | 3896.35 | 3896.35 |
| 0.2 | 2965.72 | 1542.90 | 0.2 | 2916.53 | 1000.19 |
| 0.3 | 2726.59 | 988.24 | 0.3 | 2801.87 | 466.92 |
| 0.4 | 2659.52 | 461.41 | 0.4 | 2763.52 | 340.92 |
| 0.5 | 2373.29 | 336.34 | 0.5 | 2529.97 | 247.16 |

Table : 3, gives the variation of shock velocity and shock strength with propagation distance having different values of parameters, for strong shock with strong magnetic field, from table we observed that shock velocity and shock strength are decreasing.

5. CONCLUSION

From our results we find that the applications of coriolis forces changes in shock velocity and shock strength are very slow in comparison to the results of given by Singh and Mishra [4].

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