Propagation of Rayleigh Wave in Sandy Media with Imperfect Interface

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ABSTRACT

In the present study, the propagation of Rayleigh wave in a sandy layer overlying a sandy semi-infinite media is investigated, with the interface considered imperfect. Expressions for displacement components are obtained. The dispersion frequency equation is derived using suitable boundary conditions. In particular cases, when interface is perfect and elastic media replace sandy media are also discussed. The effects of imperfectness and sandy parameter on the Rayleigh waves’ phase velocity are investigated using MATLAB software. The theoretical results obtained may find useful applications in geophysics, civil engineering and soil mechanics.

Keywords: Rayleigh wave, Sandy layer, Imperfect interface, Dispersion equation, Phase velocity.

INTRODUCTION

Theoretical studies regarding seismic wave propagation are crucial as they provide a vast amount of information about Earth’s interior. Studies involving seismic wave propagation are important to seismologists and earthquake engineers since it contributes to determining the nature and cause of earthquakes and understanding the Earth’s crust.

Materials of Earth may not always be isotropic and elastic. However, the Earth’s crust consisting of sedimentary layers is not perfectly elastic but can be considered as sandy particles. A sandy layer consists of particles not retaining moisture or water vapours. Sand boils occurred due to the 7.1 Richter scale 1989 Loma Prieta earthquake causing the liquefaction of superficial sandy materials. So, role of a sandy layer in predicting seismic behavior is very important to seismologists. Various researchers studied seismic or Rayleigh wave propagation in sandy or elastic media. Rayleigh[1] and Bromwich[2] investigated Rayleigh waves propagation considering isotropic elastic solid and stratified media. The dynamics for dry sandy soil were explored by Weiskopf.[3]

Kar et al.[4] studied Love wave propagation in sandy medium discussing the effects of irregularity. Abd-Alla[5] investigated Rayleigh wave propagation considering orthotropic material elastic half-space. Effects of the gravity field and initial stresses on Rayleigh wave propagation considering magnetoelastic half-space were discussed by Abd-Alla et al.[6]. Viswakarma and Gupta[7] investigated Rayleigh wave propagation in Earth’s crustal layer for the sandy and elastic half-space cases obtaining the effects of inhomogeneity and rigid boundary. Pal et al.[8] explored Rayleigh wave propagation considering a sandy half-space and an anisotropic layer and derived the Rayleigh wave propagation dispersion frequency equation. Sahu et al.[9] examined Rayleigh wave propagation considering an orthotropic half-space with impacts of pre-stresses and self-weight and a liquid layer. Mandi et al.[10] investigated the propagation of Rayleigh waves in a geometry containing sandy media surrounded by couple stress media and orthotropic half-space.

As Earth is a layered media, various interfacial conditions such as irregularities or imperfect interface have a significant effect on seismic wave propagation. Such studies provide rich information regarding Earth’s seismic behavior. Various researchers studied the effects of these interfacial conditions. Hua et al.[11] studied effects of the imperfectness parameter on the propagation of Love wave considering a geometry consisting of layered graded composite structures. Vishwakarma and Xu[12] investigated dispersion of Rayleigh wave investigated considering sandy layer overlying an orthotropic mantle. The effects of irregular boundaries on upper plane has been discussed observing the initial stress and sandiness parameter effect. The dispersion equation for SH wave propagation in a layer of viscoelastic overlying couple stressed substrate with interface assumed to be imperfect was derived by Sharma and Kumar[13] and observed

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the effects of imperfection, heterogeneity, friction and imperfectness parameter. Kumar et al.\textsuperscript{[14]} examined shear wave propagation considering micropolar elastic half-space and piezoelectric layer under the effects of initial stresses with imperfect interface. Kumar and Madan\textsuperscript{[15]} discussed effects of imperfectness and sandy parameter on Love wave propagation considering a layer consisting sand particles overlying an orthotropic semi-infinite media. Madan et al.\textsuperscript{[16]} investigated propagation of Rayleigh wave considering orthotropic elastic medium under effects of pre-stresses. An explicit secular equation for perfect and sliding information has been derived.

