Feedback Control of Chaos in Porous Medium under G-jitter Effects

Ajay Singh[*1], B.S. Bhadauria[2], and Prashant Kumar Gangwar[3]

1.* Assitant Professor, Department of Humanities and Applied Sciences, School of Management Sciences,, Lucknow- 226501 (UP) India.; e-mail:ajaysingh0044@gmail.com,

2. Professor, School of Physical and Decision Sciences, Babasaheb Bhimrao Ambedkar University, Lucknow, (U.P.), India. e-mail : mathsbsb@yahoo.com

3. Head of Department, Civil Engineering, School of Management Sciences, Lucknow- 226501(UP) India; e-mail: er.prashantgangwar@gmail.com,

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***Corresponding author :**
Ajay Singh
e-mail : ajaysingh0044@gmail.com

Abstract

In the present paper, we studied feedback control of chaotic convection in porous medium under gravity modulation. A non-autonomous system having three differential equations is obtained by employing the truncated Galerkin expansion method in to the modulated momentum and energy equations, called as Lorenz system in the literature. The parameter R demonstrates either periodic or chaotic behavior of the system as increasing R. It is also found that the influence of amplitude of modulation is to advance the chaotic nature in the system whereas the feedback control and frequency of modulation parameters have tendency to delay the chaotic behavior.

1. INTRODUCTION

The problem related to convection (thermal instability) in a fluid-saturated porous medium is used from last four decades till now due to its demand in various fields such as research, industry, chemical engineering sciences geothermal energy utilization, building thermal insulation, and nuclear sciences. To understand the onset of convection or mechanism of thermal instability between the fluid layers, there are some books available in the literature named as Ingham and Pop [1], Nield and Bejan [2] and Vafai [3]. The chaotic analysis provide only predictable solutions not exact and apply in typical problems as a paradigm weather sciences in the form of Lorenz system. Some other fields where chaos concept is applicable are as dynamics of satellites in the solar system, thermal insulation and geothermal energy utilization. A lot of studies on chaotic convection are performed by many researchers since last few decades which help us to research and applications. Firstly, the great man Poincaré [4] who declared the effect of chaos for a dynamical system which is very sensitive to initial conditions. After that Edward Lorenz [5] studied the system of three ordinary differential equations and gave the model for atmospheric convection, the similar results have been computed in. [6] Rossler [7], Chen and Ueta [8] and Long et al. [9] studied the chaotic system for different model. A number of papers on chaotic convection in a porous medium is studied by Vadasz. [10-14] In these papers he obtained the analytical, numerical results and seen how system moves from steady to chaotic and vice-versa. Recently, Vadasz [15], Bhadauria and Kiran [16, 17] and Gupta and Singh [18] studied chaotic convection in porous medium by using different physical model.

The control of any convective phenomenon is very necessary from the application point of view, in other word it works as a centralizer in the mechanism by which we can develop a time durable model for any industrial process. There are many method to control the chaos such as bang-bang [19], pid [20], adaptive [21], feedback

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and gravity modulation in the literature. Therefore, among these method we consider feedback and gravity modulation in the present article to see how dynamical behavior change under the control. Ott et al. [22] were the proposed the model for controlling of chaos. Their results shown system has chaotic and periodic behavior via controlling parameter. Pyragas [23] studied a time-delayed feedback control, he obtained a stabilize effect on chaos through the controller. Bessa et al. [24] proposed a robust controller to stabilize dynamical system of unstable periodic orbits and found the possibility to perform chaos control even in situations where high uncertainties are involved. Yuen and Bau [25] investigated linear and non-linear controllers to suppress chaotic motion in the loop and to stabilize periodic orbits embedded in the chaotic attractor. Mahmud and Hasim [26], Roslan et al. [27] studied the feedback control on chaotic convection in porous medium for various model and compare the results with feedback and without feedback, their results shows chaotic and periodic solutions depending upon the feedback parameter. The new contribution of this paper is gravity modulation concept because all the feedback control of chaos studies are done without gravity modulation. we suppose gravity modulation as a function of time to study time periodic vibrations of the system which is co-linear with actual gravity, moreover it gives two parameters to control the convective event in all applicable fields for instance oil cooling, chemical sciences etc. Gresho and Sani [28] were the first who studied the gravity modulated system, in which they found that the gravity modulation enable the system to get control on its instability either by suitable adjusting the values of frequency or the amplitude of modulation. For more details on gravity modulation as well as its impact and applications, we refer the interested readers to. [29- 35]

