Effect of Line Loading on An Irregular Elastic Medium Possessing Cubic Symmetry

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Abstract

In the present paper, the closed form analytical expressions due to an inclined line-load in an irregular elastic medium with cubic symmetry have been obtained. The irregularities are considered in the shape of rectangular and parabolic. Numerically, the effect of irregularities have been studied by variation of displacements with the horizontal distance by considering different sizes of irregularities (e) of parabolic and rectangular. At different values of 'e' the comparisons are made at different values of angle of inclinations.

1. INTRODUCTION

Elastic problems with irregular boundaries have gained much importance in geophysicists due to their closeness to their natural environmental conditions. It leads to a better understanding and better predictions for the seismic behaviour at continental margins and mountain roots. It is therefore interesting to study the static deformation in media with irregular boundaries. A number of researchers have studied the problem of irregular boundaries, such as Chattopadhyay et al. [1], Chattopadhyay and Pal [2],Madan et al. [3], De Noyer [4], Sato [5], Mal [6], Kar et al. [7] and others.

The behaviour of elastic materials due to line-loading is of great interest in engineering, soil mechanics and geophysics. When the source surface is very long in one direction in comparison to the others, the use of two-dimensional approximation is justified and consequently calculations are simplified to a great extent and one gets a closed-form analytical solution. A very long strip-source and a very long line-source are examples of such two-dimensional sources. The deformation due to loading such as inclined line-load, stripe-load, continuous line-load, etc., is useful in analysing the field around mining tremors and drilling into the crust of the earth. It can also contribute to the theoretical consideration of the seismic and volcanic sources since it can account for the deformation fields in the entire volume surrounding the source region. [8] Selim [9] studied the effect of irregularity on static deformation of an isotropic elastic half-space. Madan et al [10] studied the static response of transversely isotropic elastic medium with irregular boundaries present in the medium Madan and Gaba [11] have studied the effect of different type of irregularities on the variation of displacements and stresses in an orthotropic elastic media due to normal line-loading by using eigen value approach with distinct eigen values.

In the present problem, we have considered an irregular model of elastic medium possessing cubic symmetry in which the irregularity has been taken in the form of a rectangular and parabolic, i.e. free from initial stress. The closed form expressions for the displacement at any point due to line loadinghave been obtained by using the same approach with equal eigen values. Numerically, the graphs have been depicted to show the variation of displacements with the horizontal distance at different sizes of irregularity and also comparison has been madegraphically between the variation of displacements due to rectangular and parabolic irregularities. At different values of the angle of inclinations the effect of line loading is studied. The corresponding results for isotropic elastic medium can be obtained as a particular case from the obtained results.

2. BASIC EQUATIONS

From the Generalized Hooke's Law, the stress strain relation in matrix form for an elastic medium with cubic symmetry is (Love [12]):

$$\sigma_{11} = c_{11}e_{11} + c_{12}e_{22} + c_{12}e_{33}$$
$$\sigma_{22} = c_{12}e_{11} + c_{11}e_{22} + c_{12}e_{33}$$
$$\sigma_{33} = c_{12}e_{11} + c_{12}e_{22} + c_{11}e_{33} \qquad (1)$$
$$\sigma_{23} = 2c_{44}e_{23}$$
$$\sigma_{13} = 2c_{44}e_{13}$$
$$\sigma_{12} = 2c_{44}e_{12}$$

where the suffix quantity c_{ij}, σ_{ij} and e_{ij} denotes the elastic constants, stress components and strain components respectively of the isotropic elastic medium with cubic symmetry.

In the absence of body forces, the equilibrium equations in the Cartesian co-ordinate system *(x,y,z)* are:

$$\frac{\partial\sigma_{11}}{\partial x} + \frac{\partial\sigma_{12}}{\partial y} + \frac{\partial\sigma_{13}}{\partial z} = 0$$

$$\frac{\partial\sigma_{21}}{\partial x} + \frac{\partial\sigma_{22}}{\partial y} + \frac{\partial\sigma_{23}}{\partial z} = 0 \qquad (2)$$

$$\frac{\partial\sigma_{31}}{\partial x} + \frac{\partial\sigma_{32}}{\partial y} + \frac{\partial\sigma_{33}}{\partial z} = 0$$

The strain displacement relation are given by:

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad 1 \le i,j \le 3 \qquad (3)$$

where $(u_1, u_2, u_3) = (u, v, w)$ and $(x_1, x_2, x_3) = (x, y, z)$ (u_1, u_2, u_3)=(u, v, w) and (x_1, x_2, x_3)=(x, y, z).