The effects of imperfect interfacial conditions for Rayleigh wave propagation in sandy media remain unexplored. So, an effort has been made to study Rayleigh wave propagation in a dry sandy layer overlying a dry sandy semi-infinite medium. The interface is assumed to be dislocation-like imperfect and keeping upper boundary of the layer rigid. The geometry for the considered problem is shown in Figure 1.

For 2-dimensional problem (-xz plane) discussion, we must have displacements components are independent of y i.e. \( \frac{\partial}{\partial y} = 0 \) and are zero in -y-direction.

Dynamics of Sandy Layer and Semi-infinite Medium

Dynamical equation of motion without external forces for Rayleigh surface wave propagation with displacement components \( u_t \) and \( w_t \) along x and z direction for a sandy layer is given as (Biot)\textsuperscript{[17]}

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} = \rho_1 \frac{\partial^2 u}{\partial t^2} \quad (1)
\]

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} = \rho_1 \frac{\partial^2 w}{\partial t^2} \quad (2)
\]

where \( \tau_{xx}, \tau_{xz}, \tau_{zz} \) denotes stress components and \( \rho_1 \) denotes density of material in sandy layer.

Using stress-displacement relations for the sandy layer,

\[
\tau_{xx} = \eta_1 \left( \lambda_1 + 2\mu_1 \frac{\partial^2 u}{\partial x^2} + \lambda_1 \frac{\partial^2 w}{\partial z^2} \right) \quad (3)
\]

\[
\tau_{xz} = \eta_1 \mu_1 \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} \right) \left( \lambda_1 + 2\mu_1 \frac{\partial^2 u}{\partial x^2} + \lambda_1 \frac{\partial^2 w}{\partial z^2} \right) \quad (4)
\]

\[
\tau_{zz} = \eta_1 \left( \lambda_1 \frac{\partial^2 w}{\partial x^2} + \left( \lambda_1 + 2\mu_1 \right) \frac{\partial^2 u}{\partial z^2} \right) \quad (5)
\]

where \( \lambda_1 \) and \( \mu_1 \) denote Lamé’s constant, \( \eta_1 \) denotes sandiness parameter.

Using equations (3), (4) and (5) in (1) and (3), we have

\[
\eta_1 \left( \lambda_1 + 2\mu_1 \right) \frac{\partial^2 u}{\partial x^2} + \eta_1 \mu_1 \frac{\partial^2 w}{\partial x^2} + \eta_1 \lambda_1 \frac{\partial^2 w}{\partial z^2} = \rho_1 \frac{\partial^2 u}{\partial t^2} \quad (6)
\]

\[
\eta_1 \frac{\partial^2 w}{\partial x^2} + \eta_1 \left( \lambda_1 + 2\mu_1 \right) \frac{\partial^2 w}{\partial z^2} + \eta_1 \lambda_1 \frac{\partial^2 u}{\partial z^2} = \rho_1 \frac{\partial^2 w}{\partial t^2} \quad (7)
\]

Now, assume solution of equations (6) and (7) be

\[
\begin{align*}
\mathbf{u}_1 &= (Ae^{-kp_1z} + Be^{kp_1z}) e^{ikx-(ct)} \quad (8) \\
\mathbf{w}_1 &= (Ce^{-kp_2z} + De^{kp_2z}) e^{ikx-(ct)} \quad (9)
\end{align*}
\]

Using values of \( u_1 \) and \( w_1 \) in equations (6) and (7) and separating coefficients of \( e^{kp_1z} \) and \( e^{kp_2z} \), we have