The objective of this paper is to study the combined effect of feedback control and gravity modulation on chaotic convection in a porous medium. First of all, the adopted model is reduced into Lorenz system by employing trauncated Galerkin expansion method. The influence of each convective parameter has been studied extensively. The proposed Lorenz system has been analyzed by using phase portrait and time domain diagrams.

2. MATHEMATICAL MODEL OF THE PROBLEM

An infinitely extended horizontal porous layer of depth d confined between two parallel planes: the lower plane at $z = 0$ while upper plane at $z = d$. A Cartesian frame of reference is adopted in such a way that the origin lies on the lower plane and z axis is vertically upward. The porous layer is heated from below and cooled from the above. The physical configuration of the model is presented by Fig. 1. The Darcy law, Oberbeck-Boussinesq approximation is considered to solve the model equations. The non-dimensionlized system of the model equations is obtained according as [17, 27]

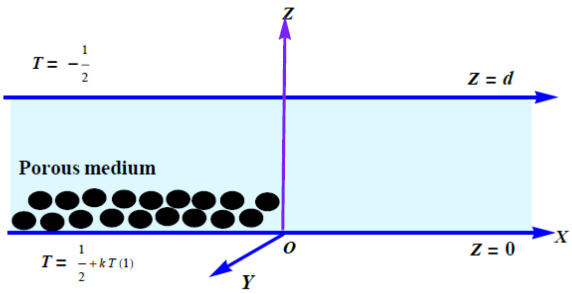

Fig.1: Physical configuration of the problem

$$\frac{1}{V_a}\frac{\partial}{\partial t}(\nabla^2\psi) = -\nabla^2\psi - R_a(1+\delta\sin(\Omega t))\frac{\partial T}{\partial x}, \quad (1)$$

$$-\frac{\partial\psi}{\partial x}\frac{\partial T_b}{\partial z} + \left(\frac{\partial}{\partial t} - \nabla^2\right)T = \frac{\partial(\psi,T)}{\partial(x,z)}, \quad (2)$$

where $V_a = \frac{\phi\nu d^2}{K\kappa_T}$ Vadasz number, $R_a = \frac{\alpha_T g_0 \Delta T d\vec{K}}{\nu\kappa_T}$

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thermal Rayleigh number, $(1 + d \sin (Wt))$ modulation term and other variables have their usual meanings as given in the nomenclature.

The externally imposed thermal boundary conditions, gravitational field are given by

$$T(t)=\begin{cases} \frac{1}{2}+kT(1); & \text{at } z=0; \\ -\frac{1}{2}; & \text{at } z=d. \end{cases} \tag{3}$$

$$\vec{g}=g_0(1+\delta\sin(\Omega t))\hat{k}. \tag{4}$$

The basic state temperature present in Eq. (2) is obtained by using the above boundary condition Eq. (3)

$$T_b=\frac{1}{2}[1-k-(2-k)z]. \tag{5}$$

3. FORMULATION OF LORENZ SYSTEM

The solution of nonlinear Eqs. (1) and (2) are obtained by using truncated Galerkin expansion method. The stream function and temperature field are taken in the forms as mentioned in [27].

$$\psi=A_{11}\sin\left(\frac{\pi x}{L}\right)\sin(\pi z), \tag{6}$$

$$T=T_b+B_{11}\cos\left(\frac{\pi x}{L}\right)\sin(\pi z)+B_{02}\sin(2\pi z). \tag{7}$$