Now using equations (1) - (3) we have obtained the equilibrium equations in terms of displacement components as:

$$\left.\begin{aligned} c_{11}\frac{\partial^2 u}{\partial x^2} + c_{44}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \left(\frac{c_{11}+c_{12}}{2}\right)\frac{\partial^2 v}{\partial x\partial y} + (c_{12}+c_{44})\frac{\partial^2 w}{\partial x\partial z} = 0 \\ c_{44}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2}\right) + c_{11}\frac{\partial^2 v}{\partial y^2} + \left(\frac{c_{11}+c_{12}}{2}\right)\frac{\partial^2 u}{\partial x\partial y} + (c_{12}+c_{44})\frac{\partial^2 w}{\partial y\partial z} = 0 \\ c_{44}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + c_{11}\frac{\partial^2 w}{\partial z^2} + (c_{12}+c_{44})\frac{\partial^2 u}{\partial x\partial z} + (c_{12}+c_{44})\frac{\partial^2 v}{\partial y\partial z} = 0 \end{aligned}\right\} \qquad (4)$$

3. FORMULATION OF THE PROBLEM

Let us consider an unbounded elastic half space with cubic symmetry with x-axis vertically downwards, where the origin of Cartesian coordinates is situated at x = 0. Assume that an inclined line load F_0 per unit length is acting downwards on a line parallel to the z-axis and passing through the point *(h, 0)* and the considered irregularity is of the form of rectangular and parabolic with length *2a* and depth *h* (fig. 1).

Let the equation of irregularity be

1. For Rectangular Irregularity

$$x = \in f(y) \qquad (5)$$

where

$$\in f(y) = \begin{cases} h & for\ |y| \le a \\ 0 & for\ |y| > a \end{cases} \qquad (6)$$

Where $\in$ is a small parameter

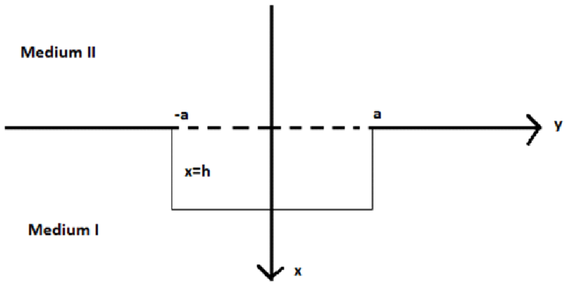

Fig.1:

2. For Parabolic Irregularity

$$\in f(y) = \begin{cases} h\left(1-\frac{y^2}{a^2}\right) & for\ |y| \le a \\ 0 & for\ |y| > a \end{cases} \quad (7)$$

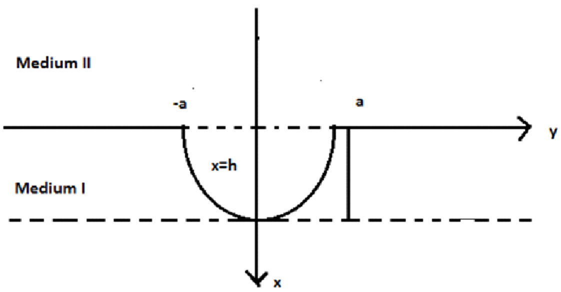

Fig.2:

Suppose that the displacement components under the condition of plain strain deformation, parallel to *xy-plane* be independent of z and are given by:

$$u = u(x,y), v = v(x,y), w = 0 \quad (8)$$

For plane strain problem, the non-zero stresses are:

$$\left.\begin{aligned} \sigma_{11} &= c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial v}{\partial y} \\ \sigma_{22} &= c_{12}\frac{\partial u}{\partial x} + c_{11}\frac{\partial v}{\partial y} \\ \sigma_{12} &= c_{44}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \end{aligned}\right\} \quad (9)$$

The equilibrium equations are correspondingly reduce to the form:

$$\left.\begin{aligned} c_{11}\frac{\partial^2 u}{\partial x^2} + c_{44}\frac{\partial^2 u}{\partial y^2} + \left(\frac{c_{11}+c_{12}}{2}\right)\frac{\partial^2 v}{\partial x \partial y} = 0 \\ c_{44}\frac{\partial^2 v}{\partial x^2} + c_{11}\frac{\partial^2 v}{\partial y^2} + \left(\frac{c_{11}+c_{12}}{2}\right)\frac{\partial^2 u}{\partial x \partial y} = 0 \end{aligned}\right\} \quad (10)$$

After applying Fourier Transformation on eq.(10), we obtain the following expressions:

$$\left.\begin{aligned} c_{11}\frac{d^2\bar{u}}{dx^2} + c_{44}(-x^2)\bar{u} - ip\left(\frac{c_{11}+c_{12}}{2}\right)\frac{d\bar{v}}{dx} = 0 \\ -ip\left(\frac{c_{11}+c_{12}}{2}\right)\frac{d\bar{u}}{dx} + c_{44}\frac{d^2\bar{v}}{dx^2} - c_{11}p^2\bar{v} = 0 \end{aligned}\right\} \quad (11)$$

where bar stands for Fourier Transform with transformed fourier parameter p.