\[
\begin{align*}
[p_1 c^2 - \eta_1 \left( \lambda_1 + 2\mu_1 \right) + \eta_1 \mu_1 p_1^2] A_1 - i \eta_1 \lambda_1 A_1 + \eta_1 \mu_1 p_1 D_1 &= 0 \quad (10) \\
[p_1 c^2 - \eta_1 \left( \lambda_1 + 2\mu_1 \right) + \eta_1 \lambda_1 p_1^2] B_1 + i \eta_1 \mu_1 B_1 + \eta_1 \lambda_1 p_1 D_1 &= 0 \quad (11) \\
[p_1 c^2 - \eta_1 \mu_1 + \eta_1 \lambda_1 + p_1 \eta_1 \mu_1] C_1 - i \eta_1 \mu_1 C_1 + \eta_1 \lambda_1 A_1 &= 0 \quad (12) \\
[p_1 c^2 - \eta_1 \mu_1 + \eta_1 \lambda_1 + p_1 \eta_1 \lambda_1] D_1 + i \eta_1 \mu_1 D_1 + \eta_1 \lambda_1 A_1 &= 0 \quad (13)
\end{align*}
\]

Writing equations (10)–(13) in determinant form in order to eliminate \( A_1, B_1, C_1 \) and \( D_1 \), we must have:

\[
\begin{vmatrix}
\lambda_1 - p_1^2 & \eta_1 - \eta_1 \mu_1 & \eta_1 - \eta_1 \lambda_1 & 0 \\
\eta_1 - \eta_1 \mu_1 & \lambda_1 - p_1^2 & \eta_1 - \eta_1 \lambda_1 & 0 \\
\eta_1 - \eta_1 \lambda_1 & \eta_1 - \eta_1 \mu_1 & \lambda_1 - p_1^2 & 0 \\
0 & 0 & 0 & \lambda_1 - p_1^2
\end{vmatrix} = 0
\]

On expanding the determinant, we obtain a biquadratic equation in \( p_1 \), given as

\[
\eta_1 \left( \frac{\lambda_1 - p_1^2}{\lambda_1 - p_1^2} \right)^2 + \left( \frac{\eta_1 - \eta_1 \mu_1}{\eta_1 - \eta_1 \mu_1} \right)^2 + \left( \frac{\eta_1 - \eta_1 \lambda_1}{\eta_1 - \eta_1 \lambda_1} \right)^2 + \left( \frac{\eta_1 - \eta_1 \mu_1}{\eta_1 - \eta_1 \mu_1} \right)^2 = 0
\]

where, \( \lambda_1 = \frac{1 + \mu_1}{\lambda_1 - p_1^2} \) and \( \eta_1 = \frac{\eta_1 - \eta_1 \mu_1}{\eta_1 - \eta_1 \mu_1} \).

Assume \( \pm p_1 \) and \( \pm p_2 \) be solution of equation (14), then displacement expression in equations (8) and (9) can be written as:

\[
\begin{align*}
\mathbf{u}_1 &= (A_1 e^{-kp_1z} + A_2 e^{kp_1z} + B_1 e^{kp_2z} + B_2 e^{-kp_2z}) e^{ikx-(ct)} \quad (15) \\
\mathbf{w}_1 &= (C_1 e^{-kp_1z} + C_2 e^{kp_1z} - n_1 B_1 e^{kp_2z} - n_2 B_2 e^{-kp_2z}) e^{ikx-(ct)} \quad (16) \\
\mathbf{u}_y &= c_2 (n_1 - n_2) \left( \frac{\partial}{\partial z} \right) \quad (j = 1, 2) \text{, with } p_1 = \sqrt{1 - \frac{c_2}{c_1^2}} \text{ and } p_2 = \sqrt{1 - \frac{c_2}{c_1^2}} \quad (17)
\end{align*}
\]

In a similar manner displacement expression for sandy semi-infinite medium can be written as:

\[
\begin{align*}
\mathbf{u}_2 &= (E_1 e^{-kp_1z} + E_2 e^{kp_1z} + F_1 e^{kp_2z} + F_2 e^{-kp_2z}) e^{ikx-(ct)} \quad (18) \\
\mathbf{w}_2 &= (n_1 E_1 e^{-kp_1z} + n_1 E_2 e^{kp_1z} - n_2 F_1 e^{kp_2z} - n_2 F_2 e^{-kp_2z}) e^{ikx-(ct)} \quad (19)
\end{align*}
\]
Also, we have displacement vanishing as the depth increases, i.e., $u_2, w_2 \to 0$ as $z \to \infty$. So, $F_1$ and $F_2$ must be zero. Then from equations (18) and (19):

$$u_2 = (E_1 e^{-k_1 z} + E_2 e^{-k_2 z}) e^{ik(x-ct)} \quad (21)$$
$$w_2 = (n_1' E_1 e^{-k_1 z} + n_2' E_2 e^{-k_2 z}) e^{ik(x-ct)} \quad (22)$$

Equations (21) and (22) represents expressions for displacement components for the semi-infinite medium.