Using Eqs. (6) and (7) in to Eqs. (1) and (2), multiplying the equations by orthogonal eigenfunctions corresponding to Eqs. (6) and (7), and then integrating them over the spatial domain, yield a set of three differential equations for the time evolution of the amplitudes, in the form of

$$\frac{dA_{11}}{d\tau}=-\frac{V_a\gamma}{\pi^2}\left(\frac{R_a}{\pi\theta}(1+\delta\sin(\Omega\tau))B_{11}+A_{11}\right), \tag{8}$$

$$\frac{dB_{11}}{d\tau}=-\frac{(2-k)}{2\pi\theta}A_{11}-\frac{1}{\theta}A_{11}B_{02}-B_{11}, \tag{9}$$

$$\frac{dB_{02}}{d\tau}=\frac{1}{2\theta}A_{11}B_{11}-4\gamma B_{02}, \tag{10}$$

where the time has been re-scaled and the following notations are introduce.

$$\tau=\frac{(L^2+1)\pi^2}{L^2}t,\quad \theta=\frac{L^2+1}{L},\quad \gamma=\frac{L^2}{L^2+1},\quad \Omega=\frac{L^2}{(L^2+1)\pi^2}\Omega^*,\ \sigma=\frac{V_a\gamma}{\pi^2}\ \text{and}\ R=\frac{R_{aD}}{\pi^2\theta^2}.$$

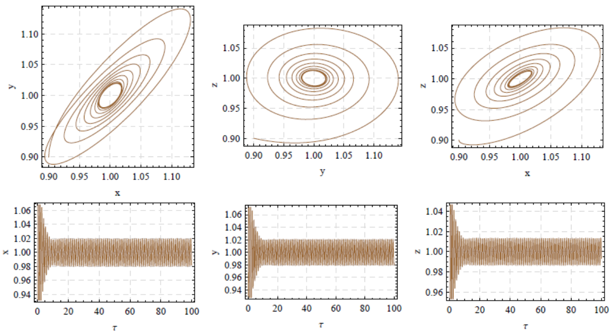

Fig.2: Phase portrait and time domain diagrams for the System (11) with parameters R = 10, δ=0.01, k = 0.01, Ω =5

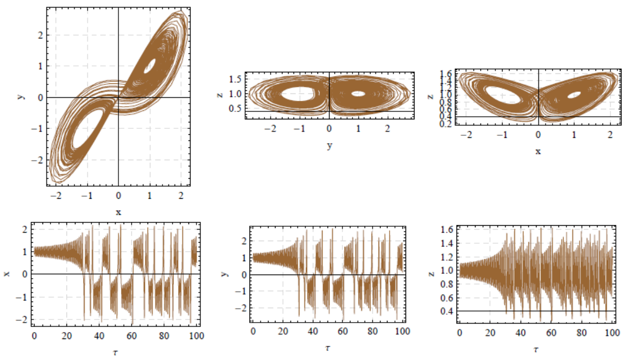

Fig.3: Phase portrait and time domain diagrams for the System (11) with parameters R = 22.2, δ=0.01, k = 0.01, Ω =5

$$X=-\frac{A_{11}}{2\theta\sqrt{2\gamma[\frac{1}{2}(2-k)R-1]}},\ Y=\frac{\pi RB_{11}}{2\sqrt{2\gamma[\frac{1}{2}(2-k)R-1]}}\ \text{and}\ Z=-\frac{2\pi RB_{02}}{(2-k)R-2}.$$

Which provide the following set of equations

$$\begin{cases} \frac{dX}{d\tau}=\sigma\left((1+\delta\ \mathrm{Sin}(\Omega\tau))\,Y-X\right), \\ \frac{dY}{d\tau}=\left(\frac{2-k}{2}\right)RX-Y-\left(\left(\frac{2-k}{2}\right)R-1\right)XZ, \\ \frac{dZ}{d\tau}=4\gamma\,(XY-Z). \end{cases} \tag{11}$$

If the amplitude of gravity modulation = 0 then the system (11) reduces in to Hashim et al.'s [27] model.