Now eq.(11) can be written in the vector matrix differential equation form :

$$E_1\frac{d^2U}{dx^2} - i\eta E_2\frac{dU}{dx} - p^2E_3U = 0 \quad (12)$$

Where

$$E_1 = \begin{pmatrix} c_{11} & 0 \\ 0 & c_{44} \end{pmatrix}, \quad E_2 = \begin{pmatrix} 0 & \frac{c_{11}+c_{12}}{2} \\ \frac{c_{11}+c_{12}}{2} & 0 \end{pmatrix},$$

$$E_3 = \begin{pmatrix} c_{44} & 0 \\ 0 & c_{11} \end{pmatrix}, U = \begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix} \quad (13)$$

Let the solutions of the matrix equations (12) be of the form

$$U(x,p) = A(p)e^{mx} \quad (14)$$

Where m is the parameter and *E(p)* is a matrix of the type 2 x 1

Substituting the values of U from eqs.(13) in eqs. (12), we get the following characteristic equation

$$m^4 - 2p^2m^2 + p^4 = 0 \quad (15)$$

Eq. (12) has repeated eigen values (m). The equilibrium equations are equivalent to the first order vector differential equation. Using the process given by Ross [13], to tackle the repeated eigen values, we obtain the following differential equation:

$$\frac{dU_1}{dx} = P_1U_1 \quad (16)$$

Where

$$U_1 = \begin{bmatrix} \bar{u} \\ \bar{v} \\ \frac{d\bar{u}}{dx} \\ \frac{d\bar{v}}{dx} \end{bmatrix}, \quad P_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{c_{44}}{c_{11}}p^2 & 0 & 0 & \frac{c_{11}+c_{12}}{2c_{11}}ip \\ 0 & \frac{c_{11}}{c_{44}}p^2 & \frac{c_{11}+c_{12}}{2c_{44}}ip & 0 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{p^2}{T_1} & 0 & 0 & \frac{T_2}{2}ip \\ 0 & T_1p^2 & T_4ip & 0 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} i|p| \\ k \\ ip^2 \\ p|p| \end{bmatrix}, A_2 = \begin{bmatrix} i\{x|p| - T_2\} \\ p\left(x - \frac{1}{|p|}\right) \\ i|p|\{x|p| - T_3\} \\ xp|p| \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -i|p| \\ p \\ ip^2 \\ -p|p| \end{bmatrix}, A_4 = \begin{bmatrix} -i\{x|p| + T_2\} \\ p\left(x + \frac{1}{|p|}\right) \\ i|p|\{x|p| + T_3\} \\ -xp|p| \end{bmatrix} \tag{17}$$

Thus, the general solution of equation (15) for elastic medium with cubic symmetry is

$$U_1 = (BE_1 + CE_2)e^{|p|x} + (DE_3 + GE_4)e^{-|p|x} \tag{18}$$

where the values of the constants B, C, D and G will be derived using the boundary conditions for line-load. After solving the eq. (15) and using eq (18), we obtain the following displacement and stresses in the transformed domain:

$$\left.\begin{aligned} \bar{u}(x,y) &= i\left[\{B|p| + C(x|p| - T_2)\}e^{|p|x} - \{D|p| + G(x|p| + T_2)\}e^{-|p|x}\right] \\ \bar{v}(x,y) &= p\left[\left\{B + C\left(x - \frac{1}{|p|}\right)\right\}e^{|p|x} + \left\{D + G\left(x + \frac{1}{|p|}\right)\right\}e^{-|p|x}\right] \\ \bar{\sigma}_{11} &= i[\{\left(2c_{44}Bp^2 + C\left(2c_{44}xp^2 + |p|(c_{12} - T_3c_{11})\right)\right\}e^{x|p|} + \\ &\quad \{2c_{44}Dp^2 + G\left(2c_{44}xp^2 - |p|(c_{12} - T_3c_{11})\right)\}e^{-x|p|}] \\ \bar{\sigma}_{12} &= pc_{44}[\{2B|p| + C(2|p|x - T_2)\}e^{x|p|} - \{2D|p| + G(2|p|x + T_2)\}e^{-x|p|}] \end{aligned}\right\} \tag{19}$$

where $T_1 = \frac{C_{11}}{C_{44}}, T_2 = \frac{4C_{11}}{C_{11}+C_{12}}, T_3 = 2\left(\frac{C_{11}+C_{44}}{C_{11}+C_{12}}\right), T_4 = \frac{C_{11}+C_{12}}{2C_{44}}$

After applying Inverse Fourier Transformation in eq. (19), we obtain the following expressions for displacement and stresses:

$$u(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i\left[\{B|p| + C(x|p| - T_2)\}e^{|p|x} - \{D|p| + G(x|p| + T_1)\}e^{|p|x}\right]e^{-ipy}\,dp$$

$$v(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} p\left[\left\{B + C\left(x - \frac{1}{|p|}\right)\right\}e^{|p|x} + \left\{D + Gp\left(x + \frac{1}{|p|}\right)\right\}e^{-|p|x}\right]e^{-ipy}\,dp$$

$$\sigma_{12}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i\big[\{2c_{44}(B + Cx)p^2 + |p|(2c_{12} - T_5c_{11}\}e^{|p|x} + \{2c_{44}(D + Gx)p^2 - |p|(c_{12} - T_5c_{11}\}e^{-|p|x}e^{-ipy}\big]dk$$

$$\sigma_{11}(x,y) = \frac{1}{2\pi}C_{44}\int_{-\infty}^{\infty} p\left[\{B|p| + C(2|p|x - T_2)\}e^{|p|x} - \{2D|p| + C(2|p|x + T_2)\}e^{-|p|x}\right]e^{-ipy}\,dk \tag{20}$$

4. SOLUTION OF THE PROBLEM

In order to find the displacement and stresses at any point of an irregular isotropic medium with cubic symmetry due to an inclined line-load F_0 per unit length, acting on the z-axis, let us consider the irregular media consisting of region $x < \in f(y)$ *(Medium I) and* $x > \in f(y)$ *(Medium II)* having identical elastic properties.