**Boundary Conditions and Dispersion Equation**

To examine Rayleigh wave propagation in a dry sandy layer overlying a sandy semi-infinite medium with dislocation like imperfection present at the interface, the boundary conditions involving vanishing of stresses, continuity of displacements and stresses may be presented as:

- Displacement vanishes at rigid upper boundary plane. i.e. $u_1 = 0$ and $w_1 = 0$ at $z = -h$.
- Displacement components for semi-infinite medium and layer are proportional at the imperfect interface, i.e., $u_2 = \eta_T u_1$ and $w_2 = \eta_N w_1$ at $z = 0$ where $\eta_T$ and $\eta_N$ denotes imperfectness parameters along tangential and normal direction.
- Stresses are continuous at the imperfect interface, i.e., $\tau_{2x} = \tau_{1x}$ and $\tau_{2z} = \tau_{1z}$ at $z = 0$.

Using these boundary conditions and expressions for displacements and stresses for sandy layer and half-space, we obtain the following set of six homogeneous equations in terms of $A_1 A_2 B_1 B_2 E_1$ and $E_2$

$$A_1 e^{\beta_1 h} + A_2 e^{\beta_2 h} + B_1 e^{\beta_1 h} + B_2 e^{\beta_2 h} = 0 \quad (23)$$
$$A_1 n_1' e^{\beta_1 h} + A_2 n_2' e^{\beta_2 h} - B_1 n_1' e^{\beta_1 h} - B_2 n_2' e^{\beta_2 h} = 0 \quad (24)$$
$$A_2 \tau_T + A_2 \tau_T - B_2 \tau_T - E_2 = 0 \quad (25)$$
$$A_2 \tau_N - A_2 \tau_N - B_2 \tau_N - B_2 \tau_N - n_2 E_2 = 0 \quad (26)$$
$$A_2 \beta_1 (\hat{a}_1 + \hat{a}_2) + A_2 \beta_1 (\hat{a}_1 + \hat{a}_2) + B_2 \beta_1 (\hat{a}_1 + \hat{a}_2) + B_2 \beta_1 (\hat{a}_1 + \hat{a}_2) = 0 \quad (27)$$
$$A_2 \beta_1 (\hat{a}_1 - \hat{a}_2) + A_2 \beta_1 (\hat{a}_1 - \hat{a}_2) + B_2 \beta_1 (\hat{a}_1 - \hat{a}_2) + B_2 \beta_1 (\hat{a}_1 - \hat{a}_2) = 0 \quad (28)$$

For non-trivial solution of this homogeneous system from (23) - (28), we must have,

$$a_{ij} = 0, \ i, j = 1 \ to \ 6. \quad (29)$$

where $a_{ij}$ are coefficients of $A_1 A_2 B_1 B_2 E_1$ and $E_2$ in the system of six equations represented by equations (23) - (28).

The real part of equation (29) represents the dispersion frequency equation for Rayleigh wave propagating in a dry sandy layer overlying a sandy semi-infinite medium with imperfect interface.

**Special Cases**

**Case I.** When $\eta_T = \eta_N = 1$, then equation (29) becomes frequency equation for propagation of Rayleigh wave in sandy layer overlying a sandy semi-infinite medium.

**Case II.** When $\eta_1 = \eta_2 = 1$, then equation (29) becomes frequency equation for propagation of Rayleigh wave in an isotropic elastic layer overlying an isotropic semi-infinite medium with in-terface assumed to be imperfect.