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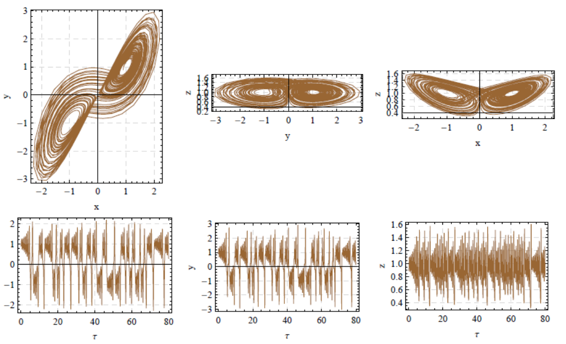

Fig.4: Phase portrait and time domain diagrams for the System (11) with parameters R = 32, δ = 0.01, k = 0.01, Ω = 5

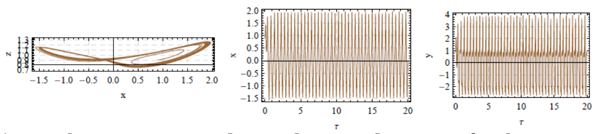

Fig.5: Phase portrait and time domain diagrams for the System (11) with parameters R = 300, δ = 0.01, k = 0.01, Ω = 5

4 RESULTS AND DISCUSSION

All the numerical simulations of the Lorenz system (11) is computed with the help of MATHEMATICA software. In this computation, we considered the initial conditions $\tau = 0$: $X = Y = Z = 0.9$ and fixed the parameters $\sigma = 10$, $\gamma = 0.5$. The convective parameters R, k, d and Ω are assumed as variable to investigate the effect of modulated chaotic system. The results are further depicted in Figs. (2-16) to analyse the Lorenz model by using phase-portrait and time domain diagrams. The effect of scaled Rayleigh number R on the system is depicted in Fig. (2-5), keeping fixed the other parameters. Fig. 2 shows a periodic solution ($R = 10$) and for ($R = 22.2$) system moves from periodic to weak chaotic solution in Fig. 3. Fig. 4 ($R = 32$) depicts a strong chaotic behavior or aperiodic solution of the Lorenz system which shows that heat transfer is more in this case in comparison to earlier two case. On increasing the higher value of $R \geq 300$ system always shows periodic solution depicts in Fig. 5. Hence we conclude that the system has either periodic or chaotic behavior depending upon the values of scaled Rayleigh number, a similar results obtained by Long et al. [9].

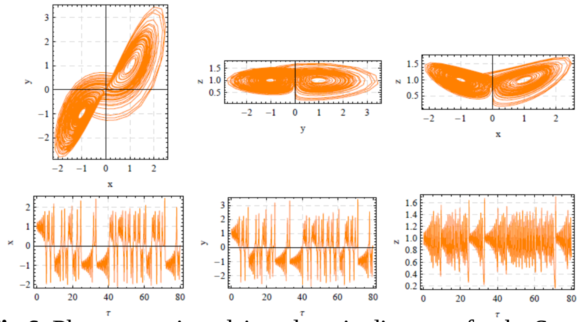

Fig.6: Phase portrait and time domain diagrams for the System (11) with parameters R = 32, δ=0.01, k = 0.01, Ω =5

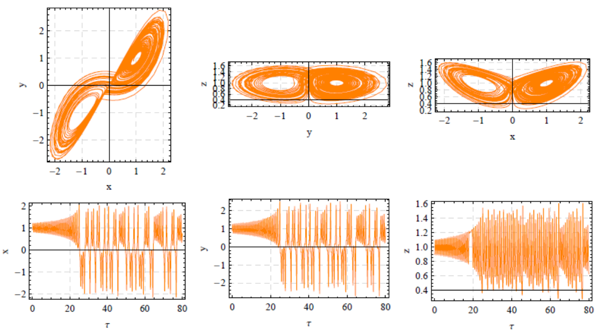

Fig.7: Phase portrait and time domain diagrams for the System (11) with parameters R = 32, δ = 0.01, k = 0.6, Ω = 5