The displacement and stresses for Medium I are:

$$u^I(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i\left[\{B|p| + C(x|p| - T_2^I)\}e^{|p|x}\right]e^{-ipy}\,dp$$

$$v^I(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} p\left[\left\{B + C\left(x - \frac{1}{|p|}\right)\right\}e^{|p|x}\right]e^{-ipy}\,dp$$

$$\sigma_{12}^I(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i\left[\{2c_{44}(B + Cx)p^2 + |p|(c_{12} - T_5^Ic_{11}\}e^{|p|x}\right]e^{-ipy}\,dp$$

$$\sigma_{12}^I(x,y) = \frac{1}{2\pi}C_{44}\int_{-\infty}^{\infty} p\left[\{B|p| + C(2|p|x - T_2^I)\}e^{|p|x})\}\right]e^{-ipy}\,dp \tag{21}$$

The displacement and stresses for Medium II are:

$$u^{II}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i\left[-\{D|p| + G(x|p| + T_2^{II})\}e^{-|p|x}\right]e^{-ipy}\,dp$$

$$v^{II}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} p\left[\left\{D + G\left(x + \frac{1}{|p|}\right)\right\}e^{-|p|x}\right]e^{-ipy}\,dp$$

$$\sigma_{11}^{II}(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} i\left[\{2c_{44}(D+Gx)p^2 + G|p|(2c_{12} - T_5^{II}c_{11}\}e^{-|p|x}\right]e^{-ipy}\,dp$$

$$\sigma_{12}^{II}(x,y) = \frac{1}{2\pi}C_{44}\int_{-\infty}^{\infty} p\left[-\{D|p| + G(2|p|x - T_2^{II})\}e^{-|p|x})\}\right]e^{-ipy}\,dp \quad (22)$$

Where $T_5 = \dfrac{3c_{11} - c_{12}}{2c_{44}}$

4.1 Normal Line-Load

Consider a normal-line load F_1 per unit length, is acting vertically downwards on the interface irregularity x = h along z-axis (Fig. 1). Then, the boundary conditions at x = h are

$$u^I(h,y) = u^{11}(h,y)$$

$$v^I(h,y) = v^{11}(h,y) \quad (23)$$

$$\sigma_{12}^{I}(h,y) = \sigma_{12}^{II}(h,y)$$

$$\sigma_{11}^{I}(h,y) - \sigma_{11}^{II}(h,y) = -F_1\delta(\mathrm{y})$$

Where h = $\in f\,(y)$ and δ(y) is the Dirac-delta satisfying the expression: (24)

$$\int_{-\infty}^{\infty}\delta(\mathrm{y})\,f(y)dy = 1, \delta(\mathrm{y}) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-ipy}\,dp = 1$$

Using the method of Erigen and Suhani [14], the arbitrary constants B,C,D and G will be calculated by expanding above equations in terms of $\in$ and retaining only the linear terms, we may express the constants as:

B = $B_0 + \in B_1$, C=$C_0 + \in C_1$, D = $D_0 + \in D_1$, and G = $G_0 + \in G_1$ (25)

And since $|\in f\,(y)| << 1$, then

$$e^{\pm|p|\in f(y)} \approx 1 \pm |k| \in f(y). \quad (26)$$

Using equation (24) in equations (22) and (23) we obtain the following values of constants B, C, D and G i.e.,

B=$\frac{F_1}{k^2}Q_1 + \in f(y)\frac{F_1K_3}{2|k|}$

C=$\frac{-F_1}{|k|}\delta_2 + \in f(y)F_1K_2$

D=$\frac{F_1}{k^2}Q_2 + \in f(y)\frac{F_1K_4}{2|k|}$

G=$\frac{-F_1}{|k|}\delta_1 + \in f(y)F_1K_1$ (27)

Where $T_5 = \frac{3c_{11} - c_{12}}{2c_{44}}$ and

$$\delta_1 = \frac{T_2^I}{T_2^{II}T_1^I - T_2^{II} - T_2^I T_6^{II} - T_2^I}, \delta_2 = \frac{\delta_1 T_2^{II}}{T_2^I},$$

$$Q_1 = \frac{\delta_2 T_2^{II} - \delta_1 T_2^I - \delta_1 - \delta_1}{2}, Q_2 = \frac{\delta_2 T_2^{II} - \delta_1 T_2^I + \delta_1 + \delta_1}{2}$$

$$\delta_3 = Q_2 + \delta_1 - \delta_2 T_2^I - \delta_1 T_2^{II}, \delta_4 = Q_2 - Q_1, Q_3 = \delta_2\left(1 - c_{12} - T_5^I c_{11}\right) - \delta_1\left(1 + c_{12} + T_5^{II} c_{11}\right)$$

$$Q_4 = 2\delta_2 - \delta_2 T_2^I + 2\delta_1 - \delta_1 T_2^{II}, K_1 = \frac{2c_{44}T_2^I\delta_4 - 2c_{44}Q_2T_2^I + 2(\delta_3 - Q_3)(1 - 2c_{44})}{T_2^{II} - T_2^I - 2c_{44}T_2^{II} - 2c_{44}T_2^I}, K_2 = \frac{T_2^{II}K_2 - 2(\delta_3 - Q_4)}{T_2^I}$$

$$K_3 = \delta_3 - T_2^I K_2 - T_2^{II}\delta_1 - \delta_4 - K_2 - K_1, K_4 = \delta_3 - T_2^I K_2 - T_2^{II}\delta_1 + \delta_4 + K_2 + K_1$$