**Case III.** When $\eta_1 = \eta_2 = \eta_T = \eta_N = 1$, then equation (13) becomes frequency equation for Rayleigh wave propagation in an isotropic elastic layer overlying an isotropic semi-infinite medium with perfect interface.

**Numerical Computations and Discussion**

Propagation of Rayleigh wave considering a geometry comprised of a sandy layer and sandy semi-infinite medium...
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has been examined with the interface assumed to be dislocation like imperfect. The derived frequency equation, involving various parameters like imperfectness and sandiness, has been derived for the considered model. To examine the effects of various parameters, following data has been considered:

For the dry sandy layer (Gubbins [18])

\[ \rho_1 = 2300000 \text{ Kg/m}^3, \mu_1 = 6 \text{ Gpa}, \lambda_1 = 9 \text{ Gpa}. \]

For the dry sandy half-space (Pal et al. [8]):

\[ \rho_2 = 7800 \text{ Kg/m}^3, \mu_2 = 5.66 \text{ Gpa}, \lambda_2 = 2.46 \text{ Gpa}. \]

Graphs are plotted to show the effects of imperfectness, sandiness parameter for layer and half-space on phase velocity variation against wave number using real part of equation (29) with help of MATLAB software.

Figure 2 shows the effects of the imperfectness parameter \( (\eta_1, \eta_2) \). Variation of phase velocity against wave number to observe the effects of imperfectness parameter is shown for the two cases, i) for the sandy layer and sandy semi-infinite medium and ii) when layer and half-space becomes isotropic, i.e. \( \eta_1 = \eta_2 = 1 \). Three different values of the imperfectness parameter \( (0.35, 0.50 \text{ and } 0.65) \) are used for plotting with values of \( \eta_1 \) and \( \eta_2 \) taken as 1.5 for Figure 2(a). It can be depicted that phase velocity increased with the increment in the imperfectness factor, for both cases.

Figure 3 shows the effects of sandiness factor \( (\eta_1) \) for the layer. Variation is shown for the imperfect and perfect interface, respectively. Phase velocity is plotted against wavenumber using three values of \( \eta_1 = 1.20, 1.50 \text{ and } 1.80 \) with value of \( \eta_1 \) and \( \eta_2 = 0.5 \) for Figure 3(a). Sandiness parameter for the layer decreased the phase velocity of the Rayleigh wave for both the imperfect and perfect interface but with a slightly different behavior.

Figure 4 shows the effects of the sandiness parameter \( (\eta_2) \) for the half-space. Variation is shown for the imperfect and perfect interface, respectively. Plotting has been done using three different values of \( \eta_2 = 1.20, 1.50 \text{ and } 1.80 \) with value of \( \eta_1 \) and \( \eta_2 = 0.5 \) for Figure 4(a). It has been found that sandiness parameter for the semi-infinite medium enhanced the Rayleigh wave phase velocity.

**Conclusion**

A mathematical analysis of Rayleigh wave propagation in dry sandy layer overlying dry sandy half-space has been studied with the interface assumed to be dislocation like imperfect. The dispersion frequency equation has been obtained by applying appropriate boundary conditions for the geometry. Graphs have been plotted to show the impacts of imperfectness and sandiness factor \( (\eta_1 = \eta_2) \) on Rayleigh waves phase velocity. The conclusions for the present study may be summarized as:

- Imperfectness parameter enhances the phase velocity of Rayleigh wave for the dry sandy layer and isotropic elastic layer case.
- Sandiness parameter for the layer affected inversely the phase velocity. Sandiness parameter increases, phase velocity decreases both for imperfect and perfect interface case.
- Sandiness parameter for the semi-infinite medium also significantly effected the phase velocity by enhancing the phase velocity with enhancement in sandiness parameter for both imperfect and perfect interface.

These theoretical studies regarding seismic wave propagation considering layered media have various applications in geophysics, civil engineering and understanding the effects and causes of earthquakes. Rayleigh wave causes more damage during earthquakes. The present results could be used to connect modeling results to field applications.

**References**