Figs. (6-8) depict the effect of different values of feedback control parameter $k = 0.01, 0.6, 0.9$, keeping fixed the other parameters. The phase-portrait diagrams and time domain solutions show that the system has chaotic nature for $k = 0.01$, for $k = 0.6$ the system transition from chaotic to periodic nature and last for $k = 0.9$, system represents a periodic behavior for a long time respectively. Thus, the system returns to periodic solution from the chaotic solution as k increases, and so, k delay the heat transfer which is compatible with the result of Hashim et al. [27]. The impact of amplitude of gravity modulation d on the system for different parametric values $\delta = 0.02, 0.1, 0.2$ keeping fixed the other parameters, is depicted in Figs. (9-11) respectively. These figures depict that the trajectories are much disturbed on increasing δ. Therefore, the chaotic behavior advances, that is, the heat transfer is increases gradually. The effect of frequency of gravity modulation Ω is depicted in Figs. (12,13,14) for

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Ω =50,100,150, keeping fixed other parameters. In this case, the system looses its chaotic behavior and shift in to periodic behavior, and so, heat transfer is delay the convection, both results are analogous to Bhadauria and kiran [16]. Lastly we also compare the our result from the result already obtained by Hashim et al. [27] depicted in Figs. (15,16). In Fig. 15 all the trajectories are moving in to fixed point and time domain solution shows a stable solution for long time for given parametric values. On the other hand in our computations all the trajectories are moving around a fixed point and time domain solution depicted a periodic system due to present the modulation term d given in Fig. 16, hence we can say that in modulated system heat transfer is more in comparison to without modulated system.

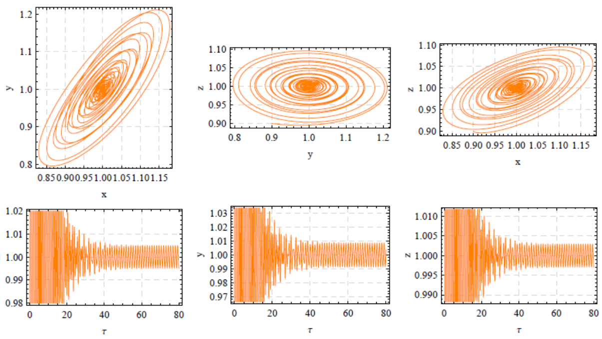

Fig.8: Phase portrait and time domain diagrams for the System (11) with parameters R = 32, δ = 0.01, k = 0.9, Ω = 5

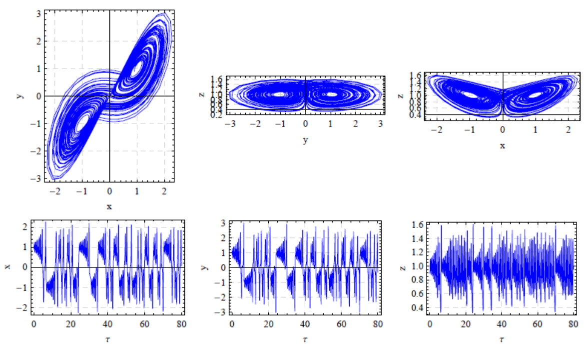

Fig.9: Phase portrait and time domain diagrams for the System (11) with parameters R = 32, δ = 0.02, k = 0.01, Ω =5