By using the values of the coefficients B, C,D and G from equations (27) in to the equations (21) we obtain the following closed form expressions with rectangular irregularity:

$$u^N(x,y) = \frac{iF_1}{2\pi}\left[\left\{(\delta_2 T_2^I - Q_1)log(x^2+y^2) + \frac{\delta_2 x^2}{x^2+y^2}\right\} + \gamma\left\{\left((K_2T_2^I - K_3)\right)\frac{2x^2}{x^2+y^2} + \frac{2K_2x^2(x^2-y^2)}{(x^2+y^2)^2}\right\}\right] \quad (28)$$

$$v^N(x,y) = -\frac{F_1}{2\pi}\left[\left\{(2\delta_2 + Q_1)log(x^2+y^2) + \frac{2i\delta_2 xy}{x^2+y^2}\right\} + \gamma\left\{(K_3 - K_2)\frac{2ixy}{x^2+y^2} + \frac{4iK_2x^3y}{(x^2+y^2)^2}\right\}\right]$$

Due to normal line-load, following are the suitable expression for the displacements with parabolic irregularity:

$$u^N(x,y) = \frac{iF_1}{2\pi}\left[\left\{(\delta_2 T_2^I - Q_1)log(x^2+y^2) + \frac{\delta_2 x^2}{x^2+y^2}\right\} + \gamma\left\{\left(1 - \frac{y^2}{a^2}\right)\left(\left((K_2T_2^I - K_3\right)\frac{2x^2}{x^2+y^2} + \frac{2K_2x^2(x^2-y^2)}{(x^2+y^2)^2}\right)\right\}\right] \quad (30)$$

$$v^N(x,y) = -\frac{F_1}{2\pi}\left[\left\{(2\delta_2 + Q_1)log(x^2+y^2) + \frac{2i\delta_2 xy}{x^2+y^2}\right\} + \gamma\left\{\left(1 - \frac{y^2}{a^2}\right)\left((K_3 - K_2)\frac{2ixy}{x^2+y^2} + \frac{4iK_2x^3y}{(x^2+y^2)^2}\right)\right\}\right] \quad (31)$$

4.2 Tangential Line-Load

Suppose that a line force F_2, per unit length is acting at orign in positively y-direction then the boundary conditions at the horizontal plane x = h are

$$u^I(h,y) = u^{11}(h,y)$$

$$v^I(h,y) = v^{11}(h,y)$$
$$\sigma_{12}^I(h,y) - \sigma_{12}^{II}(h,y) = -F_2\delta(y) \quad (32)$$

$$\sigma_{11}^I(h,y) = \sigma_{11}^{II}(h,y)$$

Using equation (24)-(27) and equation (32) in equation (21) we obtain the following values of constants B,C, D and G

$$B = \frac{F_2 P_1}{|k|} + \in f(y) F_2 P_5$$

$$C = F_2 \Omega_3 + \in f(y)|k| F_2 P_4$$

$$D = \frac{F_2 P_2}{|k|} + \in f(y) F_2 P_4 \qquad (33)$$

$$G = F_2 \Omega_2 + \in f(y)|k| F_2 P_3$$

By using the values of the coefficients B, C,D and G from equations (33)in to the equations (21) we obtain the following closed form expressions for displacements with rectangular irregularity :

$$u^T(\mathrm{x,y}) = \frac{iF_2}{2\pi}\left[\left\{-2(\Omega_3 T_2^I + P_1)\frac{2x}{x^2+y^2} + \frac{2x\Omega_3(x^2-y^2)}{(x^2+y^2)^2}\right\} + \gamma\left\{((P_5 - P_4 T_2^I + x^2 P_4))\frac{2x(x^2-y^2)}{(x^2+y^2)^2} + \right\}\right] \qquad (34)$$

$$v^T(\mathrm{x,y}) = \frac{F_2}{2\pi}\left[\left\{\Omega_3 log(x^2+y^2) - \frac{2}{x^2+y^2}(iyP_1 + x^2\Omega_3)\right\} + \gamma\left\{\frac{2x^2}{(x^2+y^2)^2}\left((x^2-y^2)P_4 - 2iyP_5\right) + \frac{2x^2}{x^2+y^2}P_4\right\}\right] \qquad (35)$$

Where

$$\Omega_1 = \frac{T_6^{II}+1}{T_6^I+1}, \Omega_2 = \frac{1}{T_2^I\Omega_1 - T_2^{II}}, \Omega_3 = \frac{\Omega_1}{\Omega_2}, P_1 = \frac{\Omega_3 T_2^I + \Omega_2 T_2^{II} + \Omega_3 - \Omega_2}{2}, P_2 = \frac{\Omega_3 T_2^I + \Omega_2 T_2^{II} - \Omega_3 + \Omega_2}{2},$$

$$\Omega_4 = \Omega_3 T_2^I + \Omega_2 T_2^{II} - \Omega_3 + P_2, \Omega_5 = P_2 - P_1, \Omega_6 = \delta_2(c_{12} - T_5^I c_{11} - 1) - \delta_1(c_{12} - T_5^{II} c_{11} - 1)$$

$$\Omega_7 = \Omega_3(T_2^I - 2) - \Omega_2(T_2^{II} - 2), P_3 = \left(\frac{(\Omega_5-\Omega_6)2c_{44}}{1+2c_{44}} + \frac{(\Omega_4-\Omega_7)2}{T_2^I}\right)\left(\frac{T_2^I}{T_2^{II}-T_2^I}\right)$$

$$P_4 = \frac{2(\Omega_4-\Omega_7) - \Omega_7 T_2^{II} T_2^I}{T_2^I}, P_5 = \frac{\Omega_4 - P_3 T_2^I + P_4 T_2^I - \Omega_5 - P_3 - P_4}{2}, P_6 = \frac{\Omega_4 - P_3 T_2^I + P_4 T_2^I + \Omega_5 + P_3 + P_4}{2}$$

Due to tangential line-load, following are the suitable expression for the displacements with parabolic irregularity:

$$u^T(\mathrm{x,y}) = \frac{iF_2}{2\pi}\left[\left\{-2(\Omega_3 T_2^I + P_1)\frac{2x}{x^2+y^2} + \frac{2x\Omega_3(x^2-y^2)}{(x^2+y^2)^2}\right\} + \gamma\left\{\left(1 - \frac{y^2}{a^2}\right)\left((P_5 - P_4 T_2^I + x^2 P_4)\frac{2x(x^2-y^2)}{(x^2+y^2)^2} + \right)\right\}\right] \qquad (36)$$

$$v^T(\mathrm{x,y}) = \frac{F_2}{2\pi}\left[\left\{\Omega_3 log(x^2+y^2) - \frac{2}{x^2+y^2}(iyP_1 + x^2\Omega_3)\right\} + \gamma\left\{\left(1 - \frac{y^2}{a^2}\right)\left(\frac{2x^2}{(x^2+y^2)^2}\left((x^2-y^2)P_4 - 2iyP_5\right) + \frac{2x^2}{x^2+y^2}P_4\right)\right\}\right] \qquad (37)$$

4.3 Inclined line-load

For an inclined line-loadF_0, per unit length, we have (Saada 1974 [15])

$$F_1 = F_0 \cos\delta \ and \ F_2 = F_0 \sin\delta$$

The normal and tangential displacements subjected to inclined line-load are obtained as the final deformation of the formulated problem i.e.

$$u^{IN}(x,y) = u^{N}(x,y) + u^{T}(x,y) \quad (38)$$

$$v^{IN}(x,y) = v^{N}(x,y) + v^{T}(x,y) \quad (39)$$

Where F_1 is for deformation due to normal line-load and F_2 is for deformation due to tangential line-load for both rectangular and parabolic irregularities.

5. NUMERICAL RESULTS

In this section we tend to examine the effect of irregularity present in anisotropic elastic half-space with cubic symmetry and inclination δ. For numerical computation, we have used the values of elastic constants given by Leibfried [16], For Medium I (Gamet).

$$c_{11} = 2.966, c_{12} = 1.085, c_{13} = 0.916$$

and

For Medium II (Mgo)

$$c_{11} = 2.97, c_{12} = 0.95, c_{13} = 1.56$$

In figures 3-6, the variation of normal displacements due to rectangular and parabolic irregularities (e = 0, 0.28, 0.56) present in the medium with inclination at $\delta = 45^0$ at x = 1 against the horizontal distance y has been depicted. Figure 3 shows that as the size of the irregularities increases displacements initially increases and beyond the horizontal distance y = 1 it decreases. In figure 4, beyond y = 1, the displacement increases as the size of irregularities increases as well the distance in magnitude between the displacement also increases. In figure 5, as the horizontal distance increases the distance in magnitude between the displacement decreases. From figure 6 we found that in case of parabolic irregularity there is a sudden increment in the displacements as the size of irregularities increases.

Figures 7-8 shows the comparison between the rectangular and parabolic irregularities (e = 0.25) for normal and tangential displacements at x = 1 against the horizontal distance y. These graphs shows that in case of normal displacement, as the horizontal distance y increases the distance between the displacements due to irregularities increases whereas in case of tangential displacement the distance between the displacements initially increases and then decreases.

Further in figures 9-12, graph has been plotted depicting the variation of normal and tangential displacements with rectangular and parabolic irregularities (e = 0.28) at different inclined line-load ($\delta = 30^0, 45^0, 60^0$) at x = 1 which shows that as the value of δ increases there is prominent effect of irregularities in the displacements. As the size of the increases the displacement decreases but in case of tangential displacement (from figure 11 and 12) it is shown that after a particular horizontal distance the displacements starts increasing.