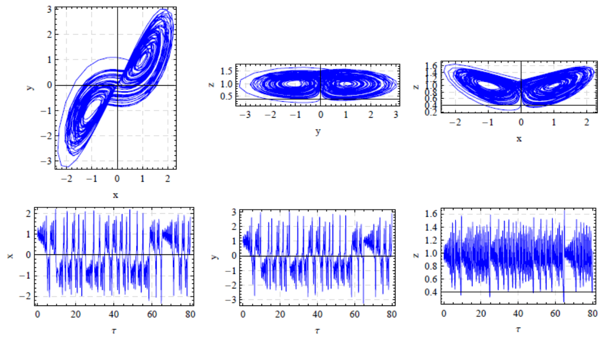

Fig.10: Phase portrait and time domain diagrams for the System (11) with parameters R = 32, δ = 0.1, k = 0.01, Ω =5

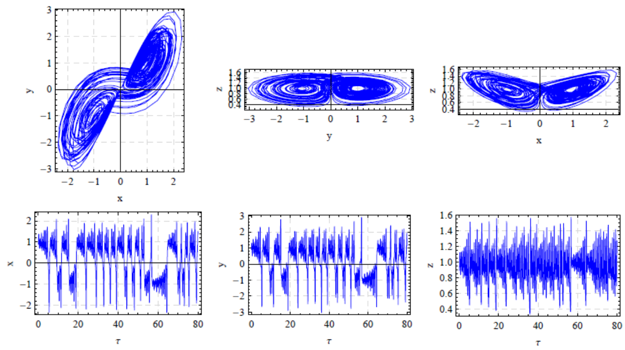

Fig.11: Phase portrait and time domain diagrams for the System (11) with parameters R = 32, δ = 0.2, k = 0.01, Ω =5

5 CONCLUSION

In this paper, feedback control and G-jitter effects on chaotic convection in a porous medium are studied. The adopted model is first reduced into Lorentz system by employing trauncated Galerkin expansion method. By using phase portrait and time domain diagrams the following findings are obtained

a) The effect of scaled Rayleigh number R is to either increase (chaotic) or decrease (periodic) the heat transport in the Lorenz system.
b) The feedback control parameter k is to delay the chaotic convection i.e. heat transfer decreases in the system.
c) The amplitude δ (frequency Ω) of modulation is to advance (delay) the heat transfer in the Lorenz system.
d) Finally, it is obtained that heat transfer is more in the modulated system in comparison to the without modulated system.

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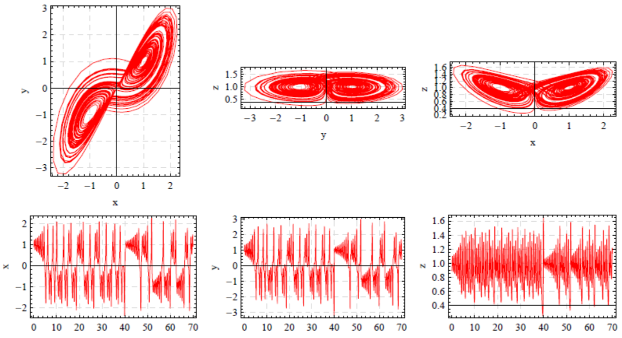

Fig.12: Phase portrait and time domain diagrams for the System (11) with parameters R = 32, δ = 0.1, k = 0.01, Ω = 50

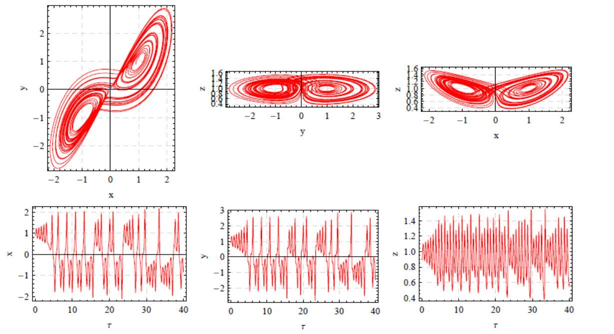

Fig.13: Phase portrait and time domain diagrams for the System (11) with parameters R = 32, δ = 0.01, k = 0.01, Ω = 100