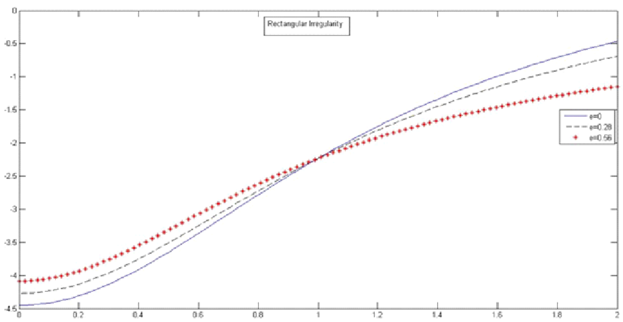

Fig. 3 : Variation of normal displacement (u) against the horizontal distance (y) for rectangular irregularities (e = 0, 0.28, 0.56) on the plane x = 1.0.

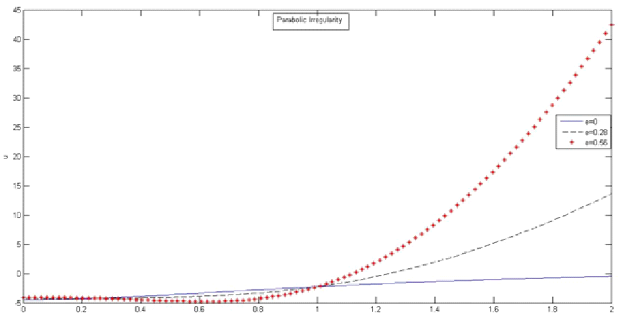

Fig.4: Variation of normal displacement (u) against the horizontal distance (y) for parabolic irregularities (e = 0, 0.28, 0.56) on the plane x = 1.0.

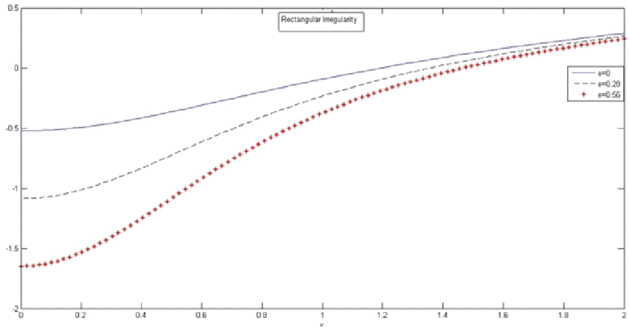

Fig.5: Variation of tangential displacement (u) against the horizontal distance (y) for rectangular irregularities (e = 0, 0.28, 0.56) on the plane x = 1.0.

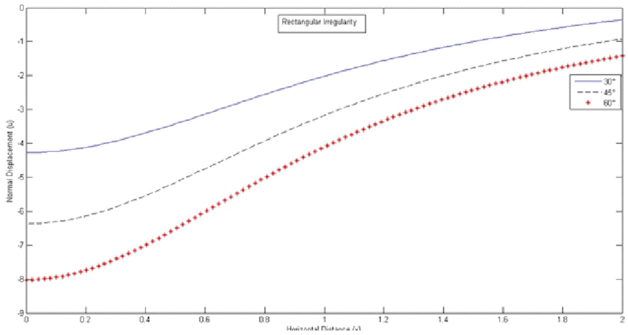

Fig.9: Variation of normal displacements with rectangular irregularity (e = 0.25) due to inclined line-load (δ = 30°,45°,60°) at x = 1

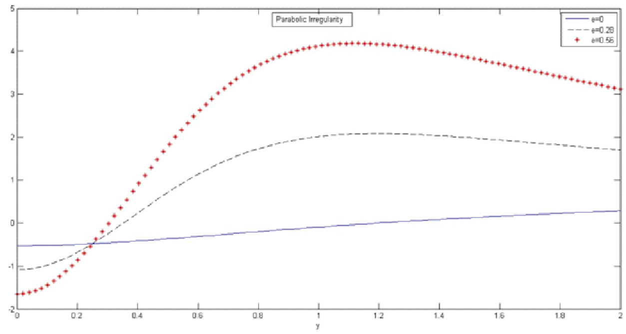

Fig.6: Variation of tangential displacement (u) against the horizontal distance (y) for parabolic irregularities (e = 0, 0.28, 0.56) on the plane x = 1.0.

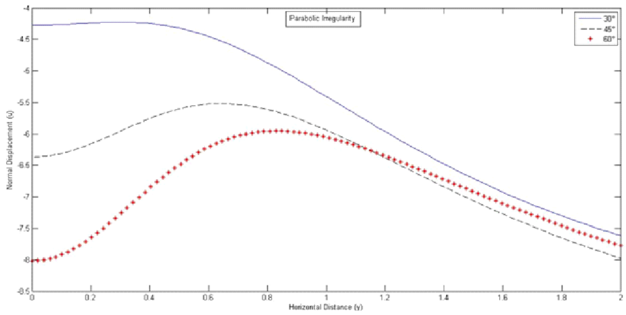

Fig.10: Variation of normal displacements with Parabolic irregularity (e = 0.25) due to inclined line-load (δ = 30°,45°, 60°) at x = 1.