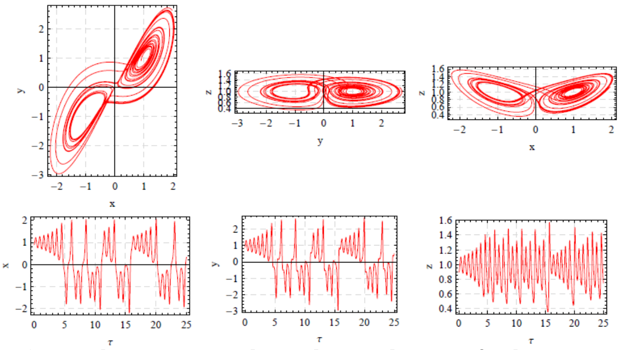

Fig.14: Phase portrait and time domain diagrams for the System (11) with parameters R = 32, δ = 0.01, k = 0.01, Ω = 150

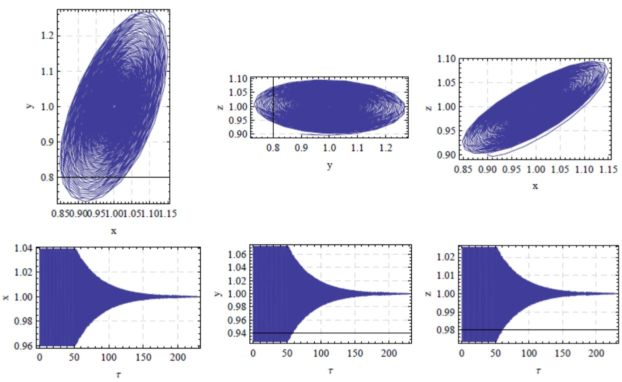

Fig.15: Phase portrait and time domain diagrams for the System (11) with parameters R = 24.9, δ = 0.0, k = 0.125, σ = 5

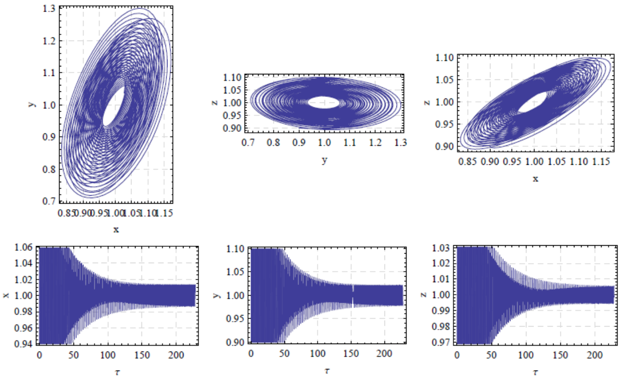

Fig.16: Phase portrait and time domain diagrams for the System (11) with parameters R = 24.9, δ = 0.03, k = 0.125, σ = 5

6. NOMENCLATURE

Latin symbols

k	feedback parameter
d	depth of fluid layer
L	length of porous layer
$\vec{g}$	acceleration due to gravity
p	reduced pressure
R_a	thermal Darcy-Rayleigh number
R	scaled Rayleigh number
T	temperature
ΔT	temperature difference across the porous layer
t	time
$\vec{q}$	fluid velocity(u,v,w)
(x,z)	horizontal and vertical co-ordinates

Greek symbols

α_T	coefficient of thermal expansion
κ_T	effective thermal diffusivity
$\vec{K}$	permeability
δ	amplitude of gravity modulation
Ω	frequency of modulation
μ	dynamic viscosity of the fluid
ϕ	porosity
γ	heat capacity ratio
ν	kinematic viscosity
ρ	fluid density
ψ	stream function
τ	rescaled time
X	rescaled amplitude
Y	rescaled amplitude
Z	rescaled amplitude

Other symbols

∇^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$

subscripts

b	basic state
0	reference value

superscripts

'	perturbed quantity
*	dimensionless quantity

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