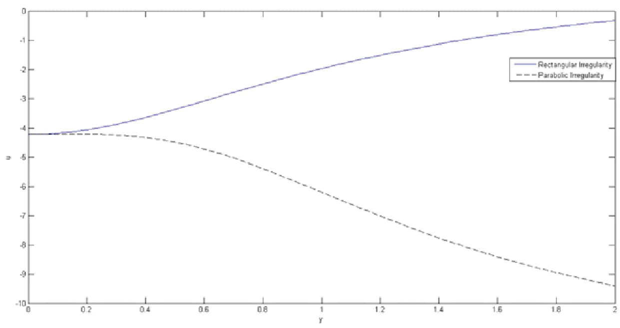

Fig.7: Comparison of normal displacement due to rectangular and parabolic irregularity (e = 0.25) at x = 1 and δ = 45^0.

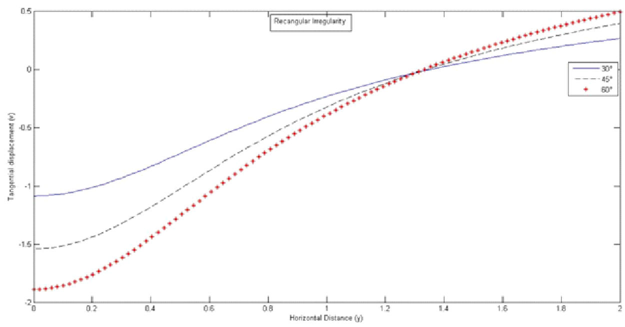

Fig.11: Variation of normal displacements with rectangular irregularity (e = 0.25) due to inclined line-load (δ = 30°,45°,60°) at x = 1

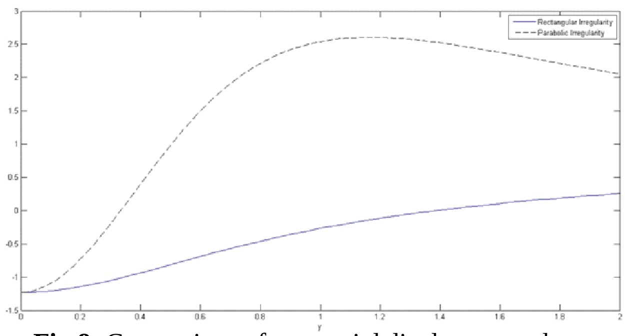

Fig.8: Comparison of tangential displacement due to rectangular and parabolic irregularity (e = 0.25) at x = 1 and δ = 45°

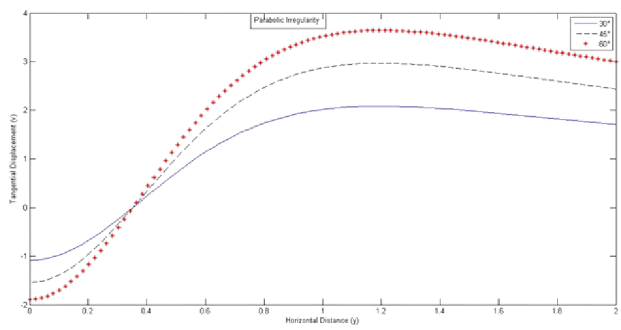

Fig.12: Variation of normal displacements with Parabolic irregularity (e = 0.25) due to inclined line-load (δ = 30°,45°,60°) at x = 1

6. CONCLUSION

For understanding the effect of irregularities present in the medium, we have obtained the deformation at any point of irregular elastic medium possessing cubic symmetry caused by normal, tangential and inclined line-loading. The irregularities has been considered in the form of rectangular and parabolic. For governing equations, the method of eigen value approach with equal eigen values has been used to obtain the analytical solution in an explicit closed form. Numerically results have been obtained by using the values given by [15] and graphs are plotted in Matlab to gain more perspective view of the problem. There is a prominent effect of inclination on line-loading in the displacements in an irregular elastic medium.it is pointed out that with minor substitution the present results can be used to analyse the effect of irregularity and line-loading in isotropic elastic medium.

Appendix A. (x>0)

$$\int_{-\infty}^{\infty} \exp(-|k|x)\exp(-iky)\,dk = \frac{2x}{y^2+x^2}$$

$$\int_{-\infty}^{\infty} \frac{k}{|k|}\exp(-|k|x)\exp(-iky)\,dk = \frac{-2iy}{y^2+x^2}$$

$$\int_{-\infty}^{\infty} (|k|)^{-1}\exp(-|k|x)\exp(-iky)\,dk = -\log(y^2+x^2)$$

$$\int_{-\infty}^{\infty} |k|\exp(-|k|x)\exp(-iky)\,dk = \frac{2(x^2-y^2)}{(y^2+x^2)^2}$$

$$\int_{-\infty}^{\infty} k\exp(-|k|x)\exp(-iky)\,dk = \frac{-4iyx}{(y^2+x^2)^2}$$

$$\int_{-\infty}^{\infty} \frac{1}{k}\exp(-|k|x)\exp(-iky)\,dk = -2itan^{-1}\left(\frac{y}{x}\right)$$

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